KHANMAMEDOV Ag.Kh.

TRIANGULAR REPRESENTATIONS OF THE SOLUTION OF A DIFFERENCE EQUATIONS SYSTEM

Abstract

The transformations operator is constructed for difference equations with a condition on infinity. The real class of coefficients for the existence of the transformation operator is noted.

1. Introduction.

The transformations operator arise in different questions of operators theory [1-2]. Special interest to these operators appeared since they were applied for investigation of inverse problems of spectral analyses [2].

At the given paper the transformation operator for a system of difference equations is constructed

$$\begin{cases} a_{1,n}y_{2,n+1} + a_{2,n}y_{2,n} = \lambda y_{1,n}, \\ a_{1,n-1}y_{1,n-1} + a_{2,n}y_{1,n} = \lambda y_{2,n} \end{cases} \quad n = 1,2,...., \tag{1}$$

with the coefficients of the class

$$\sum_{n=1}^{\infty} n\left(\left|a_{1,n} - A_1\right| + \left|a_{2,n} - A_2\right|\right) < \infty, \tag{2}$$

where $A_i a_{i,n} > 0$, $i = 1,2$. At $A_1 = 1$, $A_2 = -1$ in more restricted class of coefficients this problem is considered in the paper [3].

2. The transformation operator.

Let Γ be a complex λ plane with the sections on the segments $A_1A_2(A_1 - A_2)^2 \le A_1A_2\lambda^2 \le A_1A_2(A_1 + A_2)^2$. Let's consider in plane Γ the function

$$z = \frac{\lambda^2 - A_1^2 - A_2^2}{2A_1A_2} + \sqrt{\left(\frac{\lambda^2 - A_1^2 - A_2^2}{2A_1A_2}\right)^2 - 1},$$

choosing the branches of the root so that $z(\infty) = 0$.

Let's introduce the notations

$$\sigma(n) = \sum_{k=n}^{\infty}\left\{\left|\frac{A_1^2 - \alpha_{1,k-1}^2}{A_1A_2}\right| + \left|\frac{A_2^2 - \alpha_{2,k}^2}{A_1A_2}\right|\right\}, \quad \sigma_1(n) = \frac{A_1^2 + A_2^2}{|A_1A_2|}\sum_{k=n}^{\infty}\sigma(k)$$

Theorem. *At condition (2) the system of equations (1) has the solution* $\{f_{1,n}(\lambda)\}_{n=0}^{\infty}$, $\{f_{2,n}(\lambda)\}_{n=1}^{\infty}$, *represented in form*

$$\begin{aligned} f_{1,n}(\lambda) &= \alpha_n^{11}\left(1 + \sum_{m=1}^{\infty} A_{nm}^{11} z^m\right)\frac{A_1 z + A_2}{\lambda} z^n, \\ f_{2,n}(\lambda) &= \alpha_n^{22}\left(1 + \sum_{m=1}^{\infty} A_{nm}^{22} z^m\right) z^n. \end{aligned} \tag{3}$$

The kernel A_{nm} *satisfies the conditions*

$$\left|A_{nm}^{ii}\right| \le \frac{A_1^2 + A_2^2}{|A_1 A_2|}\sigma\left(n+1+\left[\frac{m}{2}\right]\right)\left\{1+\sigma\left(n+\left[\frac{m}{2}\right]\right)\right\}\exp\{\sigma_1(n)\},\ \ i=1,2\,, \tag{4}$$

where $[\,\cdot\,]$ *means the whole part.*

Proof. Let's note, that the expression (3) wittingly satisfies the condition (1) if the next relations are fulfilled

$$\left(\alpha_{n-1}^{11}\right)^{-1} = \prod_{k=n}^{\infty}\frac{a_{1,k-1}a_{2,k}}{A_1 A_2},\ \ \left(\alpha_n^{22}\right)^{-1} = \prod_{k=n}^{\infty}\frac{a_{1,k}a_{2,k}}{A_1 A_2},\quad n=1,2,\dots,$$

$$A_{n1}^{ii} = \sum_{k=n+1}^{\infty}\left\{b_{1,k+1-i} + b_{2,k}\right\} \stackrel{def}{=} \varepsilon_i(n), \qquad i=1,2\,,$$

$$A_{n2}^{ii} = \sum_{k=n+1}^{\infty}\left\{b_{1,k+1-i}\sum_{s=k+3-i}^{\infty}\left\{b_{1,s-1}+b_{2,s}\right\}\right\} + b_{2,k}\sum_{s=k+1}^{\infty}\left\{b_{1,s}+b_{2,s}\right\}\Bigg\} + \frac{A_2}{A_1}\sum_{k=n+1}^{\infty}b_{1,k} + \frac{A_1}{A_2}\sum_{k=n+3-i}^{\infty}b_{2,k} \stackrel{def}{=} \delta_i(n)$$

$$A_{n,2k+1}^{ii} = \sum_{r=1}^{k}\sum_{s=n+1+k-r}^{\infty}\left\{b_{i,s+1-i}A_{s+2-i,2r}^{22} + b_{2,s}A_{s,2r}^{11}\right\} + \frac{A_1}{A_2}\sum_{r=0}^{k-1}\sum_{s=n+1+k-r}^{\infty}b_{2,s}A_{s,2r+1}^{11} +$$

$$+\frac{A_2}{A_1}\sum_{r=0}^{k-1}\sum_{s=n+1+k-r}^{\infty}b_{1,s-1}A_{s,2r+1}^{22} + \varepsilon_i(n+k) \stackrel{def}{=} \varphi_i\left(A^{11},A^{22}\right) + \varepsilon_i(n+k)\,,$$

$$A_{n,2k}^{ii} = \sum_{r=1}^{k-1}\sum_{s=n+k-r}^{\infty}\left\{b_{1,s+1-i}A_{s+2-i,2r+1}^{22} + b_{2,s}A_{s,2r+1}^{11}\right\} + \frac{A_1}{A_2}\sum_{r=1}^{k-1}\sum_{s=n+2-i+k-r}^{\infty}b_{2,s}A_{s,2r}^{11} +$$

$$+\frac{A_2}{A_1}\sum_{r=1}^{k-1}\sum_{s=n+1+k-r}^{\infty}b_{1,s-1}A_{s,2r}^{22} + \delta_i(n-1+k) \stackrel{def}{=} \psi_i\left(A^{11},A^{22}\right) + \delta_i(n-1+k),$$

where

$$b_{i,s} = \frac{A_i^2 - a_{i,s}^2}{A_1 A_2},\quad i=1,2.$$

We look for $A_{nm}^{ii}\,(m \ge 3)$ in the form

$$A_{nm}^{ii} = \sum_{l=0}^{\infty}A_{nm}^{ii}(l)\,, \tag{5}$$

where

$$A_{n,2k+1}^{ii}(0) = \varepsilon_i(n+k),\ \ A_{n,2k+1}^{ii}(l+1) = \varphi_i\left(A^{11}(l),\ A^{22}(l)\right),$$
$$A_{n,2k}^{ii}(0) = \delta_i(n-1+k), A_{n,2k}^{ii}(l+1) = \psi_i\left(A^{11}(l), A^{22}(l)\right).$$

Let

$$\xi(n,k) = \frac{A_1^2 + A_2^2}{|A_1 A_2|}\sum_{s=n}^{\infty}(s-k)\left\{\left|b_{1,s-1}\right| + \left|b_{2,s}\right|\right\}.$$

Let's prove that

$$\left|A_{n,2k+1}^{ii}(l)\right|,\left|A_{n,2k}^{ii}(l)\right| \le \frac{A_1^2 + A_2^2}{|A_1 A_2|}\sigma(n+1+k)\left\{1+\sigma(n+k)\right\}\frac{\xi^l(n+1,n)}{l!}\,. \tag{6}$$

The truth of the estimations (6) is determined by induction. At $l=0,1$ such estimations evidently are true and if are true for l, then

$$\left|A''_{n,2k+1}(l+1)\right|,\left|A''_{n,2k}(l+1)\right|\le\frac{A_1^2+A_2^2}{|A_1A_2|}\frac{\sigma(n+1+k)\{1+\sigma(n+k)\}}{l!}\sum_{r=n+1}^{m}\sum_{s=r}^{\infty}\frac{A_1^2+A_2^2}{|A_1A_2|}\{|b_{1,s-1}|+$$

$$+|b_{2,s}|\}\xi^l(s+1,s)=\frac{A_1^2+A_2^2}{|A_1A_2|}\frac{\sigma(n+1+k)\{1+\sigma(n+k)\}}{l!}\times$$

$$\times\sum_{s=n+1}^{\infty}\frac{A_1^2+A_2^2}{|A_1A_2|}(s-n)\{|b_{1,s-1}|+|b_{2,s}|\}\xi^l(s+1,s)\ .$$

Further

$$\frac{A_1^2+A_2^2}{|A_1A_2|}\sum_{s=n+1}^{\infty}(s-n)\{|b_{1,s-1}|+|b_{2,s}|\}\xi^l(s+1,s)=\sum_{s=n+1}^{\infty}\{\xi(s,n)-\xi(s+1,n)\}\xi^l(s+1,s)\le$$

$$\le\sum_{s=n+1}^{\infty}\{\xi(s,n)-\xi(s+1,n)\}\xi^l(s+1,n)\,.$$

On the other hand

$$\sum_{s=n+1}^{\infty}\{\xi(s,n)-\xi(s+1,n)\}\xi^l(s+1,n)\le\sum_{s=n+1}^{\infty}\{\xi(s,n)-\xi(s+1,n)\}\xi^l(s,n)=$$

$$=\xi^{l+1}(n+1,n)-\sum_{s=n+1}^{\infty}\{\xi^l(s,n)-\xi^l(s+1,n)\}\xi(s+1,n)\le\xi^{l+1}(n+1,n)-$$

$$-l\sum_{s=n+1}^{\infty}\{\xi(s,n)-\xi(s+1,n)\}\xi^l(s+1,n)\ .$$

whence comparing the beginning and the end we'll get

$$\sum_{s=n+1}^{\infty}\{\xi(s,n)-\xi(s+1,n)\}\xi^l(s+1,n)\le\frac{1}{l+1}\xi^{l+1}(n+1,n)\,.$$

Consequently

$$\left|A''_{n,2k+1}(l+1)\right|,\left|A''_{n,2k}(l+1)\right|\le\frac{A_1^2+A_2^2}{|A_1A_2|}\sigma(n+1+k)\{1+\sigma(n+k)\}\frac{\xi^{l+1}(n+1,n)}{(l+1)!}\,.$$

Now we must note that (4) follows from (5) and (6).

The theorem is proved.

References

[1]. Левитан Б.М. *Операторы обобщенного сдвига.* М., Наука, 1973.

[2]. Марченко В.А. *Некоторые вопросы теории дифференциального оператора второго порядка.* Дан СССР, 1950, т. 72, №3, с.457-460.

[3]. Азимова Г.М., Гусейнов И.М. *Прямая и обратная задачи теории рассеяния для системы разностных уравнений первого порядка.* Изв. АН Аз.ССР, 1987, №3, с.3-8.

Khanmamedov Ag.Kh.
Baku State University.
23, Z.I. Khalilov str., 370148, Baku, Azerbaijan.

Received September 5, 2000; Revised January 11, 2001.
Translated by Mamedova V.A.