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THE SOLUTION OF ONE DIFFERENCE PROBLEM FOR THE EQUATION OF VIBRATION OF A STICK WITH THE SEPARATED VARIABLES METHOD

Abstract

In this work one difference problem for the equation of vibration of a stick is solved by the method of separation of variables and the stability of obtaining solution is proved.

The mixed problem is considered for the equation of vibration of a stick. The difference problem is constructed by the explained way [1], which is solved with the Fourier's separated variables method. The eigen-values and eigen-functions of corresponding difference problems are defined, their properties are studied, the solution of difference problem is constructed and the stability of this solution is proved.

§1. The difference problem on eigen-values and its solution:

$$y^{IV}(x)+\lambda y(x)=0, \quad 0<x<l, \tag{1}$$
$$y(0)=y''(0)=y(l)=y''(l)=0. \tag{2}$$

It is clear, that

$$\lambda_k=-\left(\frac{k\pi}{l}\right)^4, \qquad y_k(x)=\sin\frac{k\pi x}{l}, \quad k=1,2,3,... \tag{3}$$

are eigen-values and eigen-functions of this problem.

Let $N\geq 4$ be fixed natural number. Let's define the uniform grid on interval $[0,l]$

$$\overline{\omega}_h=\{x_i=ih, i=0,1,...,N, h=l/N, \ x_0=0, x_N=l\}.$$

According to [1], the difference problem

$$\begin{gathered}\frac{1}{h^4}(5y_1-4y_2+y_3)+\lambda y_1=0,\\ \frac{1}{h^4}(y_{i-2}-4y_{i-1}+6y_i-4y_{i+1}+y_{i+2})+\lambda y_i=0,\\ y_0=y_N=0, \ i=2,3,...,N-2,\\ \frac{1}{h^4}(y_{N-3}-4y_{N-2}+5y_{N-1})+\lambda y_{N-1}=0\end{gathered} \tag{4}$$

approximate the problem (1)-(2) to within $O(h^2)$.

The solution of this difference problem we'll find in the form

$$y_i=\sin\alpha x_i, \quad i=0,1,...,N, \tag{5}$$

where α is still unknown parameter. The value of this parameter we'll define later.

Substituting this expression y_i in difference equation (4) when $i=2,3,...,N-2$, after the simple transformations we'll get:

$$\left[\frac{2}{h^4}(\cos 2\alpha h-4\cos\alpha h+3)+\lambda\right]\cdot\sin\alpha x_i=0, \quad i=2,3,...,N-2.$$

From this equality, because of $\sin\alpha x_i\neq 0$, the truth of the equality follows

$$\frac{2}{h^4}(\cos 2\alpha h - 4\cos\alpha h + 3) + \lambda = 0 .$$

From here we've

$$\lambda = -\frac{16}{h^4}\sin^4\frac{\alpha h}{2} . \qquad (6)$$

Let's show that, in this value of λ, the grid function y_i, which is defined by the equality (5), satisfies the first and the last equation in (4)

$$\frac{1}{h^4}(5y_1 - 4y_2 + y_3) + \lambda y_1 = \frac{1}{h^4}(5\sin\alpha h - 4\sin 2\alpha h + \sin 3\alpha h) + \lambda\sin\alpha h =$$

$$= \frac{1}{h^4}(\sin\alpha h + \sin 3\alpha h - 4\sin 2\alpha h + 4\sin\alpha h) + \lambda\sin\alpha h =$$

$$= \frac{1}{h^4}(2\sin 2\alpha h\cdot\cos\alpha h - 4\sin 2\alpha h + 4\sin\alpha h) + \lambda\sin\alpha h =$$

$$= \frac{1}{h^4}\left(4\sin\alpha h\cdot\cos^2\alpha h - 8\sin\alpha h\cdot\cos\alpha h + 4\sin\alpha h\right) + \lambda\sin\alpha h =$$

$$= \left[\frac{4}{h^4}(1-\cos\alpha h)^2 + \lambda\right]\sin\alpha h = \left(\frac{16}{h^4}\sin^4\frac{\alpha h}{2} + \lambda\right)\sin\alpha h = 0 .$$

Let's pass to proof of fulfilling the last equality in (4). By this aim we'll use the equality:

$$\sin(n+1)\theta = 2\sin n\theta\cdot\cos\theta - \sin(n-1)\theta$$

and the condition $y_N = \sin\alpha x_N = \sin\alpha l = 0$, for which the grid function (5) must fulfill.

Let's suppose that $\theta = \alpha h$. Then

$$\frac{1}{h^4}(y_{N-3} - 4y_{N-2} + 5y_{N-1}) = \frac{1}{h^4}[\sin\alpha(N-3)h - 4\sin\alpha(N-2)h + 5\sin\alpha(N-1)h] =$$

$$= \frac{1}{h^4}[\sin(N-3)\theta - 4\sin(N-2)\theta + 5\sin(N-1)\theta] =$$

$$= \frac{1}{h^4}[2\sin(N-2)\theta\cdot\cos\theta - \sin(N-1)\theta - 4\sin(N-2)\theta + 5\sin(N-1)\theta] =$$

$$= \frac{1}{h^4}[(2\cos\theta - 4)\sin(N-2)\theta + 4\sin(N-1)\theta] =$$

$$= \frac{1}{h^4}\left[\left(4\cos^2\theta - 8\cos\theta + 4\right)\sin(N-1)\theta - (2\cos\theta - 4)\sin N\theta\right] =$$

$$= \frac{4}{h^4}(\cos\theta - 1)^2\sin(N-1)\theta + \frac{2}{h^4}(2-\cos\theta)\sin N\theta =$$

$$= \frac{4}{h^4}(1-\cos\alpha h)^2\sin\alpha(N-1)h + \frac{2}{h^4}(2-\cos\theta)\sin\alpha l =$$

$$= \frac{16}{h^4}\sin^4\frac{\alpha h}{2}\sin\alpha(N-1)h = -\lambda\sin\alpha(N-1)h.$$

That is why

$$\frac{1}{h^4}(y_{N-3} - 4y_{N-2} + 5y_{N-1}) + \lambda y_{N-1} = -\lambda y_{N-1} + \lambda y_{N-1} = 0 .$$

Let's define the parameter α from fulfilling the conditions $y_0 = y_N = 0$. The condition $y_0 = 0$ is fulfilled for any value of α. From condition $y_N = 0$ we'll get $\sin\alpha l = 0$, from here follows $\alpha = \alpha_k = \frac{k\pi}{l}$, $k \in Z$.

The problem (4) has no more than $N-1$ eigen-values and eigen-functions. Substituting the found values of α in equalities (5) and (6) we'll get all eigen-values and eigen-functions of the problem (4):

$$\lambda_k = -\frac{16}{h^4}\sin^4\frac{k\pi h}{2l}, \quad y_i^{(k)} = \sin\frac{k\pi x_i}{l}, \;\; k = 1,2,\dots,N-1. \tag{7}$$

Let's transfer their properties:

1. The eigen-values of λ_k satisfy the next inequalities:

$$\lambda_1 = -\frac{16}{h^4}\sin^4\frac{\pi h}{2l} > \lambda_2 > \dots > \lambda_{N-1} = -\frac{16}{h^4}\cos^4\frac{\pi h}{2l} > -\frac{16}{h^4}.$$

The eigen-value of λ_1 we can estimate from above.

For it let's copy its expression in the next form:

$$\lambda_1 = -\frac{\pi^4}{l^4}\cdot\left(\frac{\sin\frac{\pi h}{2l}}{\frac{\pi h}{2l}}\right)^4.$$

It's obvious that $0 < \frac{\pi h}{2l} \le \frac{\pi}{8}$ so as $h \le \frac{l}{4}$. That is why, if we consider the function $f(\alpha) = \frac{\sin\alpha}{\alpha}$ when $\alpha \in \left(0, \frac{\pi}{8}\right]$ then it is easy to check that this function in the considered interval achieves its minimum at the point $\alpha = \frac{\pi}{8}$. That is why

$$f(\alpha) = \frac{\sin\alpha}{\alpha} \ge \frac{\sin\frac{\pi}{8}}{\frac{\pi}{8}} = \frac{8\sqrt{\sqrt{2}-1}}{\pi\sqrt[4]{8}}, \;\; \alpha \in \left(0, \frac{\pi}{8}\right].$$

Taking into account this inequality for λ_1, we get:

$$\lambda_1 \le -\frac{512\left(3-2\sqrt{2}\right)}{l^4}.$$

So for eigen-values λ_k we have:

$$-\frac{16}{h^4} < \lambda_{N-1} < \lambda_{N-2} < \dots < \lambda_2 < \lambda_1 \le -\frac{512\left(3-2\sqrt{2}\right)}{l^4}.$$

Let $\overset{\circ}{\Omega}$ be a space of grid functions, given on the grid $\bar{\omega}_h$ and equal to zero when $i = 0$ and $i = N$. Let's define the scalar product and norm by equalities

$$(y, v) = \sum_{i=1}^{N-1} y_i v_i h, \;\; \|y\| = \sqrt{(y, y)}. \tag{8}$$

2. The eigen-functions $y_i^{(n)}$, $y_i^{(m)}$ answering different eigen-values, are orthogonal in the meaning of scalar product (8).

Let's prove it. Let operator A acting at space $\overset{\circ}{\Omega}$ be given by equality

$$(Ay)_i = \begin{cases} \frac{1}{h^4}(5y_1 - 4y_2 + y_3), \ i = 1, \\ \frac{1}{h^4}(y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}), \ i = 2,3,..,N-2, \\ \frac{1}{h^4}(y_{N-3} - 4y_{N-2} + 5y_{N-1}), \ i = N-1. \end{cases}$$

Then we can copy difference problem (4) as

$$Ay + \lambda y = 0. \tag{9}$$

Let $y_i^{(n)}$ and $y_i^{(m)}$ be eigen-functions of difference problem (4) or (9), corresponding to different eigen-values λ_n and λ_m:

$$Ay^{(n)} + \lambda_n y^{(n)} = 0, \quad Ay^{(m)} + \lambda_m y^{(m)} = 0. \tag{10}$$

The truth of equality

$$(y, Av) = (y_{\bar{x}x}, v_{xx}),$$

is proved in [1] for operator A and in particular self-adjoining of operator A follows from this equality.

Using this equality and taking (10) into account it is easy to get the truth of next equalities:

$$\lambda_n\left(y^{(n)}, y^{(m)}\right) = -\left(y^{(n)}_{\bar{x}x}, y^{(m)}_{\bar{x}x}\right), \ \ \lambda_m\left(y^{(n)}, y^{(m)}\right) = -\left(y^{(n)}_{\bar{x}x}, y^{(m)}_{\bar{x}x}\right).$$

From this two equalities we have:

$$(\lambda_n - \lambda_m)\cdot\left(y^{(n)}, y^{(m)}\right) = 0$$

or

$$\left(y^{(n)}, y^{(m)}\right) = 0 \text{ since } \lambda_n \neq \lambda_m.$$

3. Norm of eigen-function $y_i^{(k)}$ is defined by equality [2]

$$\left\|y^{(k)}\right\| = \sqrt{\frac{l}{2}}, \ k = 1,2,...,N-1.$$

That is why grid functions

$$\bar{y}_i^{(k)} = \sqrt{\frac{2}{l}}\sin\frac{k\pi x_i}{l}, \ \ k = 1,2,...,N-1$$

form an ortonormal system in the meaning of scalar product (8).

4. Let grid function $f(x)$, $x = x_i \in \bar{\omega}_h$, satisfy condition $f(0) = f(l) = 0$. Then [2] this function can be distributed on eigen-functions of problem (4):

$$f(x) = \sum_{k=1}^{N-1} f_k \bar{y}^{(k)}_{(x)}, \ \ x = x_i \in \bar{\omega}_h,$$

where coefficients f_k of the distribution are defined by equalities

$$f_k = \left(f(x), \bar{y}^{(k)}(x)\right), \quad k = 1,2,...,N-1.$$

In this case the equation

$$\|f\|^2 = \sum_{k=1}^{N-1} f_k^2.$$

is true.

§2. Applying the method of the separated variables to the solution of difference problem for the equation of vibration of a stick.

Let's consider the next problem for the equation of vibration of a stick:

$$\frac{\partial^2 u}{\partial t^2}+a^2\frac{\partial^4 u}{\partial x^4}=0,\quad 0<x<l,\ \ 0<t\le T, \tag{11}$$

$$u(0,t)=\frac{\partial^2 u(0,t)}{\partial x^2}=0,\qquad u(l,t)=\frac{\partial^2 u(l,t)}{\partial x^2}=0,\quad 0\le t\le T, \tag{12}$$

$$u(x,0)=\varphi_0(x),\ \ \frac{\partial u(x,0)}{\partial t}=\overline{\varphi}_1(x),\ \ 0\le x\le l. \tag{13}$$

In closed domain $\overline{D}=\{0\le x\le l,\,0\le t\le T\}$ let's define grid domain $\overline{\omega}_{h\tau}=\{(x_i,t_j),\,x_i=ih,t_j=j\tau,i=0,1,...,N,\,j=0,1,...,M,\,Nh=l,\,M\tau=T\}$. Let's denote by y_i^j the value in bundle (x_i,t_j) of the grid function y defined on grid $\overline{\omega}_{h\tau}$.

Let's compare problem (11)-(13) with the next difference problem [1] which approximates to within $O(h^2+\tau^2)$:

$$\begin{gathered}y_{\bar{t}t,i}^j+a^2A\left(\sigma y_i^{j+1}+(1-2\sigma)y_i^j+\sigma y_i^{j-1}\right)=0,\\ i=1,2,...,N-1,\,j=1,2,...,M-1,\end{gathered} \tag{14}$$

$$y_i^0=\varphi_0(x_i),\ \ y_{t,i}^0=\varphi_1(x_i),\ i=0,1,...,N, \tag{15}$$

where σ is an arbitrary real parameter, $\varphi_1(x)=\overline{\varphi}_1(x)-\frac{a^2\tau}{2}\varphi_0^{IV}(x)$ and operator A for each fixed value j moves at space $\overset{\circ}{\Omega}$ and defined by formula of the previous paragraph.

We'll seek the solution of difference problem (14)-(15) in form $y_i^j=X(x_i)\cdot T(t_j)$ or $y_i^j=X_i\cdot T^j$. Substituting this expression into equation (14) we'll get:

$$\frac{AX_i}{X_i}=-\frac{T_{\bar{t}t}^j}{a^2\left(\sigma T^{j+1}+(1-2\sigma)T^j+\sigma T^{j-1}\right)}=-\lambda\,.$$

From here and conditions $y_0^j=y_N^j=0$ we get for X_i the problem on eigen-values

$$\begin{gathered}AX_i+\lambda X_i=0,\ \ i=1,2,...,N-1,\\ X_0=X_N=0,\end{gathered} \tag{16}$$

and the next equation for T^j:

$$T_{\bar{t}t,i}^j=a^2\lambda\left(\sigma T^{j+1}+(1-2\sigma)T^j+\sigma T^{j-1}\right). \tag{17}$$

The eigen-values and eigen-functions of problem (16) are defined by next equalities:

$$\lambda_k=-\frac{16}{h^4}\sin^4\frac{k\pi h}{2l},\quad X_i^{(k)}=\sqrt{\frac{2}{l}}\sin\frac{k\pi x_i}{l},\ \ k=1,2,...,N-1. \tag{18}$$

Substituting this values $\lambda=\lambda_k$ into (17) and denoting the solution of this difference equation by T_k^j, after simple transformations we get the equation of this kind:

$$T_k^{j-1} - 2\left(1 + \frac{0{,}5a^2\tau^2\lambda_k}{1 - a^2\tau^2\lambda_k\sigma}\right)T_k^j + T_k^{j-1} = 0 . \qquad (19)$$

The solution of this equation we seek as $T_k^j = q_k^j$ where q_k is still an unknown number. Substituting this expression T_k^j in (19) we get the next square equation relative to q_k:

$$q_k^2 - 2\left(1 + \frac{0{,}5a^2\tau^2\lambda_k}{1 - a^2\tau^2\lambda_k\sigma}\right)q_k + 1 = 0 .$$

The roots of this equation are defined by equalities

$$q_k^{(1,2)} = 1 + \frac{0{,}5a^2\tau^2\lambda_k}{1 - a^2\tau^2\lambda_k\sigma} \pm \frac{a\tau\sqrt{\lambda_k\left(1 + a^2\tau^2\lambda_k(0{,}25 - \sigma)\right)}}{1 - a^2\tau^2\lambda_k\sigma} .$$

Let condition

$$1 + a^2\tau^2\lambda_k(0{,}25 - \sigma) > 0$$

or the same

$$\sigma > \frac{1}{4} + \frac{1}{a^2\tau^2\lambda_k} \qquad (20)$$

be fulfilled.

As $-\frac{16}{h^4} < \lambda_k$ then $-\frac{h^4}{16a^2\tau^2} > \frac{1}{a^2\tau^2\lambda_k}$ and consequently the last inequality will be fulfilled if

$$\sigma \ge \frac{1}{4} - \frac{h^4}{16a^2\tau^2} . \qquad (21)$$

In this case roots $q_k^{(1,2)}$ are complex and it is easy to check, that

$$\left|q_k^{(1,2)}\right| = 1 .$$

That is why these roots we can represent as

$$q_k^{(1)} = \cos\alpha_k + i\sin\alpha_k = e^{i\alpha_k}, \quad q_k^{(2)} = \cos\alpha_k - i\sin\alpha_k = e^{-i\alpha_k} ,$$

where

$$\cos\alpha_k = 1 + \frac{0{,}5a^2\tau^2\lambda_k}{1 - a^2\tau^2\lambda_k\sigma}, \quad \sin\alpha_k = \frac{a\tau\sqrt{\lambda_k\left(a^2\tau^2\lambda_k(\sigma - 0{,}25) - 1\right)}}{1 - a^2\tau^2\lambda_k\sigma} .$$

The common solution of equation (19) in this case is

$$T_k^j = C_k\left(e^{i\alpha_k}\right)^j + D_k\left(e^{-i\alpha_k}\right)^j = A_k\cos j\alpha_k + B_k\sin j\alpha_k$$

where A_k and B_k are arbitrary constants.

We seek the solution of difference equation (14)-(15) as

$$y_i^j = \sum_{k=1}^{N-1} x_i^{(k)}T_k^j = \sum_{k=1}^{N-1}\left(A_k\cos j\alpha_k + B_k\sin j\alpha_k\right)X_i^{(k)} . \qquad (22)$$

Requiring fulfilling of initial conditions (15), we'll get for A_k and B_k

$$A_k = \varphi_{0k},\; B_k = \frac{1-\cos\alpha_k}{\sin\alpha_k}\varphi_{0k} + \frac{\tau}{\sin\alpha_k}\varphi_{1k}\ ,$$

where φ_{0k} and φ_{1k} are coefficients of distribution of functions $\varphi_0(x_i)$ and $\varphi_1(x_i)$ on eigen-functions $X_i^{(k)}$, $k=1,2,...,N-1$.

Substituting founded expressions for A_k and B_k in equality (22) we'll get [2]:

$$y_i^j = \sum_{k=1}^{N-1}\left(\frac{\cos(j-0{,}5)\alpha_k}{\cos 0{,}5\alpha_k}\varphi_{0k} + \frac{\tau\sin j\alpha_k}{\sin\alpha_k}\varphi_{1k}\right)X_i^{(k)}. \tag{23}$$

This is the solution of difference problem (14)-(15).

§3. Investigation of stability.

Let $\varepsilon>0$ be an arbitrary number. Suppose that parameter σ satisfies condition

$$\sigma \ge \frac{1}{a^2\tau^2\lambda_k} + \frac{1+\varepsilon}{4}. \tag{24}$$

In this case condition (20)is fulfilled automatically. On the other hand the last condition will be fulfilled if parameter σ satisfies condition

$$\sigma \ge \frac{1+\varepsilon}{4} - \frac{h^4}{16a^2\tau^2}. \tag{25}$$

So let condition (25) be fulfilled. Then we have from (24) that

$$\frac{a^2\tau^2\lambda_k}{4(1-a^2\tau^2\lambda_k\sigma)} \ge -\frac{1}{1+\varepsilon}.$$

Using this inequality for $|\cos 0{,}5\alpha_k|$ we get:

$$|\cos 0{,}5\alpha_k| = \sqrt{\frac{1+\cos\alpha_k}{2}} = \sqrt{1+\frac{a^2\tau^2\lambda_k}{4(1-a^2\tau^2\lambda_k\sigma)}} \ge \sqrt{1-\frac{1}{1+\varepsilon}} = \sqrt{\frac{\varepsilon}{1+\varepsilon}}. \tag{26}$$

For $\sin\alpha_k$ we have:

$$\sin\alpha_k = 2\sin 0{,}5\alpha_k\cdot\cos 0{,}5\alpha_k \ge 2\sin 0{,}5\alpha_k\sqrt{\frac{\varepsilon}{1+\varepsilon}}.$$

On the other hand

$$2\sin 0{,}5\alpha_k = \sqrt{2(1-\cos\alpha_k)} = \sqrt{-\frac{a^2\tau^2\lambda_k}{1-a^2\tau^2\lambda_k\sigma}} = \sqrt{-\lambda_k}\,\frac{a\tau}{\sqrt{1-a^2\tau^2\lambda_k\sigma}}.$$

Let's consider two events.

1) Let $\sigma\le 0$. Then $1-a^2\tau^2\lambda_k\sigma\le 1$ and

$$2\sin 0{,}5\alpha_k \ge \sqrt{-\lambda_k}\cdot a\tau \ge \frac{32a\tau}{l^2\sqrt{6+4\sqrt{2}}}.$$

That is why

$$\sin\alpha_k \ge \frac{32a\tau}{l^2\sqrt{6+4\sqrt{2}}}\sqrt{\frac{\varepsilon}{1+\varepsilon}}.$$

2) Let $0<\sigma<\frac{1+\varepsilon}{4}$. In this case from (25) we'll get:

$$\frac{16a^2\tau^2}{h^4} \le \frac{4}{1+\varepsilon-4\sigma}.$$

Taking this inequality into account we have:

$$1 - a^2\tau^2\lambda_k\sigma \le \frac{1+\varepsilon}{1+\varepsilon-4\sigma} \quad \text{and} \quad \frac{1}{\sqrt{1-a^2\tau^2\lambda_k\sigma}} \ge \sqrt{\frac{1+\varepsilon-4\sigma}{1+\varepsilon}}.$$

That is why

$$2\sin 0{,}5\alpha_k \ge \sqrt{-\lambda_k}\,a\tau\sqrt{\frac{1+\varepsilon-4\sigma}{1+\varepsilon}} \ge \frac{32a\tau}{l^2\sqrt{6+4\sqrt{2}}}\sqrt{\frac{1+\varepsilon-4\sigma}{1+\varepsilon}}$$

and

$$\sin\alpha_k \ge \frac{32a\tau}{l^2\sqrt{6+4\sqrt{2}}}\sqrt{\frac{1+\varepsilon-4\sigma}{1+\varepsilon}}\sqrt{\frac{\varepsilon}{1+\varepsilon}}.$$

So we have

$$\frac{\tau}{\sin\alpha_k} \le \frac{l^2\sqrt{6+4\sqrt{2}}}{32a}\sqrt{\frac{1+\varepsilon}{\varepsilon}} \quad \text{when} \quad \sigma \le 0, \tag{27}$$

$$\frac{\tau}{\sin\alpha_k} \le \frac{l^2\sqrt{6+4\sqrt{2}}}{32a}\sqrt{\frac{1+\varepsilon}{1+\varepsilon-4\sigma}}\sqrt{\frac{1+\varepsilon}{\varepsilon}} \quad \text{when} \quad 0 < \sigma < \frac{1+\varepsilon}{4}. \tag{28}$$

Using (26)-(28) we'll get the estimation for $\|y^j\|$

$$\|y^j\| \le \left\|\sum_{k=1}^{N-1}\frac{\cos(j-0{,}5)\alpha_k}{\cos 0{,}5\alpha_k}\varphi_{0k}X^{(k)}\right\| + \left\|\sum_{k=1}^{N-1}\frac{\tau\sin j\alpha_k}{\sin\alpha_k}\varphi_{1k}X^{(k)}\right\| =$$

$$= \left(\sum_{k=1}^{N-1}\frac{\cos^2(j-0{,}5)\alpha_k}{\cos^2 0{,}5\alpha_k}\varphi_{0k}^2\right)^{\frac{1}{2}} + \left(\sum_{k=1}^{N-1}\frac{\tau^2\sin^2 j\alpha_k}{\sin^2\alpha_k}\varphi_{1k}^2\right)^{\frac{1}{2}} \le$$

$$\le \left(\sum_{k=1}^{N-1}\frac{1}{\cos^2 0{,}5\alpha_k}\varphi_{0k}^2\right)^{\frac{1}{2}} + \left(\sum_{k=1}^{N-1}\frac{\tau^2}{\sin^2\alpha_k}\varphi_{1k}^2\right)^{\frac{1}{2}} \le$$

$$\le \sqrt{\frac{1+\varepsilon}{\varepsilon}}\left(\|\varphi_0\| + K\|\varphi_1\|\right),$$

where

$$K = \begin{cases} \dfrac{l^2\sqrt{6+4\sqrt{2}}}{32a} & \text{for } \sigma \le 0, \\ \dfrac{l^2\sqrt{6+4\sqrt{2}}}{32a}\sqrt{\dfrac{1+\varepsilon}{1+\varepsilon-4\sigma}} & \text{for } 0 < \sigma \le \dfrac{1+\varepsilon}{4}. \end{cases} \tag{29}$$

So the next theorem is true:

Theorem. *Let* $\varepsilon > 0$ *be an arbitrary number and parameter* σ *satisfy condition (25). Then difference problem (14)-(15) is stable with initial data and in this case the next inequality has place:*

$$\|y^j\| \le \sqrt{\frac{1+\varepsilon}{\varepsilon}}\left(\|\varphi_0\| + K\cdot\|\varphi_1\|\right)$$

where constant K *is defined by equality (29).*

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