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ROBUST COMPUTATIONAL TECHNOLOGIES OF SPECTRAL ANALYSIS

1. Analysis Of Features Of Spectral Analysis Algorithms.

It will be shown below that in spectral analysis when the measured information consists of the useful signal and interference the error of requed estimates depends on the difference between the sum of errors of positive and negative products of samples of the total signal and samples of cosinusoids and sinusoids respectively.

There, below various varianty of algorithms of balancing mentioned errors are suggested. The mentioned algorithms at the expense of making the process of processing of analyzing signal more difficult provide the robustness of required estimates [1,2].

In practice the principle of superposition of signals ca.ı be used during the analysis of the operation of linear elements and systems. This principle is based on the following. If the input signal is represented as the sum of two signals then the output signal is determined as the sum of the output signals which we would have at the output of the system if each of the input signals acted separately. It is the exsiest method of determination of the reaction of the linear system or the linear part of the non linear system to the input signal with unspecified form. Here the harmonic analysis is applied. Just because of this fact the methods and the algorithms of spectral analysis are widely applied in experimental works.

As shown above in spectral analysis the analyzed signal is represented as the sum of harmonic components - sinusoids and cosinusoids the sum of ordinats of which at each moment t gives the magnitude of function.

$$x(t)=\frac{a_0}{2}+\sum_{i=1}^{\infty}(a_n \sin n\omega t+b_n \cos n\omega t), \tag{1}$$

where $\frac{a_0}{2}$ is the average value of the function $x(t)$ for the period T, a_n and b_n are the amplitudes of the sinusoid and cosinusoid with frequency $n\omega$.

The following inequality must take place to provide the sufficient accuracy of representation of the signal as the sum of sinusoids and cosinusoids:

$$\sum_{i=1}^{n}\lambda_i^2 \le S, \tag{2}$$

where λ_i^2 are the squares of deviations between the sum of the right-hand side of the equality (1) and samples of signal $x(t)$ in the moments of sampling $t_0, t_1, \ldots, t_i, \ldots t_m$ with the sampling step Δt; S is the permissible value of mean-root-square deviation.

In formula (1) in decomposing the function $x(t)$ in trigonometric Fourier series the value ω is taken equal to $2\pi/T$ and the coefficients a_n and b_n are determined so:

$$a_n=\frac{2}{T}\int_0^T x(t)\cos n\omega t\,dt \qquad \text{for} \quad n=1,2,\ldots; \tag{3}$$

$$b_n=\frac{2}{T}\int_0^T x(t)\sin n\omega t\,dt \quad \text{for} \quad n=1,2,\ldots. \tag{4}$$

Here the first harmonic has the frequency $2\pi / T$, its period and the period T of the function $x(t)$ are the same. The coefficients a_1 and b_1, a_2 and b_2, a_3 and b_3 are the amplitudes of cosinusoids and sinusoids obtained for $n=1$, $n=2$, $n=3$ and etc.

In the theory the condition (2) for the given value S holds true for the periodic signals $x(t)$ without the interference $\varepsilon(t)$. But in practice the legitimate signal $x(t)$ is accompanied by the certain interference $\varepsilon(t)$, i.e. it is the sum $g(t)=x(t)+\varepsilon(t)$. Because of this the condition (2) does not always hold true. Nevertheless many important problems are successfully solved by means of the application of the algorithms (3) and (4) in many areas of science and engineering when the value of interference changes in certain limits. But when the interference has the considerable value and the inequality (2) does not hold true, the solving of the problems by means of spectral methods seems to be impossible.

In practice when the analyzed signal $g(t)$ is the sum of the useful signal $x(t)$ and the interference $\varepsilon(t)$ i.e.

$$g(t)=x(t)+\varepsilon(t) \tag{5}$$

the formula (3) can be represented as follows:

$$a_n=\frac{2}{T}\int_0^T[x(t)+\varepsilon(t)]\cos n\omega t\,dt=$$

$$=\frac{2}{T}\left[\int_0^T x(t)\cos n\omega t dt+\int_0^T \varepsilon(t)\cos n\omega t dt\right]. \tag{6}$$

In this case the fulfillment of the condition (2) can be real when

$$\sum_{i=1}^{N^-}\int_{t_i}^{t_{i+1}}\varepsilon(t)\cos n\omega t dt=\sum_{i=1}^{N^-}\int_{t_{i+1}}^{t_{i+2}}\varepsilon(t)\cos n\omega t dt. \tag{7}$$

Here N^+, t_i, t_{i+1} are the quantity, the beginning and the end of the positive half-periods of the $\cos n\omega t$ observed in time T; N^-, t_{i+1}, t_{i+2} are the quantity, the beginning and the end of the negative half-periods of the $\cos n\omega t$ observed in time T;

Otherwise when that equality does not take place the difference

$$\lambda_{a_n}=\sum_{i=1}^{N^+}\int_{t_i}^{t_{i+1}}\varepsilon(t)\cos n\omega t dt-\sum_{i=1}^{N^-}\int_{t_{i+1}}^{t_{i+2}}\varepsilon(t)\cos n\omega t dt \tag{8}$$

leads to the error of the estimate of the coefficient a_n. The determination of the estimate b_n is analogous. At the same time as it follows from the expression (8) the difference λ_{a_n} increases in increasing the dispersion of the $\varepsilon(t)$. The difference λ_{a_n} also increases if there is the correlation between the useful signal $g(t)$ and the interference $\varepsilon(t)$ and when the distribution law of the analyzed signal $g(t)$ differs from the normal. From this point of view the errors of the estimates $\lambda_{a_n}, \lambda_{b_n}$ can be commensurable with the unknown coefficients a_n, b_n.

In this connection it is necessary to develop the algorithms allowing to provide the inequality $S_n >> \lambda_{a_n}, S_n >> \lambda_{b_n}$ and the condition (2) by means of the elimination of the cause of the appearance of the errors $\lambda_{a_n}, \lambda_{b_n}$ for increasing the reliability of the results of the analysis of the experimental data. At the same time it is necessary that these algorithms must be robust i.e. they must allow to eliminate the connection between the values λ_{a_n}, λ_{b_n} and the dispersion of the error $\varepsilon(t)$ and it is necessary that the error of the estimate does not depend on the change of the form of the distribution law of the analyzed signal, the coefficient of the correlation between the useful signal $x(t)$ and the interference $\varepsilon(t)$ and etc.

2. Causes of Appearance of Difference Between Positive and Negative Errors Caused by Interference.

Let us assume that the time T of observing the realization of total signal $g(t)$ consisting from the legitimate signal $x(t)$ and the interference $\overset{\circ}{\varepsilon}(t)$ is great enough. Here, assuming that the function $x(t)$ is a sampled stationary non-centered random signal $x(i\Delta t)$ with normal distribution law and $\overset{\circ}{\varepsilon}(t)$ is centered random signal with the mathematical expectation which is equal to zero, $m_\varepsilon = 0$, then the formula of determining coefficient a_n^* is represented as

$$a_n^* = \frac{2}{N}\sum_{i=1}^{N}[x(i\Delta t) + \overset{\circ}{\varepsilon}(i\Delta t)]\cos n\omega(i\Delta t) =$$

$$= \frac{2}{N}\sum_{i=1}^{N^+}[x(i\Delta t) + \overset{\circ}{\varepsilon}(i\Delta t)]\cos^+ n\omega(i\Delta t) +$$

$$+ \frac{2}{N}\sum_{i=1}^{N^-}[x(i\Delta t) + \overset{\circ}{\varepsilon}(i\Delta t)]\cos^- n\omega(i\Delta t). \tag{9}$$

Here for stationary random processes with normal distribution the result without error is only when the errors of positive and negative products $\left[x(i\Delta t) + \overset{\circ}{\varepsilon}(i\Delta t)\right]\cos n\omega(i\Delta t)$ are balanced and the equations fulfilled.

$$\frac{2}{N}\sum_{i=1}^{N^+}\overset{\circ}{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t) = \frac{2}{N}\sum_{i=1}^{N^-}\left|\overset{\circ}{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\right|, \tag{10}$$

$$\frac{2}{N}\sum_{i=1}^{N^+}\overset{\circ}{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t) = \frac{2}{N}\sum_{i=1}^{N^-}\left|\overset{\circ}{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\right|. \tag{11}$$

In practice the positive and negative errors compensate one another in most cases. So as it was mentioned above many important problems can be solved in the experimental research when the high accuracy of the obtained results is not required and the equalities (10) and (11) do not take places. But due to the equality (6) the interference causes the considerable influence on the result of the analysis for the wide class of the objects. At the same time the result of the calculation has considerable errors. That causes appearance of the difference between the sums of the positive and negative errors of the pair multiplications i.e.

$$\frac{2}{N}\sum_{i=1}^{N^+}\overset{\circ}{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t) \neq \frac{2}{N}\sum_{i=1}^{N^-}\left|\overset{\circ}{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\right|, \tag{12}$$

$$\frac{2}{N}\sum_{i=1}^{N^-}\overset{\circ}{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t) \neq \frac{2}{N}\sum_{i=1}^{N^-}\left|\overset{\circ}{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\right|, \tag{13}$$

$$\lambda_{a_n} = \frac{2}{N}\sum_{i=1}^{N^+}\overset{\circ}{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t) - \frac{2}{N}\sum_{i=1}^{N}\left|\overset{\circ}{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\right|, \tag{14}$$

$$\lambda_{b_n} = \frac{2}{N}\sum_{i=1}^{N^+}\overset{\circ}{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t) - \frac{2}{N}\sum_{i=1}^{N}\left|\overset{\circ}{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\right|, \tag{15}$$

The values λ_{a_n}, λ_{b_n} are the errors of the obtained estimates. The influence of the interference $\varepsilon(t)$ on the obtained results can be estimated by means of the values $\lambda_{a_n}, \lambda_{b_n}$.

They can be considerably greater than the given value S and in some cases can be commensurable with the unknown value

$$\sum_{n=1}^{r}(\lambda_{a_n} + \lambda_{b_n}) \geq S. \tag{16}$$

It is obvious that in practice solving the numerous problems by means of the spectral method is not satisfactory.

3. Algorithms for Providing Robustness of Estimates a_n, b_n.

As it follows from the above mentioned for the considered cases it is necessary to determine the difference between the sums of the negative and positive microerrors and to provide the condition (2) by means of the balancing of the microerrors for obtaining the satisfactory result of the spectral analysis.

At the same time taking into account the formula (8) the determination of the errors $\lambda_{a_n}, \lambda_{b_n}$ of the estimates a_n, b_n can be represented as follows:

$$\begin{aligned}\lambda_{a_n} = {} & \frac{2}{N}\left[\sum_{i=1}^{N^{++}}\lambda_g^+(i\Delta t)\cos^+ n\omega(i\Delta t) + \sum_{i=1}^{N^{--}}\lambda_g^-(i\Delta t)\cos^- n\omega(i\Delta t)\right] - \\ & -\frac{2}{N}\left[\sum_{i=1}^{N^{+-}}\left|\lambda_g^+(i\Delta t)\cos^- n\omega(i\Delta t)\right| + \sum_{i=1}^{N^{-+}}\left|\lambda_g^-(i\Delta t)\cos^+ n\omega(i\Delta t)\right|\right],\end{aligned} \tag{17}$$

$$\begin{aligned}\lambda_{b_n} = {} & \frac{2}{N}\left[\sum_{i=1}^{N^{++}}\lambda_g^+(i\Delta t)\sin^+ n\omega(i\Delta t) + \sum_{i=1}^{N^{--}}\lambda_g^-(i\Delta t)\sin^- n\omega(i\Delta t)\right] - \\ & -\frac{2}{N}\left[\sum_{i=1}^{N^{+-}}\left|\lambda_g^+(i\Delta t)\sin^- n\omega(i\Delta t)\right| + \sum_{i=1}^{N^{-+}}\left|\lambda_g^-(i\Delta t)\sin^+ n\omega(i\Delta t)\right|\right].\end{aligned} \tag{18}$$

Here $\cos^+ n\omega(i\Delta t)$, $\sin^+ n\omega(i\Delta t)$, $\cos^- n\omega(i\Delta t)$, $\sin^- n\omega(i\Delta t)$ are the samples of the positive and negative half-periods of the n-th cosinusoid and sinusoid respectively; N^{++}, N^{-+}, N^{+-}, N^{--} are the quantity of the errors having sign ++,-+,+-,-- respectively.

As it follows from the expressions (17) and (18) it is necessary to determine the absolute errors $\lambda_g(i\Delta t)$ for balancing the errors of the pair multiplications $\overset{\circ}{g}(i\Delta t)\cdot\cos n\omega t(i\Delta t)$

$$\lambda_g(i\Delta t) = \overline{\lambda}_{\text{rel}}\cdot\overset{\circ}{g}(i\Delta t). \tag{19}$$

Here the arithmetic mean value of relative errors $\overline{\lambda}_{rel}$ of the samples $\overset{\circ}{g}(i\Delta t)$ is determined as follows

$$\overline{\lambda}_{rel} = \frac{\frac{1}{N}\sum_{i=1}^{N}\overset{\circ}{\varepsilon}(i\Delta t)}{\frac{1}{N}\sum_{i=1}^{N}\overset{\circ}{g}(i\Delta t)} . \tag{20}$$

It is obvious that the determination of the relative errors $\overline{\lambda}_{rel}$ by means of that formula is impossible because here only samples $\overset{\circ}{g}(i\Delta t)$ are known but the value of the interference $\overset{\circ}{\varepsilon}(i\Delta t)$ is unknown and its determination is impossible. At the same time the value of the relative error of the samples $\lambda_{rel}(i\Delta t)$ can be determined as follows:

$$\lambda_{\text{rel}}(i\Delta t) = \frac{\overset{\circ}{\varepsilon}(i\Delta t)}{\overset{\circ}{g}(i\Delta t)} = \frac{\sqrt{\overset{\circ}{\varepsilon}{}^2(i\Delta t)}}{\sqrt{\overset{\circ}{g}{}^2(i\Delta t)}} . \tag{21}$$

If we replace the values $\overset{\circ}{\varepsilon}{}^2(i\Delta t)$ and $\overset{\circ}{g}{}^2(i\Delta t)$ with their arithmetic mean values the value of the relative error can be represented as follows:

$$\overline{\lambda}_{\text{rel}} = \frac{1}{N}\sum_{i=1}^{N}\lambda_{\text{rel}}(i\Delta t) \approx \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\overset{\circ}{\varepsilon}{}^2(i\Delta t)}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\overset{\circ}{g}{}^2(i\Delta t)}} . \tag{22}$$

At the same time according to the expression (22) it is necessary to determine the dispersion D_ε of the interference $\overset{\circ}{\varepsilon}(i\Delta t)$ for determining the relative error $\overline{\lambda}_{rel}$. For that purpose can be used by means of the formula

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\left(\overset{\circ}{g}{}^2(i\Delta t) + \overset{\circ}{g}(i\Delta t)\overset{\circ}{g}(i\Delta t) - 2\overset{\circ}{g}(i\Delta t)\overset{\circ}{g}(i\Delta t)\right).$$

Then $\overline{\lambda}_{rel}$ can be represented as follows

$$\overline{\lambda}_{rel} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\overset{\circ}{g}(i\Delta t)g(i\Delta t) + g((i+2)\Delta t)\overset{\circ}{g}(i\Delta t) - 2\overset{\circ}{g}(i\Delta t)g((i+1)\Delta t)\right]}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\overset{\circ}{g}{}^2(i\Delta t)}} . \tag{23}$$

So the arithmetic mean relative error $\overline{\lambda}_{rel}$ of the samples $\overset{\circ}{g}(i\Delta t)$ is determined as follows

$$\overline{\lambda}_{\text{rel}} = \frac{\sqrt{D_\varepsilon}}{\sqrt{D_g}} . \tag{24}$$

It is obvious that the value of the microerrors can be easily determined by the $\lambda_g(i\Delta t)$ by means of the formula (19) as follows

$$\lambda_{a_n}(i\Delta t) = \lambda_g(i\Delta t)\cdot\cos n\omega(i\Delta t),$$

$$\lambda_{b_n}(i\Delta t) = \lambda_g(i\Delta t)\cdot\sin n\omega(i\Delta t)$$

for the sinusoids and cosinusoids.

At that time it is possible to determine the value of the improvement of the robustness

$$\lambda_a^R = \lambda_{a_n}^+ - \lambda_{a_n}^- = \left[\sum_{i=i_1^{++}}^{N^{++}} \lambda_{a_n}^{++}(i\Delta t) + \sum_{i=i_1}^{N^{--}} \lambda_{a_n}^{--}(i\Delta)\right] - $$

$$- \left[\sum_{i=i_1^{+-}}^{N^{+-}} \lambda_{a_n}^{+-}(i\Delta t) + \sum_{i=i_1^{-+}}^{N^{-+}} \lambda_{a_n}^{-+}(i\Delta)\right], \qquad (25)$$

$$\lambda_b^R = \lambda_{b_n}^+ - \lambda_{b_n}^- = \left[\sum_{i=i_1^{++}}^{N^{++}} \lambda_{b_n}^{++}(i\Delta t) + \sum_{i=i_1^{--}}^{N^{--}} \lambda_{b_n}^{--}(i\Delta)\right] - $$

$$- \left[\sum_{i=i_1^{+-}}^{N^{+-}} \lambda_{b_n}^{+-}(i\Delta t) + \sum_{i=i_1^{-+}}^{N^{-+}} \lambda_{b_n}^{-+}(i\Delta)\right]. \qquad (26)$$

Here $i^{++}, i^{-+}, i^{+-}, i^{--}$ and N^{++}, N^{-+}, N^{+-}, N^{--} are the indexes of the summing and the quantity of the multipliers having signs ++,-+,+-,--.

So the robust formulae of the determination of the coefficients of the Fourier series can be represented as follows

$$a_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N}[\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t) - \lambda_a^R]\right\}, \qquad (27)$$

$$b_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N}[\overset{\circ}{g}(i\Delta t)\sin n\omega(i\Delta t) - \lambda_b^R]\right\}. \qquad (28)$$

It is understood that it is necessary to determine the sign of the error of the samples beforehand for the realization of the expressions (25), (26). It is obvious that the information about the sign of the error can not be obtained in the process of the calculation. At the same time the sign of the error of the sample $\overset{\circ}{g}(i\Delta t)$ can be approximately determined by the sign of the increment of the sample i.e.

$$\Delta g(i\Delta t) = \overset{\circ}{g}(i\Delta t) - \overset{\circ}{g}[(i-1)\Delta t]. \qquad (29)$$

But the sign of the increment can be formed by the influence of the error and by increasing or decreasing the useful signal. In that connection the obtained value of the robustness depend on the character of the change of the useful signal and the interference. Otherwise it is necessary to know the sign of the interference $\varepsilon(i\Delta t)$ for the realization of that algorithm but that is impossible. Due to that the efficiency of the use of the algorithms (25)-(28) is not high.

So it is necessary to change the robust algorithms and eliminate that disadvantage. At the same time those algorithms must be 'technological' i.e. they must be convenient for the mass use by means of the PC.

4. Robust Technology of Determining Coefficients of Fourier Series.

It is easy to show that assuming the equalities

$$\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} = \frac{1}{N}\sum_{i=1}^{N} \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t), \qquad (30)$$

$$\overline{\overset{\circ}{g}(i\Delta t)\sin n\omega(i\Delta t)} = \frac{1}{N}\sum_{i=1}^{N}\overset{\circ}{g}(i\Delta t)\sin n\omega(i\Delta t) \tag{31}$$

and taking into account the signs of the samples of the signal $g(i\Delta t)$ and cosinusoids $\cos n\omega(i\Delta t)$ the difference between the microerrors of the coefficients a_n of the Fourier series can be represented as follows [2,3]

$$\lambda_a^R = \lambda_{a_n}^+ - \lambda_{a_n} = \left[N_{a_n}^{++}\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} + N_{a_n}\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} \right] -$$

$$-\left[N_{a_n}^{+-}\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} + N_{a_n}^{-+}\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} \right]. \tag{32}$$

Taking into account the equalities

$$N^+ = N^{++} + N^{--}, \tag{33}$$

$$N^- = N^{+-} + N^{+-}, \tag{34}$$

that difference can be represented as follows

$$\lambda_{a_n}^R = \alpha_n (N_{a_n}^+ - N_{a_n}^-)\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}, \tag{35}$$

$$\lambda_{b_n}^R = \beta_n (N_{b_n}^{++} - N_{b_n}^-)\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\sin n\omega(i\Delta t)}, \tag{36}$$

$$\alpha_n = \beta_n \approx \frac{1}{2} \div \frac{1}{4}.$$

Taking into account that the algorithms of the spectral analysis are widely used in many areas of the science and engineering it is advisable to create the technology of the robust spectral analysis allowing to obtain more accurate estimates a_n and b_n than obtained by the formulae (35) and (36) for the mass use of that algorithms.

It is connected with that there is the part of the error in the expressions (35) and (36) caused by the inequality $\Pi^+ \neq \Pi^-$ that was not taken into account. For that purpose it is advisable to determine the mean value of the multiplication $\Pi = \overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}$ and the mean value of the positive $\Pi^+ = \overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^+$ and negative $\Pi^- = \overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^-$ multiplication simultaneously in the process of the calculation of the sum $\sum_{i=1}^{N}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)$ and also to determine their quantities N, N^+ and N^- for the realization of the traditional algorithms.

At the same time for the case when $N^+ = N^-$ and $\Pi^+ = \Pi^-$ the ignored error is equal to $\alpha_n N\left[\overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^+ - \overline{\lambda}_{rel}\overline{\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^- \right]$.

If $N^+ > N^-$ and $\Pi^+ > \Pi^-$ that value is equal to $\alpha_n\left[N-\left(N^+_{a_n}-N^-_{a_n}\right)\right]\left[\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} - \overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right]$. And if $N^+ < N^-$ and $\Pi^+ < \Pi^-$ that value is equal to

$$\alpha_n\left[N-\left(N^-_{a_n}-N^+_{a_n}\right)\right]\left[\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} - \overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right].$$

Taking into account those equalities the formula (35) for determining $\lambda^R_{a_n}$ can be represented as follows:

$$\lambda^R_{a_n}=\begin{cases}0 & \text{for } N^+=N^- \text{ and } \Pi^+=\Pi^-, \\ \alpha_n(N^+_{a_n}-N^-_{a_n})\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}+ & \\ +\alpha_n N\left[\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\right. & \\ \left.-\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] & \text{for } N^+=N^- \text{ and } \Pi^+\neq\Pi^-, \\ \alpha_n\left(N^+_{a_n}-N^-_{a_n}\right)\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}+ & \\ +\alpha_n\left[N-\left(N^+_{a_n}-N^-_{a_n}\right)\right]\left[\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\right. & \\ \left.-\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] & \text{for } N^+>N^- \text{ and } \Pi^+>\Pi^-, \\ \alpha_n\left(N^-_{a_n}-N^+_{a_n}\right)\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}+ & \\ +\alpha_n\left[N-\left(N^-_{a_n}-N^+_{a_n}\right)\right]\left[\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\right. & \\ \left.-\overline{\overline{\lambda}_{rel}\overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] & \text{for } N^+<N^- \text{ and } \Pi^+<\Pi^-. \end{cases} \quad (37)$$

The combination of the sequence of the procedures presenting the robust technology of the spectral analysis is represented below by means of the expression (37).

1. The dispersion of the interference D_ε and the arithmetic mean value of the relative error of the samples $\overline{\lambda}_{rel}$ are determined.
2. The values Π^+, Π^-, N^+ и N^- are determined.
3. The conditions $N^+ = N^-$ and $\Pi^+ = \Pi^-$ for which the use of the traditional algorithms is recommended are checked.
4. If $N^+ \neq N^-$ and $\Pi^+ = \Pi^-$ the following forjmula is used for determining the robust estimates a^R_n

$$a_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N} \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t) - \alpha_n \left| N_{a_n}^{+} - N_{a_n}^{-} \right| \overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}\right\}.$$

5. If $N^+ > N^-$ and $\Pi^+ \neq \Pi^-$ the estimates a_n^R are determined by means of the following expression

$$a_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N} \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t) - \alpha_n \left(N_{a_n}^{+} - N_{a_n}^{-}\right) \overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} - \right.$$
$$\left. - \alpha_n \left[N - \left(N_{a_n}^{+} - N_{a_n}^{-}\right)\right]\left[\overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} - \overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right]\right\}.$$

6. If $N^+ < N^-$ and $\Pi^+ \neq \Pi^-$ the estimates a_n^R are determined by means of the following expression

$$a_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N} \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t) - \alpha_n \left(N_{a_n}^{-} - N_{a_n}^{+}\right) \overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-} - \right.$$
$$\left. - \alpha_n \left[N - \left(N_{a_n} - N_{a_n}^{-}\right)\right]\left[\overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)} - \overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right]\right\}.$$

7. If $N^+ = N^-$ and $\Pi^+ \neq \Pi^-$ the estimates a_n^R are determined by means of the following formula

$$a_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N} \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t) - \right.$$
$$\left. - \alpha_n N\left[\overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} - \overline{\lambda_{rel}\, \overset{\circ}{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right]\right\}.$$

The determination of the robust estimates b_n is analogous.

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