MANAFOV M.J.

DESCRIPTION OF DOMAIN OF DEFINITION OF A DIFFERENTIAL OPERATOR WITH COEFFICIENTS IN HIGHER ORDER SINGULARITY GENERALIZED FUNCTIONS

Abstract

The domain of definition of the operator generated by the two-term differential expression with coefficients in a higher order singularity generalized functions and Dirichlet's boundary conditions is described.

A lot of problems of mathematical physics lead to the investigation of spectral properties of singular differential operators. For example, of operators with generalized potentials. Here first of all the question about correct definition of operators in Hilbert space generated by differential expressions with generalized potentials, arises. Most complete statement of results and also examples leading to the study of second order differential operators with generalized potentials and detailed bibliography are contained in [1,2], and higher order operators with coefficients in zero order singularity generalized functions are investigated in papers [3,4].

The present work is devoted to the description of domain of definition of the operator L generated by the differential expression

$$l[.]=(-1)^n\frac{d^{2n}}{dx^{2n}}+q(x),\quad 0\le x\le b,$$

where $q(x)=\sum_{p=0}^{\infty}\sum_{m=0}^{n-1}\alpha_{pm}\delta^{(m)}(x-x_{pm})$, α_{pm} are real numbers.

Note that $q_m(x)=\sum_{p=0}^{\infty}\alpha_{pm}\delta(x-x_{pm})$ is a zero order real-valued generalized function, which is generated by the continuous from the right function $F_m(x)$ with bounded variation. For simplicity we assume that $F_m(x)=F_m(x+0)$.

The description of the domain of definition $D(L)$ of the operator L is connected with the solution of the following Volterra integro-differential equation with Stieltjes integral

$$y(x,\rho)=\frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^k}c_k\sum_{p=0}^{2n-1}\omega_p^{-k}e^{i\omega_p\rho x}-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+1}}{2n\rho^{2n-m-1}}\int_0^x e^{i\omega_p\rho(x-t)}y^{(m)}(t,\rho)dF_m(t)\quad(1)$$

the associated with differential equation $l[y]=\rho^{2n}y$, where c_k, $k=0,1,2,...,2n-1$ are arbitrary constants, ω_p, $p=0,1,2,...,2n-1$ are roots of the $2n$-th order from 1.

Now we study the integral equation (1). Such "integral generalization" of Sturm-Liouville type problems is considered in [1, ch.11, 12].

Let's formulate a theorem on the existence and uniqueness of the solution of the equation (1).

Theorem 1. *If $F_m(x)$ $(m=0,1,2,...,n-1)$ is a continuous from the right function of bounded variation in the interval $[0,b]$, then for given c_k, $k=0,1,2,...,2n-1$, the equation (1) has a unique solution in the class $\mathbf{C}^{(n-1)}[0,b]$.*

Let's divide a complex λ-plane into $2n$ equal sectrums S_h, $h=1,2,...,2n$, determined by the inequality

$$\frac{h-1}{n}\pi \le \arg\rho < \frac{h}{n}\pi .$$

Then in every sector S_h we can numerate the numbers ω_j, $j=0,1,2,...,2n-1$, such that

$$\mathrm{Re}(i\omega_0\rho)\le ...\le \mathrm{Re}(i\omega_{n-1}\rho)\le 0\le \mathrm{Re}(i\omega_n\rho)\le ...\le \mathrm{Re}(i\omega_{2n-1}\rho).$$

Let $|\rho|\le N$. Assume

$$y^{(j)}(x,\rho)=A^{(j)}(x)\exp[\mathrm{Re}(i\omega_{2n-1}\rho)x],\quad j=0,1,...,n-1. \tag{2}$$

Then differentiating the equality (1) j $(j=0,1,2,...,n-1)$- times from (1) we obtain

$$\begin{gathered}A^{(j)}(x)=\frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^{k-j}}c_k\sum_{p=0}^{2n-1}\omega_p^{j-k}e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)]x}-\\ -\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}\int_0^x e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)](x-t)}A^{(m)}(t)dF_m(t).\end{gathered} \tag{3}$$

Now we pass to the proof of the theorem. At first we prove the uniqueness of the solution of the equation (1). Let's assume the inverse, i.e. let the equation (1) has two different solutions $y_1(x,\rho)$ and $y_2(x,p)$.

Then

$$z^{(j)}(x,\rho)=y_1^{(j)}(x,\rho)-y_2^{(j)}(x,\rho),\quad j=0,1,2,...,n-1$$

satisfies the equation

$$z^{(j)}(x,\rho)=-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}\int_0^x e^{i\omega_p\rho(x-t)}z^{(m)}(t,\rho)dF_m(t). \tag{4}$$

Taking into account (2) in the equation (4) we obtain

$$B^{(j)}(x)=-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}\int_0^x e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)](x-t)}B^{(m)}(t)dF_m(t), \tag{5}$$

where

$$B^{(j)}(x)=z^{(j)}(x,\rho)e^{-\mathrm{Re}(i\omega_{2n-1}\rho)x}=\left[y_1^{(j)}(x,\rho)-y_2^{(j)}(x,\rho)\right]e^{-\mathrm{Re}(i\omega_{2n-1}\rho)x}.$$

Let's prove that $B^{(m)}(x)\equiv 0$, $m=0,1,2,...,n-1$ in some right hand neighborhood of the point 0. As far as $F_m(x)$ is a function of bounded variation, we can choose $x_1>0$, such that

$$\sum_{m=0}^{n-1}\frac{x_1^{2n-m-j-1}}{(2n-m-j-1)!}\int_0^{x_1}|dF_m(t)|<\frac{1}{2},\quad j=0,1,2,...,n-1, \tag{6}$$

here $\int_0^x|dF_m(t)|=\operatorname*{var}_{0\le t\le x}F_m(t)$ is a complete variation of the function $F_m(t)$ in the interval $[0,x]$ and tends to zero when $x\to +0$. Denote by $B=\sup_m\max_{0\le t\le x_1}\left|B^{(m)}(t)\right|$. Then by virtue of (6) and

$$\left|\sum_{j=0}^{2n-1}\frac{(i\omega_j)^{k+1}}{2n\rho^{2n-k-1}}e^{[i\omega_j\rho-\mathrm{Re}(i\omega_{2n-1}\rho)]x}\right|\le\frac{x^{2n-k-1}}{(2n-k-1)!},\quad k=0,1,2,...,2n-1,\quad (x\ge 0,\ \rho\in S_h), \tag{7}$$

for $x=x_1$, from (5) we have

$$\left|B^{(j)}(x_1)\right| = \left|-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}\int_0^{x_1} e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)](x_1-t)}B^{(m)}(t)dF_m(t)\right| \le$$

$$\le B\cdot\sum_{m=0}^{n-1}\frac{x_1^{2n-m-j-1}}{(2n-m-j-1)!}\int_0^{x_1}\left|dF_m(t)\right| \le \frac{1}{2}B,$$

i.e. $B\le\frac{1}{2}B$. From here $B=0$. We have $B^{(m)}(x)\equiv 0,\ m=0,1,2,...,n-1$ in $[0,x_1]$. Let further b' be an upper bound of that x form $[0,b]$ for which $B^{(m)}(x)\equiv 0,\quad m=0,1,2,...,n-1$. Then the equation (5) adopts the following form

$$B^{(j)}(x) = -\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}\int_{b'}^{x} e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)](x-t)}B^{(m)}(t)dF_m(t).$$

Now for $x\in[b',b]$ learning those reasonings we obtain that $B^{(m)}(x)$, $m=0,1,2,...,n-1$ is equal to zero in right hand vicinity of the point b'. So the function $B^{(m)}(x),\ m=0,1,2,...,n-1$ is identically equal to zero in $[0,b]$.

Now we prove the existence of the solution of the integral equation (1). At first we assume that $F_m(x)$ is a step-function with finite number jumps at the point x_{rm}, where $0=x_{0m}<x_{1m}<x_{2m}<...<x_{qm}=b$. In this case we can solve the equation (1) or (3) recurrently. For this we write the equation (3) substituting the Stieltjes integral by the sum

$$A^{(j)}(x) = \frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^{k-j}}c_k\sum_{p=0}^{2n-1}\omega_p^{j-k}e^{[i\omega_p\rho x-\mathrm{Re}(i\omega_{2n-1}\rho)]x} -$$

$$-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\sum_{x_{rm}<x}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}\int_0^x e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)](x-x_{rm})}A^{(m)}(x_{rm})[F_m(x_{rm})-F_m(x_{rm}-0)]. \tag{8}$$

In particular

$$A^{(j)}(x_{s+1,q}) = \frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^{k-j}}c_k\sum_{p=0}^{2n-1}\omega_p^{j-k}e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)]x_{s+1,q}} -$$

$$-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\sum_{r\le s}\frac{(i\omega_p)^{m+j+1}}{2n\rho^{2n-m-j-1}}e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)](x_{s+1,q}-x_{r,m})}A^{(m)}(x_{rm})[F_m(x_{rm})-F_m(x_{rm}-0)], \tag{9}$$

and

$$A^{(j)}(0)=A^{(j)}(x_{0,m})=c_j,\ A^{(j)}(x_{1,m}) = \frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^{k-j}}c_k\sum_{p=0}^{2n-1}\omega_p^{j-k}e^{[i\omega_p\rho-\mathrm{Re}(i\omega_{2n-1}\rho)]x_{1,m}}. \tag{10}$$

With the help of the equalities (9), (10) we can find all $A^{(m)}(x_{rm})$ and from here on the basis of (8) and $A^{(j)}(x)$ - for all x.

Using the inequality (7) and estimating (9) by module we obtain

$$\left|A^{(j)}(x_{s+1,q})\right| \le \sum_{k=0}^{2n-j-1}\frac{|c_k|}{k!}b^k + \sum_{r=1}^{s}\sum_{m=0}^{n-1}b^{2n-m-j-1}\left|A^{(m)}(x_{rm})\right|\left|F_m(x_{rm})-F_m(x_{rm}-0)\right|. \tag{11}$$

Denote

$$f_j=\sum_{k=0}^{2n-j-1}\frac{|c_k|}{k!}b_k,\ \ \omega(x)=\sum_{m=0}^{n-1}\sum_{x_{rm}\le x}\left|F_m(x_{rm})-F_m(x_{rm}-0)\right|,\ \ b_1=\begin{cases}b^{2n-1}, & if\ \ b>1\\ 1, & if\ \ b\le 1\end{cases} \tag{12}$$

and we show that

$$\left|A^{(j)}\left(x_{s,q}\right)\right| \le f_0 \cdot \exp\left[b_1\omega\left(x_{s-1,q}\right)\right], \quad s=2,3,\dots \tag{13}$$

Since

$$\left|A^{(j)}\left(x_{1,q}\right)\right| \le f_j \le f_0,$$

then by virtue of (11) for $s=1$

$$\left|A^{(j)}\left(x_{2,q}\right)\right| \le f_0 + b_1 f_0 \cdot \omega\left(x_{1,q}\right) \le f_0\left[1+\int_0^{\omega(x_{1,q})}\exp(b_1u)d(b_1u)\right] \le f_0\exp\left[b_1\omega\left(x_{1,q}\right)\right],$$

and that is the inequality (13) for $s=2$. We prove the inequality (13) by the method of mathematical induction assume that (13) is valid for all $A^{(j)}\left(x_{r,q}\right)$, $r=1,2,\dots,s$. Then from (11) we obtain

$$\left|A^{(j)}\left(x_{s+1,q}\right)\right| \le f_0 + f_0 b_1 \sum_{r=1}^{s}\sum_{m=0}^{n-1}\exp\left[b_1\omega\left(x_{r-1,q}\right)\right]\left|F_m\left(x_{rm}\right)-F_m\left(x_{rm}-0\right)\right|,$$

where $\omega(0)=\omega\left(x_{0,q}\right)=0$. Consequently

$$\left|A^{(j)}\left(x_{s+1,q}\right)\right| \le f_0\left\{1+b_1\sum_{r=1}^{s}\exp\left[b_1\omega\left(x_{r-1,q}\right)\right]\cdot\left[\omega\left(x_{r,q}\right)-\omega\left(x_{r-1,q}\right)\right]\right\} \le$$

$$\le f_0\left\{1+\sum_{r=1}^{s}\int_{\omega(x_{r-1,q})}^{\omega(x_{r,q})}\exp\left(b_1u\right)d\left(b_1u\right)\right\} = f_0\exp\left[b_1\omega\left(x_{s,q}\right)\right],$$

that proves the validity of the estimation (13) for any s. Taking into account the right hand side of the equality (8) this estimation in the form

$$\left|A^{(m)}\left(x_{r,m}\right)\right| \le f_0\exp\left[b_1\omega(b)\right]$$

we obtain

$$\left|A^{(j)}(x)\right| \le f_0 + f_0\exp\left[b_1\omega(b)\right]b_1\omega(b) \le f_0\exp\left[2b_1\omega(b)\right]. \tag{14}$$

Repeating the reasonings mentioned above, for $A(x)$ we get the following estimation

$$\left|A(x)\right| \le f_0 + f_0\exp\left[b_1\omega(b)\right]b_1\omega(b) \le f_0\exp\left[3b_1\omega(b)\right], \tag{15}$$

or taking into account the replacement (2)

$$\left|y(x,\rho)\right| \le f_0\exp\left[\operatorname{Re}\left(i\omega_{2n-1}\rho\right)x+3b_1\omega(b)\right] \le f_0\exp\left[\operatorname{Re}\left(i\omega_{2n-1}\rho\right)b+3b_1\omega(b)\right]. \tag{16}$$

We'll use the estimation (16) for passage to the limit, i.e. for increasing the number of points of potential's jumps in the equation (1).

Let $F_m(x)$ be functions of bounded variations and continuous from the right. In this case we approximate the function $F_m(x)$ with the help of a sequence of the step functions F_{ml}, $l=1,2,\dots$, chosen such that the number of their jumps are at most l and they coincide with $F_m(x)$ at the points obtained as a result of division of the interval $[0,b]$ by l equal parts, and between these points they are constants. Thus, for $\eta=0,1,2,\dots,l-1$ we have

$$F_{ml}(x)=F_m\left(\frac{b}{l}\eta\right), \quad \frac{b}{l}\eta \le x < \frac{b}{l}(\eta+1)$$

and we construct the corresponding solutions $y_l(x,\rho)$ in the form of

$$y_l(x,\rho)=\frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^k}c_k\sum_{p=0}^{2n-1}\omega_p^{-k}e^{i\omega_p\rho x}-\sum_{p=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_p)^{m+1}}{2n\rho^{2n-m-1}}\int_0^x e^{i\omega_p\rho(x-t)}y_l^{(m)}(t,\rho)dF_{ml}(t). \quad (17)$$

Substituting in the inequality (16) $\omega(b)$ by $\omega_l(b)$, from (16) we get

$$|y_l(x,\rho)|\le f_0\exp[\mathrm{Re}(i\omega_{2n-1}\rho)b+3b_1\omega_l(b)]. \quad (18)$$

As $F_{ml}(x)$ is a family of step functions having uniformly bounded variations, then the complete variation of each of them doesn't exceed variation of the function $F_m(x)$. In other words

$$\omega_l(b)=\sum_{m=0}^{n-1}\int_0^b|dF_{ml}(t)|\le\sum_{m=0}^{n-1}\int_0^b|dF_m(t)|=\omega(b)$$

then from (18) we have

$$|y_l(x,\rho)|\le f_0\exp[\mathrm{Re}(i\omega_{2n-1}\rho)b+3b_1\omega(b)].$$

Thus the family of the functions $y_l(x,\rho)$, $l=1,2,\ldots$ is uniformly bounded.

From the equality (17) it is easy to conclude that this family of functions is also equicontinuous. From the equality (17) we have

$$|y_l(x_2,\rho)-y_l(x_1,\rho)|\le|x_2-x_1|\left\{e^{\mathrm{Re}(i\omega_{2n-1}\rho)b}\left[f_0+b_1\sum_{m=0}^{n-1}\max_x\left|y_l^{(m)}(x,\rho)\right|\cdot\int_0^b|dF_{ml}(t)|\right]\right\}.$$

Here the coefficient for $|x_2-x_1|$ in the right hand side of the inequality is uniformly bounded by l, so that all $y_l(x,\rho)$ satisfy the uniform Lipschitz condition and are equicontinuous.

Now applying Artcela's compactness principle we conclude that $y_l(x,\rho)$ uniformly converges to the limit function $y(x,\rho)$. Thus, theorem 1 is completely proved.

The following theorem establishes the smoothness property of solutions of the integro-differential equation (1) in the class $\mathbf{C}^{n-1}[0,b]$.

Theorem 2. *The solution of the equation (1) has the n-th right hand derivative, where the n-th derivative is two-sided at points, where $F_{n-1}(x)$ is continuous or $y^{(n-1)}(x)=0$, $(n+k)$-th derivative is the $(k-1)$-th order singularity generalized function, for which*

$$y^{(n+k)}(x)-\sum_{p=0}^{\infty}\sum_{m=0}^{n-1}(1)^m\alpha_{p,n-m-2}\delta^{(m)}(x-x_{p,n-m-2})y^{(m+1)}(x),\quad k=1,2,\ldots,n-1$$

is a function of jumps, which is continuous from right.

We plan the scheme of the proof of the theorem.

Consider the following difference:

$$y^{(n-1)}(x_2,\rho)-y^{(n-1)}(x_1,\rho)=\frac{1}{2n}\sum_{k=0}^{2n-1}\frac{1}{(i\rho)^{k+1-n}}\sum_{p=0}^{2n-1}\omega_p^{n-k-1}\left[e^{i\omega_p\rho x_2}-e^{i\omega_p\rho x_1}\right]-$$

$$-\sum_{k=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_k)^{n+m}}{2n\rho^{n-m}}\int_0^{x_1}\left[e^{i\omega_k\rho(x_2-t)}-e^{i\omega_k\rho(x_1-t)}\right]y^{(m)}(t,\rho)dF_m(t)- \quad (19)$$

$$-\sum_{k=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_k)^{n+m}}{2n\rho^{n-m}}\int_{x_1}^{x_2}e^{i\omega_k\rho(x_2-t)}y^{(m)}(t,\rho)dF_m(t).$$

Using the estimation (7) for $k=n+m-1$, dividing (19) by x_2-x_1 and tending x_2 from the right to x_1 for the fixed x_1 we obtain

$$(x_2-x_1)^{-1}\left[\sum_{k=0}^{2n-1}\sum_{m=0}^{n-1}\frac{(i\omega_k)^{n+m}}{2n\rho^{n-m}}\int_{x_1}^{x_2}e^{i\omega_k\rho(x_2-t)}y^{(m)}(t,\rho)dF_m(t)\right]\to 0 .$$

It is valid for $x_2\to x_1+0$, since the function $F_{n-1}(x)$ is continuous from right, and it is also valid, when x_2 tends to x_1 from any side, if $F_{n-1}(x)$ is continuous at the point x_1 or $y^{(n-1)}(x_1)=0$.

The other statements of the theorem are analogously proved.

From theorem 1 and 2 follows that $y(x)$ is the solution of the equation (1), then $c_k=y^{(k)}(0)$ for $k=0,1,2,...,n$,

$$c_{n+k}=\left\{y^{(n+k)}(x)-\sum_{p=0}^{\infty}\sum_{m=0}^{k-1}(-1)^m\alpha_{p,n-m-1}\delta^{(m)}(x-x_{p,n-m-1})y^{(m)}(x)\right\}\Bigg|_{x=0}$$

for $k=1,2,...,n-1$.

With the help of theorem 2 describing the smoothness of solutions of the integro-differential equation (1) we introduce the next operator.

Denote by D the set of all functions $y(x)\in L_2[0,b]$ such that

a) $y^{(\nu)}(x)$, $\nu=0,1,2,...,n$ exists, are continuous for $\nu=0,1,2,...,n-1$, $y^{(n)}(x)$ is continuous from right, and $(n+k)$-th derivatives are $(k-1)$-th order singularity generalized functions;

b) $y^{(n+k)}(x)-\sum_{p=0}^{\infty}\sum_{m=0}^{n-1}(-1)^m\alpha_{p,n-m-2}\delta^{(m)}(x-x_{p,n-m-2})y^{(m+1)}(x)$, $k=1,2,...,n-1$ is a function of jumps which is continuous from right, where

$$\left\{y^{(n+k)}(x)-\sum_{p=0}^{\infty}\sum_{m=0}^{n-1}(-1)^m\alpha_{p,n-m-2}\delta^{(m)}(x-x_{p,n-m-2})y^{(m+1)}(x)\right\}\Bigg|_{x_{p,n-k-1}-0}^{x_{p,n-k-1}}=$$

$$=\alpha_{p,n-k-1}y^{(n-k-1)}(s_{p,n-k-1}),\ k=1,2,...,n-1;\ p=0,1,2,...,$$

$$y^{(n)}(x_{p,n-1})-y^{(n)}(x_{p,n-1}-0)=\alpha_{p,n-1}y^{(n-1)}(x_{p,n-1});$$

c) $(-1)^n y^{(2n)}(x)+q(x)y(x)\in L_2[0,b]$.

Further denote by D_0 a set of all functions $y(x)\in D$ satisfying the boundary conditions

$$y^{(m)}(0)=0\ ,\ y^{(m)}(b)=0\ ,\ m=0,1,2,...,n-1. \quad (20)$$

Let's determine the operator L in the space $L_2[0,b]$ in the following form: $D(L)=D_0$ and $Ly=(-1)^n y^{(2n)}(x)$, $y(x)\in D_0$.

Thus we obtained description of the operator L generated by the differential expression (1) and the boundary conditions (20).

Remark. We can consider an analogous problem for more general boundary conditions.

References

[1]. Аткинсон Ф. *Дискретные и непрерывные граничные задачи.* Изд. «Мир», 1968, 749с.
[2]. Mingarelli A. *Volterra-Stieltjes integral equations and generalized ordinary differencial expressions.* Lecture Notes in Mathematics, 1983, v.989, №14, 317c.
[3]. Manafov M. *Study of spectral properties of an operator with generalized potentials.* Сборник трудов I Респ. Конф. по мат. и мех., Баку, 1995, ч. II, с. 152-154.
[4]. Манафов М. *Описание области определения обыкновенного дифференциального оператора с обобщенными потенциалами.* Дифф. ур-я, 1996, т.32, №5, с.706-707.

Manafov M.J.
Institute of Mathematics and Mechanics of AS Azerbaijan.
9, F.Agayev str., 370141, Baku, Azerbaijan.
Tel.: 39-47-20.

Received November 16, 2000; Revised February 19, 2001.
Translated by Mirzoyeva K.S.