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LOCALIZATION OF SPECTRUM AND ITS APPLICATIONS II.

NUMERICAL RANGE AND STRUCTURE OF A SPECTRUM

Abstract

At the given paper received by us the theorem of localization of residual spectrum by means of a numerical range is Banach space is applied to the series problems of the spectral theory of the operators. This is a criterion of the closeness of a numerical range in the terms of spectrum, description of a spectrum of the operator through on interior of a numerical range; conclusions about structure of a spectrum Hermitian operators in Banach spaces.

The present paper being the continuation of [18e], has the aim to show how the theorem about localization of residual spectrum of the operator an interior of Baner's numerical range [18e, theorem 11] finds applications to the problems of the spectral theory of operators. Other applications of localization of the parts of operators spectrum are planned to consider at the following parts of the paper.

Here first of all we apply the theorem of localization of residual spectrum to the problem on the conditions of closeness of a numerical range of the operator in the terms of spectrum secondly by mean of localization theorem we receive the description of the spectrum of the operator through the interior of a numerical range and we derive series of corollaries about properties of spectrum of the various operators, and thirdly we show what conclusions about structure of spectrum of Hermitian operators in Banach space may be drown from the above-mentioned description of spectrum.

Let's denote especially that all stated spectral properties of Hermitian operators are true for the normal operators in Banach space.

In this paper the used notations and concepts, connected with numerical ranges and with spectrum operators and with geometry of Banach spaces are in first part of paper [18e]. Other necessary informations are given by the way of exposition.

§3. Numerical range and description of spectrum.

3.1. Studying the numerical range of operators in Hilbert space P.Halmos [1, p.116] puts the question about determination of such operators whose numerical range is closed. Let's give the application of the theorem about localization of residual spectrum [18e, th.1.1] to the problem about closeness of numerical range of the operators in terms of spectrum. This problem for the various classes of operators in Hilbert space was studied by the authors: Meng C-H., Hildebrandt S., Lin S-C., de Barra G. and others. The discussion of general criterion for the Hausgorff's numerical range and the literature about the question we can find in [18f].

The first result in this direction has been got by Meng [9] on the basis of spectral decomposition for the normal operator in Hilbert space he proved the criterion of closeness of numerical range by means of the continuous spectrum. At the same place he shows that for the unitary operator in Hilbert space the closeness of a numerical range is equivalence to coincidence of spectrum with the point spectrum of this operator.

At the papers [18b,d] both of these results are extended on Bauer's numerical range accordingly to normal and isoabelian operators in Banach space whereupon for the Hilbert spaces new proofs are got not based on spectral decomposition. Incidentally emptyness of the residual spectrum following from the previously received information on thin structure of a spectrum of normal and isoabelian operators was used (for the Hermitian operators about this see p.4.1 of the given paper).

However at transition to wider classes of the operators the property of emptyness of a residual spectrum is lost, that was an obstacle to attain the exact analogoue of Meng criterion for the extended classes of operators even in Hilbert space. Received by us [18e, th.1.1] localization relation allows to by pass this obstracle and to give full-blooded criterion of closeness of a numerical range for various classes of operators in Banach space. Let's illustrate the told by one particalar result and we will show possible generalizations.

Let's consider the translatable paranormal or transparanormal (t.p.n.) operators, i.e. such $T \in B(X)$ for $T-\lambda$ is paranormal at any complex λ. Let's remind that operator $A \in B(X)$ is called paranormal, if for any unit vector $x \in X$ the inequality $\|Ax\|^2 \leq \|A^2 x\|$ is true.

Theorem 3.1. *For t.p.n. operator $T \in B(X)$ in uniformly rotund Banach space X the Bauer's numerical range $V(T)$ is closed iff set of exposed points on convex hull of spectrum doesn't intersect with continuous spectrum of T.*

Scheme of the proof. The proof of necessary condition for the closeness of a numerical range is carried out on a few steps. First of all, as in case of Hilbert space, it is proved that in any Banach space the spectral radius and the norm of paranormal operators are equal each other. The next step is in getting the result of Forster's variant [10], proved by them in Hilbert space: if T - t.p.n. is the operator in uniformly rotund Banach space, then every exposed point of the set $\overline{V}(T)$ (lying on $V(T)$) is approximate eigenvalues (is eigenvalues) t.p.n. of the operator T. Finally it is proved that convex hull of spectrum, t.p.n. of the operator T coincides with closure of its Bauer's numerical range. In the proof the sufficiency condition of closeness of a numerical range besides previously mentioned we use: theorem 1.1 from [18e] about localization of residual spectrum and theorem of Milman-Pettis [3, p.182]; variant the theorem Straszwecz [11], concerning the exposed points and being analogy of the Krein-Milman theorem on extreme points and finally the Zenger's theorem on localization of convex hull of point spectrum [4, §19].

Corollary 3.1. *For t.p.n. of the operators $T \in B(X)$ in uniformly rotund Banach space X the numerical range $V(T)$ is closed iff exposed points of convex hull of spectrum lie on the point spectrum of the operator T.*

The simple illustration to the criterion of the closeness of a numerical range is the operator of unilateral shift 21 [1, p.48], in if the set $ex\,pco\sigma(U)$, being the unit circle, coincides with continuous spectrum and the point spectrum is empty. So necessary conditions of the closeness of a numerical range $W(U)$ from theorem and its corollary are broken. Indeed $W(U)$ is an open unit circle [1, p.308].

Degrees not for long from the applying of the theorem on the localization of residual spectrum in order to describe the essential spectrum of Weyl for the above-mentioned class of operators.

Remind that the essential spectrum of Weyl (Browder) $\sigma_W(T)$ $(\sigma_b(T))$ of the operators $T \in B(X)$ is called the maximal part of the spectrum $\sigma(T)$, that is invariant with respect to any additive perturbation $T+K$ of operator T with compact operator K (commutating with T). Let be said that for essential spectrum $\sigma_e(T)$, $e = w, b$, Weyl's theorem holds (theorem of Weyl's type), if $\sigma_e(T) = \sigma(T) - \pi_{00}(T)\,(\sigma_e(T) =$

$=\sigma(T)-\hat{\pi}_{00}(T))$, where $\pi_{00}(T)\,(\hat{\pi}_{00}(T))$ is a set of isolated eigenvalues with finite geometrical (algebrical) multiplicity [2, p.211 and 229].

Theorem 3.2. *Weyl's spectrum of t.p.n. operator $T \in B(X)$ in Banach space X satisfies Weyl's theorem.*

The scheme of proof. First the fulfillment of Weyl's theorem for essential Browder's spectrum of t.p.n. operator T is proved. For this it is enough to show the justice of inclusion $\pi_{00}(T) \subset \hat{\pi}_{00}(T)$, which follows from equality to null of operator $T_R - \lambda$ for any $\lambda \in \pi_{00}(T)$, where T_R is a restriction of T onto image R of spectral projector corresponding to λ, and this draws inclusion of R into kernel of operator $T-\lambda$. Then using finiteness of ascent of operator $T-\lambda$, according to Werner's theorem [12, p. 469] the coincidence of Browder's and Weyl's spectra of t.p.n. operator T is shown.

Remark. Theorem 3.1 together with the corollary is true for wider class of transaloids, moreover, in more general spaces; the necessary of closeness condition uses only rotundness, but for deriving the sufficiency only the reflexivity of the space is necessary. A variant of this criterion is true even for convexoids and in particular, for operators with first order growth condition on the resolvent with respect to spectrum. Besides, theorem 3.2 remains in force for subclasses of the enumerated operators.

3.2. Let's now apply the theorem about localization of residual spectrum [18e, theorem 1.1] to the problem of description of operator spectrum via interior of numerical range of this operator. For this let's remind that essential spectrum of Goldberg of operator $T \in B(X)$ in the Banach space X is called the set $\sigma_g(T)$ of all such complex numbers λ, for which range $Ran(T-\lambda)$ of operator $T-\lambda$ is not closed in X.

Due to Hilbert situation [13] we call the eigenvalue $\lambda \in \sigma_p(T)$ normal-isolated for the operator $T \in B(X)$ in Banach space X, if, first, λ is an isolated point of spectrum $\sigma(T)$; secondly, λ is normal eigenvalue, i.e. the pair of subspaces $\ker(T-\lambda)$ and $\overline{Ran}(T-\lambda)$ completely reduces the operator T: $X=\ker(T-\lambda)\oplus\overline{Ran}(T-\lambda)$, $T(\ker(-\lambda))\subset(T-\lambda)$, $T(\overline{Ran}(T-\lambda))\subset\overline{Ran}(T-\lambda)$, and thirdly, the geometrical and algebraically multiplicity of number λ are co-ordinate. It is well known that for normal operator in Hilbert space all isolated eigenvalues are normal-isolated. We note that this fact is also true in Banach space [18d].

Using theorem 1.1 [18e] on localization of residual operator spectrum via the interior of its numerical range and proposition about points of compression spectrum, lying on the border of numerical range [18e, proposition 1.2] it is possible to give the next classification of points of spectrum.

Theorem 3.3. *In reflexive Banach space X any the points of spectrum $\sigma(T)$ of operator $T \in B(X)$ lie either in interior $\operatorname{int} V(T)$ of Bauer numerical range $V(T)$, or in the Goldberg spectrum $\sigma_g(T)$, or in the set $\pi_{0v}(T)$ of normal isolated eigenvalues.*

The scheme of proof. The localization relation for the residual spectrum $\sigma_r(T)$ from [18e, theorem 1.1] and inclusion of continuous spectrum $\sigma_c(T)$ in Goldberg's spectrum $\sigma_g(T)$ shows that point $0 \in \sigma(T)$ not lying in interior $V(T)$ and in $\sigma_g(T)$ will be eigenvalue of operator T. The proof of normality of eigenvalue $\lambda=0$ is carried out by the following way. According to Crabb-Sinclair theorem [4, §20, theorem 1] the kernel $\ker T$ of operator T is orthogonal according by Birkhoff to its range $RanT$. It draws their intersection only by null vector and considering the closeness of subspace $\ker T$ and $RanT$, it is possible to show that their sum is also closed subspaces in X. Hence, it follows that the pair of subspaces $\ker T$ and $RanT$ completely reduces operator T, since otherwise there would be existed non-trivial annihilator for both of these subspaces, lying in the intersection of kernel and range of adjoint operator T^*. And this gives contradiction, since the application of proposition 1.2 from [18e] and the above-mentioned theorems of Grabb-Sinelair to the adjoint operator T^* would reduce to the null intersection of its kernel and range. The isolatedness of eigenvalue $\lambda = 0$ is obtained from consideration of restriction of operator to the reducing subspaces $\ker T$ and $RanT$. Moreover, this eigenvalue is a first order pole of resolvent of operator T which reduces the agrement of its geometrical and algebraic multiplicity.

In particular, when the space is Hilbert's the previous theorem gives main result of paper [13, theorem 1].

Let's note some corollaries from above-proved theorems, first of which in the case of Hilbert's space is a second main result from [13, theorem 2]. In all of these corollaries the Banach space is reflexive.

Corollary 3.2. *For convexoid operator any extremal point of closure of numerical range lies, either in Goldberg's spectrum, or is normal isolated eigenvalue of this operator.*

From such classification of extremal points the following criterion follows.

Corollary 3.3. *In finite dimensional Banach space an operator is convexoid iff any extremal point of numerical range of this operator will be its normalisolated eigenvalue.*

The following two corollaries also are derived from theorem 3.3.

Corollary 3.4. *Any non-zero point of spectrum of compact operator which lies on the boundary of its numerical range is normal-isolated eigen-value.*

Corollary 3.5. *The spectrum of nonzero quasi-nilpotent operator with closed range lies in interior numerical range of this operator.*

Note that two variants of theorem 3.3 and corollaries 3.2-3.5 are true in any Banach space. One of (weaker) by means of algebraic numerical range shows itself in the proof of theorem 4.3. It leads to the characterization of Goldberg's spectrum of Hermitian operator. But before describe thin structure of spectrum of Hermitian operator and give some conclusions from it.

§ 4. The structure of spectrum of Hermitian operator and its application.

4.1. Before we describe the thin structure of spectrum of Hetmitian operators by means of states [14] we note that the complete exposition of properties of Hermitian operators is given in [4, §§5-8 and §§26-29].

Now a few words about notion of states (according to Taylor-Halberg) of operator (detailly see [14]). For the range $RanT$ the any operator $T \in B(X)$ in normed space X the following cases are possible: I. $RanT = X$; II. $RanT \neq X$, but $\overline{RanT} = X$; III $\overline{RanT} \neq X$. For inverse mapping T^{-1} from $RanT$ into X the following cases are possible: 1. T^{-1} exists and is continuous; 2. T^{-1} exists and is discontinuous; 3. T^{-1} doesn't exist. The record $T \in I_1$, means that T satisfies the conditions I and 1, simultaneously (analogously, for other combinations of conditions I, II, III with conditions 1, 2, 3). This classification scheme is applicable also to the adjoint operator

T^*. If we take ordered pair (T,T^*), then condition on T and condition on T^* give ordered pair of conditions which is called state of pair (T,T^*). For example, $(T,T^* \in (II_3, III_1))$ means that $T \in II_2$ and $T^* \in III_1$. It is known [14] that when X is arbitrary Banach space and $T \in B(X)$ is arbitrary operator then the number of possible states of pair T,T^* is equal to nine: (I_1,I_1); (II_2,II_2); (III_3,III_3); (I_3,III_1); (III_1,I_3); (II_3,III_2); (III_2,II_3); (II_2,III_3); (III_2,III_3). If X is reflexive then number of possible states of pair (T,T^*) equal to seven since states (II_2,III_3), (III_2,III_3) are impossible.

The next theorem describes the state of Hermitian operators.

Theorem 4.1. *The Hermitian operator $T \in B(X)$ in any Banach space X has the next possible states (I_1,I_1); (II_2,II_2); (III_3,III_3); (II_2,III_3);(III_2,III_3), but in reflexive X - (I_1,I_1); (II_2,II_2); (III_3,III_3).*

A brief proof. The possible states of operator is convenient to formulate in terms of parts of spectrum. Codensity of numerical range of operator T and Sincliar's theorem [4, §20] remove the conditions I_3 and II_3 for T, and the localizability of spectrum by algebraic numerical range $\mathcal{V}(T)$ of operator T gives the impossibility of state III_1. In a reflexive X the behaviour of parts of spectrum under adjoining remove for T of state III_2. It only remains use invariance of $\mathcal{V}(T)$ with respect to Banach conjugation and complete the proof.

The previous theorem at once gives the next information about thin structure of spectrum of Hermitian operator [18a].

Corollary 4.1. *For Hermitian operator $T \in B(X)$ in any Banach space it is true the next relations: a) $\sigma_p(T) \subset \sigma_\gamma(T)$; b) $\sigma(T) = \sigma_\pi(T) = \sigma_\delta(T)$; c) $\sigma_p(T) = \sigma_\gamma(T)$, if X is reflexive.*

Besides Banach analogue of Weyl's theorem about emptiness of residual spectrum [18e, Corollary 1.3] for Hermitian operator it is true the number of other properties of selfadjoint operators. For example, for generalized derivation Δ_{TS} defined by relation $\Delta_{TS} = TA - AS$ for $T,S,A \in B(X)$ it is true.

Gorollary 4.2. *If $T,S \in B(X)$ are Hermitian operators then there exists semi-inner produce [,] giving the norm in X, for which $[R,K] = 0$ for any $K \in \ker\Delta_{TS}$ and $R \in Ran\Delta_{TS}$.*

Let's note that in Hilbert space X, when $T = S$ this corollary gives the result [15, theorem 1.5] proved by another way. The other corollary of theorem 4.1 is related with invariant subspaces.

Corollary 4.3. *In a reflexive X all T invariant subspaces of injective Hermitian operator $T \in B(X)$ is regular (no Sz-Nagy-Foiaş [5, p.91]).*

This a key fact in the solution of problems about approximation of inverse operator [18b, p.74] which will be considered in the following part of our paper for a wider class of dissipative operators. Therefore, for illustration we give only formulation of the next criterion.

Proposition 4.1. *Let $A \in B(X)$ be invertible operator in reflexive X and $\mathcal{A}$ be algebra of polynomials from A. Then A^{-1} is approximated by operators from $\mathcal{A}$, iff* *there exists such injective operator T from weak closure $\mathcal{A}$, that operator TA is Hermitian.*

Returning not for long to the criterion of closeness of numerical range from point 3.1, we note that, the proof of analogous fact for Hermitian operators has the same scheme, but by virtue of point c) of corollary 4.1 it is not required theorem 1.1 from [18e] about localization of residual spectrum. Besides the following proposition enable to use extreme points of numerical range: in a rotund Banach space every extremal point of numerical range of Hermitian operator is eigenvalue. All that leads to the following criterion which is special case of our result [18b, p.80] and therefore we omit the details.

Theorem 4.2. *Let $T \in B(X)$ be Hermitian operator in Banach space X. Then for closeness of numerical range $V(T)$ it is sufficient, but for rotund X it is necessary the relation $extco\sigma(T) \subset \sigma_p(T)$. If also X is reflexive then the previous condition can be substituted for the next $\sigma_c(T) \cap extco\sigma(T) = \varnothing$.*

It remains to note that unlike the case of Hilbert space theorem 4.2 doesn't follow from theorem 3.1, since the Hermitian operator in Banach space can not to be paranormal. Let's note also that the example of multiplication operator from theorem 1.1 [18e] shows that the condition of reflexivity and rotundness of space in theorem 4.2 mustn't be omitted.

4.2. Let's move on consideration of essential spectra of Hermitian operator in Banach space. From thin structure of spectrum $\sigma(T)$ of Hermitian operator $T \in B(X)$ (see corollary 4.1) follows that $\sigma(T) = \sigma_g(T) \cup \sigma_p(T)$. In reflexive X the theorem 3.3 at once makes this fact more precise by substituting $\sigma(T)$ for $\pi_{0v}(T)$. The following statement which shows the justice of this verification in any Banach space gives also two characterization of Goldbery's spectrum noted in [18e]. For this let's remind that the descent (ascent) of operator $T \in B(X)$ called the least non-negative integer number $\delta(T)$ $(\alpha(T))$ such that the ranges (kernels) of operators T^k and T^{k+1} coincide for all $k \geq \delta(T)$ $(k \geq \alpha(T))$. By $\sigma(T)^\alpha$ denote the set of all points of accumulation of spectrum $\sigma(T)$.

Theorem 4.3. *For Hermitian operator $T \in B(X)$ the following equalities are true:*

a) $\sigma(T) = \sigma_g(T) \cup \pi_{0v}(T)$; *b)* $\sigma_g(T) = \sigma(T)^d$; *c)* $\sigma_g(T) = \{\lambda \in \sigma(T) : \delta(T - \lambda \geq 2)\}$.

A brief proof. a) has the same scheme of proof as theorem 3.3, but with participation of algebraic numerical range $\mathcal{V}(T)$ we omit it; b) follows from a) if we show that every isolated point of spectrum $\sigma(T)$ will be simple pole for T and apply the theorem 24 from [6, p.616]; c) is proved by the following scheme; inclusion of the set $\sigma_1 = \{\lambda \in \sigma(T) : \delta(T - \lambda) \geq 2\}$ to the Goldberg's spectrum $\sigma_g(T)$ is obtained by restriction of operator $T - \lambda$ on its range and utilization of corollary 4.1, and also M-paranormality of Hermitian operator [4, §10, theorem 1]. The inverse inclusion $\sigma_g(T)$ to the set $\sigma_1(T)$ is based on Riesz's decomposition for operators with finite ascent and finite descent.

Now a few words about analogue of theorem 3.2 for Hermitian operators. It is well known that for selfadjoint operator T in Hilbert space all four essential spectra: Kato $\sigma_k(T)$, Fredholm $\sigma_f(T)$ (see [2, p.305]) Weyl $\sigma_W(T)$ and Browder $\sigma_b(T)$ (see p.3.1 of the given paper) coincide and for them Weyl's theorem holds. From the previous theorem 4.3 follows proposition which makes the result of paper [16] more precise (in [16] for Weyl's spectrum of Hermitian operator of Weyl's type theorem is proved).

Corollary 4.4. *For Weyl's spectrum $\sigma_W(T)$ (coinciding with $\sigma_k(T),\sigma_f(T)$ and $\sigma_b(T)$) of Hermitian operator $T \in B(X)$ Weyl's theorem holds.*

Moreover, for any polynomial $p(T)$ from Hermitian operator T holds

Corollary 4.5. $p(\sigma_W(T)) = \sigma_W(p(T))$ and $\sigma_W(p(T)) = \sigma(p(T)) - \pi_{00}(p(T))$.

Before deriving spectral characterization of compact operators we do remark about Hermitian operators of meromorphic type. Let $T \in B(X)$ be an operator with enumerable spectrum for which only zero can be point of accumulation (for example, compact operator). If for such operator T every non-zero points of spectrum is a pole then T is called operator of meromorphic type (for example, Hermitian compact operator).

Well known spectral expansion of compact self-adjoint operators for Hermitian compact operators in Banach space was obtained by a series of authors [4, §28] in various complementary conditions. For Hermit operators of meromorphic type the similar results are in paper [17]. From the previous considerations follows the proof of auxiliary key facts from [17], suitable for normal operators. Thus, its is possible to obtain main theorems 3.5 and 3.6 from [17] for normal operators of meromorphic type in reflexive Banach space.

At last, let's note two corollaries of Theorem 4.3 motivated by classical results from spectral theory, of compact self-adjoint operators in Hilbert space and extending them to the case of Banach spaces.

First of these corollaries belongs to spectral characteristics of compact self-adjoint operators in Hilbert space [7, p.162] or [8, p.351, theorem 12.30]. It is well known that for compact operator T in Banach space for any non-zero complex λ the operator $T-\lambda$ has closed range and finite-dimensional kernel [8, theorems 4.23 and 4.25]. It turns out that when T is a Hermitian operator in Banach space these two necessary conditions are also sufficient for compactness of T.

Corollary 4.6. *Hermitian operator $T \in B(X)$ in Banach space X is compact iff for any nonzero $\lambda \in \sigma(T)$ the following conditions holds: a) λ doesn't lie in Goldberg's spectrum $\sigma_g(T)$; b) the kernel of operator $T-\lambda$ is finite-dimensional.*

Let's note that in Hilbert space this corollary exactly coincides with theorem about spectral characterization of compact operators [7, p.162] if according to previous theore 4.3 instead of condition a) we take equivalent condition a') the spectrum $\sigma(T)$ cannot have nonzero points of accumulation.

From the same theorem it follows precides generalization to Banach space of well known classical result about self-adjoint compact operators in Hilbert space.

Corollary 4.7. *In Banach space X for Hermitian compact operator $T \in B(X)$ there exists normal-isolated eigenvalue of finite multiplicity whose modulus coincides with the norm of T.*

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