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ASYMPTOTIC ANALISYS OF A MIXED PROBLEM OF ELASTICITY THEORY FOR RADIAL-INHOMOGENEOUS CYLINDER OF SMALL THICKNESS

Abstract

In the present paper the spatial stress-strain state of a radial inhomogeneous cylinder of small thickness is investigated by the method of direct asymptotic integration of elasticity theory equations. Inhomogeneous and homogeneous solutions are constructed. The character of stress-strain state is explained on the base of qualitative analysis.

Spatial stress-strain state of a radial inhomogeneous cylinder of small thickness is investigated by the method of direct asymptotic integration of elasticity theory equations. Inhomogeneous and homogeneous solutions are constructed. The character of stress-strain state is explained on the base of qualitative analysis.

Consider the axisymmetric problem of elasticity theory for a radial inhomogeneous cylinder of small thickness. Relate the cylinder to the cylindrical system of coordinates r, θ, z:

$$r_1 \le r \le r_2,\ 0 \le \theta \le 2\pi,\ -L \le z \le L$$

Introduce the new dimensionless variables ρ and ξ:

$$\rho = \frac{1}{\varepsilon}\ln\left(r/r_0\right); \qquad \xi = \frac{z}{r_0},$$

where $\varepsilon = \frac{1}{2}\ln\left(r_2/r_1\right)$ is a small parameter characterizing the thickness of the cylinder $r_0 = \sqrt{r_1 r_2}$. Note that $\rho \in [-1;1]$; $\xi \in [-l;l]$ $\left(l = \frac{L}{r_0}\right)$.

We'll assume that the elastic Lame parameters $G = G(\rho)$, $\lambda = \lambda(\rho)$ are positive piecewise continuous functions of the variable ρ.

The equilibrium equations in the displacements have the form:

$$\left(L_0 + \varepsilon\partial\left(L_1 + L_2\right) + \varepsilon^2\partial^2 L_3\right)\bar{u} = 0 \tag{1.1}$$

Here $u = \left(u_\rho, u_\xi\right)^T$, u_ρ, u_ξ are the components of permutations vector, L_k are matrix differential operators of the form:

$$L_0 = \begin{pmatrix} \partial_1\left(\left(H\partial_1 + \varepsilon\lambda\right)e^{-\varepsilon\rho}\right) + 2G\left(\varepsilon\partial_1 - \varepsilon^2\right)e^{-\varepsilon\rho} & 0 \\ 0 & \partial_1\left(Ge^{-\varepsilon\rho}\partial_1\right) + \varepsilon Ge^{-\varepsilon\rho}\partial_1 \end{pmatrix}$$

$$L_1 = \begin{pmatrix} 0 & G\partial_1 + \partial_1(\lambda) \\ 0 & 0 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 0 & 0 \\ \partial_1 (G) + \lambda \partial_1 + (G+\lambda)\varepsilon & 0 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} Ge^{\varepsilon\rho} & 0 \\ 0 & He^{\varepsilon\rho} \end{pmatrix}$$

$$\partial = \frac{\partial}{\partial \xi};\ \partial_1 = \frac{\partial}{\partial \rho};\ H = 2G + \lambda$$

Assume that on lateral surfaces of the cylinder the following mixed boundary conditions are given:

$$\left.\bar{\sigma}\right|_{\rho=\pm 1} = (M_0 + \varepsilon \partial M_1) \left.\bar{u}\right|_{\rho=\pm 1} = \bar{q}^{\pm}(\xi), \tag{1.2}$$

where $\bar{\sigma} = (u_\rho, \sigma_{\rho\xi})^T$; $\bar{q}^{\pm}(\xi) = (h^{\pm}(\xi); f^{\pm}(\xi))^T$;

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & Ge^{-\varepsilon\rho}\varepsilon^{-1}\partial_1 \end{pmatrix};\ M_1 = \begin{pmatrix} 0 & 0 \\ G & 0 \end{pmatrix}.$$

Assume that $h^{\pm}(\xi)$, $f^{\pm}(\xi)$ are sufficiently smooth functions and relative to ε have order $O(1)$.

2. Consider construction of partial solutions of equations (1.1), satisfying the boundary conditions (1.2), i.e., nonhomogeneous solutions.

Assuming, that the value ε is sufficiently small for constructing nonhomogeneous solutions we use the asymptotic method [1].

We'll find solution (1.1)-(1.2) in the following way:

$$\begin{aligned} u_\rho &= u_{\rho 0} + \varepsilon u_{\rho 1} + \ldots \\ u_\xi &= \varepsilon^{-1}(u_{\xi 0} + \varepsilon u_{\xi 1} + \ldots) \end{aligned} \tag{2.1}$$

Substitution (2.1) into (1.1), (1.2) leads to the system whose sequential integration by ρ gives the relation for the coefficients of the expansion u_ρ, u_ξ:

$$u_{\rho 0} = e_1(\rho)\, d_1'(\xi) - \frac{h(\xi)}{l_0} \int_{-1}^{\rho} \frac{1}{H(x)} dx + h^{-}(\xi)$$

$$u_{\xi 0} = d_1(\xi)$$

$$u_{\rho 1} = \left(t_1(\rho) - \int_{-1}^{\rho} \frac{\lambda(x)}{H(x)} x dx + \frac{p_0}{l_0} e_2(\rho) + t_7 \int_{-1}^{\rho} \frac{1}{H(x)} dx \right) \times$$

$$\times d_1'(\xi) + e_1(\rho)\, d_2'(\xi) - \frac{1}{l_0} h(\xi)\, e_2(\rho) - \frac{t_6 h(\xi)}{l_0^2} \int_{-1}^{\rho} \frac{1}{H(x)} dx + e_1(\rho)\, h^{-}(\xi);$$

$$u_{\xi 1} = d_2(\xi);$$

where $\left(g_0+\frac{p_0^2}{l_0}\right)d_1''(\xi)=f(\xi)+\frac{p_0}{l_0}h'(\xi)\,;$

$$\left(g_0+\frac{p_0^2}{l_0}\right)d_2''(\xi)=\frac{p_0t_6}{l_0^2}h'(\xi)-\left(t_0+\frac{p_0^2}{l_0}\right)\left(h^-(\xi)\right)'-f^-(\xi)-$$

$$-f^+(\xi)-\frac{2p_1+t_4}{p_0}f(\xi)-\frac{l_0f(\xi)+p_0h'(\xi)}{p_0^2+l_0g_0}\left(2g_1+t_3+p_0t_7-\frac{g_0}{p_0}(2p_1+t_4)\right);$$

$$h(\xi)=h^-(\xi)-h^+(\xi)\,;\ f(\xi)=f^-(\xi)-f^+(\xi)\,;\ g_k=\int_{-1}^{1}\frac{4G(G+\lambda)}{H}\rho^kd\rho;$$

$$l_k=\int_{-1}^{1}\frac{1}{H(\rho)}\rho^kd\rho;p_k=\int_{-1}^{1}\frac{\lambda(\rho)}{H(\rho)}\rho^kd\rho$$

$$t_1(\rho)=\int_{-1}^{\rho}\frac{1}{H(y)}\left(\int_{-1}^{y}\frac{2G(x)\lambda(x)}{H(x)}dx\right)dy+\int_{-1}^{\rho}\frac{\lambda(y)}{H(y)}\left(\int_{-1}^{y}\frac{\lambda(x)}{H(x)}dx\right)dy$$

$$t_2(\rho)=\int_{-1}^{\rho}\frac{1}{H(y)}\left(\int_{-1}^{y}\frac{2G(x)}{H(x)}dx\right)dy+\int_{-1}^{\rho}\frac{\lambda(y)}{H(y)}\left(\int_{-1}^{y}\frac{1}{H(x)}dx\right)dy$$

$$t_3(\rho)=\int_{-1}^{1}\frac{\lambda(\rho)}{H(\rho)}\left(\int_{-1}^{\rho}\frac{2G(x)\lambda(x)}{H(x)}dx\right)d\rho-\int_{-1}^{1}\frac{2G(\rho)\lambda(\rho)}{H(\rho)}\left(\int_{-1}^{\rho}\frac{\lambda(x)}{H(x)}dx\right)d\rho$$

$$t_4(\rho)=\int_{-1}^{1}\frac{2G(\rho)\lambda(\rho)}{H(\rho)}\left(\int_{-1}^{1}\frac{1}{H(x)}dx\right)d\rho-\int_{-1}^{1}\frac{\lambda(\rho)}{H(\rho)}\left(\int_{-1}^{\rho}\frac{2G(x)}{H(x)}dx\right)d\rho$$

$$t_5=t_1(1)-p_1;\ t_6=t_2(1)-l_1;\ t_7=\frac{p_0}{l_0^2}t_6-\frac{1}{l_0}t_5$$

$$e_1(\rho)=\frac{p_0}{l_0}\int_{-1}^{\rho}\frac{1}{H(x)}dx-\int_{-1}^{\rho}\frac{\lambda(x)}{H(x)}dx;\ e_2(\rho)=\int_{-1}^{\rho}\frac{x}{H(x)}dx-t_2(\rho)$$

3. Consider the question on construction of homogeneous solutions. Assume in (1.2) $\bar{q}^{\pm}(\xi)=\bar{0}$. Finding the solution of homogeneous systems in the form:

$$u_\rho(\rho,\xi)=u(\rho)\,m'(\xi)\,;\qquad u_\xi(\rho,\xi)=w(\rho)\,m(\xi)$$

where $m''(\xi)-\mu^2m(\xi)=0$ after the division of variables we obtain the not self-adjoint spectral problem:

$$\begin{array}{c}\left(L_0+\varepsilon L_1+\mu^2\varepsilon\left(L_2+\varepsilon L_3\right)\right)\bar{\vartheta}=\bar{0}\\ \left(M_0+\mu^2 M_1\right)\vartheta|_{\rho=\pm1}=\bar{0}\end{array} \tag{3.1}$$

Here $\bar{\vartheta}(\rho)=(u,w)^T$.

Homogeneous solutions, corresponding to the first iteration process we can obtain from the formulas for inhomogeneous solutions if in them we put $\bar{q}^{\pm}(\xi)=\bar{0}$. We have

$$u_\rho^{(1)}=\varepsilon C_0\left\{\frac{l_0}{p_0}e_1(\rho)+\varepsilon\left[\frac{l_0}{p_0}\left(t_1(\rho)-\int_{-1}^{\rho}\frac{\lambda(x)}{H(x)}xdx\right)+\right.\right.$$

$$\left.\left.+\int_{-1}^{\rho}\frac{x}{H(x)}dx-t_2(\rho)-\frac{l_0^2t_7}{p_0^2}\int_{-1}^{\rho}\frac{\lambda(x)}{H(x)}dx\right]+O\left(\varepsilon^2\right)\right\}$$

$$u_\xi^{(1)}=\left(C_0\xi+D_0\right)\left[\frac{l_0}{p_0}-\varepsilon\frac{l_0^2t_7}{p_0^2}+O\left(\varepsilon^2\right)\right] \tag{3.2}$$

Here the constant D_0 corresponds to the permutation of the cylinder as a solid body. Therefore we can assume $D_0=0$.

The eigen-values $\mu=0$ correspond to these solutions.

There is no the second iteration process here.

According to the third iteration process the solution of spectral problem (3.1) we find in the following form:

$$\begin{array}{c}u^{(3)}=\varepsilon^2\mu_0^{-1}\left(a_0+\varepsilon a_1+...\right)\\ w^{(3)}=\varepsilon\left(b_0+\varepsilon b_1+...\right)\\ \mu=\varepsilon^{-1}\left(\mu_0+\varepsilon\mu_1+...\right)\end{array} \tag{3.3}$$

After substitution (3.3) into (3.1) for the first members of expansion we obtain the spectral problem:

$$A\left(\mu_0\right)\bar{f}_0=\bar{0}, \tag{3.4}$$

where $A\left(\mu_0\right)\bar{f}_0=\left\{t\left(\mu_0\right)\bar{f}_0,\ M_2\bar{f}_0\big|_{\pm1}=\bar{0}\right\}$; $t\left(\mu_0\right)\bar{f}_0=\left(A_0+\mu_0A_1+\mu_0^2A_2\right)\bar{f}_0$.

$$A_0=\begin{pmatrix}\partial_1\left(H\partial_1\right) & 0\\ 0 & \partial_1\left(G\partial_1\right)\end{pmatrix},\qquad A_1=\begin{pmatrix}0 & \partial_1\left(\lambda\right)+G\partial_1\\ \partial_1\left(G\right)+\lambda\partial_1 & 0\end{pmatrix}$$

$$A_2=\begin{pmatrix}G & 0\\ 0 & H\end{pmatrix};\qquad M_2=\begin{pmatrix}1 & 0\\ 0 & G\partial_1\end{pmatrix};\qquad \bar{f}_0=\left(a_0;b_0\right)^T$$

This spectral problem describes the potential solution of inhomogeneous by the thickness plate which is studied in [2-4].

At the next stage of asymptotic integration for $\bar{f}_1=\left(a_1;b_1\right)^T$ and μ_1 we obtain:

$$\left\{\begin{array}{c}t\left(\mu_0\right)\bar{f}_1=t_1\left(\mu_0,\mu_1\right)\bar{f}_0\\ M_2\bar{f}_1\big|_{\pm1}=\bar{0}\end{array}\right.$$

where $t_1(\mu_0,\mu_1)=\rho A_0+A_3-\mu_0 A_4-\rho\mu_0^2 A_2-2\mu_1(\mu_0 A_2+A_5)$

$$A_3=\begin{pmatrix}\lambda\partial_1-\partial_1(\lambda) & 0\\ 0 & 0\end{pmatrix};\quad A_4=\begin{pmatrix}0 & 0\\ (G+\lambda) & 0\end{pmatrix};$$

$$A_5=\begin{pmatrix}0 & 0\\ \partial_1(G)+\lambda\partial_1 & 0\end{pmatrix}$$

The solvability condition of this problem is orthogonality of the right side of solution to conjugate problem:

$$A^*(\mu_0)\bar{f}_0^*=A(-\bar{\mu}_0)\bar{f}_0^*=\bar{0},$$

where $\bar{f}_0^*=\left(\bar{a}_0^*;\bar{b}_0^*\right)^T$.

For μ_1 we determine:

$$\mu_1=\frac{E_2}{E_1}$$

$$E_1=\int_{-1}^{1}\left[2\mu_0\left(Ga_0\bar{a}_0^*+Hb_0\bar{b}_0^*\right)+2\lambda a_0'\bar{b}_0^*-2Ga_0\left(\bar{b}_0^*\right)'\right]d\rho;$$

$$E_2=\int_{-1}^{1}\Big[\rho\left(Ha_0'\right)'\bar{a}_0^*+\lambda a_0'\bar{a}_0^*-\mu_0(G+\lambda)a_0\bar{b}_0^*+\lambda a_0\left(\bar{a}_0^*\right)'-$$

$$-\rho Gb_0'\left(\bar{b}_0^*\right)'-Gb_0'\bar{b}_0^*-\mu_0^2\rho\left(Ga_0\bar{a}_0^*+Hb_0'\bar{b}_0^*\right)\Big]d\rho$$

Solutions corresponding to the third iteration process have the form:

$$u_\rho^{(3)}=\varepsilon^2\sum_{k=1}^{\infty}\left[-\mu_{0k}^{-4}\left(\tau_0\psi_k''\right)'-2\mu_{0k}^{-2}\tau_1\psi_k'+\mu_{0k}^{-2}\left(\tau_2\psi_k\right)'+O(\varepsilon)\right]m_k'(\xi) \qquad (3.5)$$

$$u_\xi^{(3)}=\varepsilon\sum_{k=1}^{\infty}\left[\tau_0\mu_{0k}^{-2}\psi_k''-\tau_2\psi_k+O(\varepsilon)\right]m_k(\xi).$$

Here $m_k(\xi)=D_k e^{\mu_k\xi}+C_k e^{-\mu_k\xi}$; $\psi_k(\eta)$ is a solution of generalized spectral Papkovich theorem for inhomogeneous case [2-4]:

$$\left(\tau_0\psi_k''\right)''+\mu_{0k}^2\left(2\left(\tau_1\psi_k'\right)'-\left(\tau_2\psi_k\right)''-\tau_2\psi_k''\right)+\mu_{0k}^4\tau_0\psi_k=0$$

$$\begin{cases}\left(\tau_0\psi_k''\right)'-\mu_{0k}^2\left(\tau_2\psi_k\right)'=0\\ \psi_k'=0\end{cases}\quad \text{at } \rho=\pm 1,$$

where $\tau_0=\frac{H}{4G(G+\lambda)}$; $\tau_1=\frac{1}{2G}$; $\tau_2=\frac{\lambda}{4G(G+\lambda)}$.

4. On the basis of above mentioned analysis we indicate the character of constructed problems.

Investigate the connection of homogeneous solutions with the main stresses vector P, acting in the section $\xi = const$.

We have

$$P = 2\pi \int_{r_1}^{r_2} (\sigma_{zz} + \sigma_{rz})\, r dr = 2\pi r_0^2 \varepsilon \int_{-1}^{1} (\sigma_{\xi\xi} + \sigma_{\rho\xi})\, e^{2\varepsilon\rho} d\rho; \tag{4.1}$$

We represent the permutations in the following form:

$$u_\rho = u_\rho^{(1)} + u_\rho^{(3)}; \quad u_\xi = u_\xi^{(1)} + u_\xi^{(3)} \tag{4.2}$$

$$u_\rho^{(3)} = \sum_{k=1}^{\infty} u_k(\rho)\, m_k'(\xi); \quad u_\xi^{(3)} = \sum_{k=1}^{\infty} w_k(\rho)\, m_k(\xi) \tag{4.3}$$

For the stresses we obtain:

$$\sigma_{\xi\xi} = \sigma_{\xi\xi}^{(1)} + \sigma_{\xi\xi}^{(3)}; \quad \sigma_{\rho\xi} = \sigma_{\rho\xi}^{(1)} + \sigma_{\rho\xi}^{(3)} \tag{4.4}$$

$$\sigma_{\xi\xi}^{(1)} = C_0 \left[\frac{\lambda}{H} + \frac{l_0}{\tau_0 p_0} + O(\varepsilon) \right]; \quad \sigma_{\rho\xi}^{(1)} = 0 \tag{4.5}$$

$$\sigma_{\xi\xi}^{(3)} = \sum_{k=1}^{\infty} \sigma_{1k}(\rho)\, m_k'(\xi); \; \sigma_{\rho\xi}^{(3)} = \sum_{k=1}^{\infty} \sigma_{2k}(\rho)\, m_k(\xi) \tag{4.6}$$

where $\sigma_{1k}(\rho) = Hw_k + \lambda \left(\frac{1}{\varepsilon} u_k' + u_k \right) e^{-\varepsilon\rho}$; $\sigma_{2k}(\rho) = G \left(\mu_k^2 u_k + \frac{e^{-\varepsilon\rho}}{\varepsilon} w_k' \right)$.

Substituting (4.4) into (4.1) we obtain

$$P = 2\pi r_0^2 \varepsilon C_0 w_0 + 2\pi r_0^2 \varepsilon \sum_{k=1}^{\infty} w_k \tag{4.7}$$

$$w_0 = p_0 + \frac{g_0 l_0}{p_0} + O(\varepsilon); \quad w_k = b_{1k} m_k(\xi) + b_{2k} m_k'(\xi)$$

$$b_{1k} = \int_{-1}^{1} \sigma_{1k}(\rho)\, e^{2\varepsilon\rho} d\rho; \qquad b_{2k} = \int_{-1}^{1} \sigma_{2k}(\rho)\, e^{2\varepsilon\rho} d\rho;$$

Let's prove that all $w_k = 0 \quad (k = 1, 2, ...)$. For this let's consider the mixed boundary-value problem:

$$\sigma_{\xi\xi}\big|_{\xi=\xi_j} = \sigma_{1k}(\rho)\, m_k'(\xi_j); \quad \sigma_{\rho\xi}\big|_{\xi=\xi_j} = \sigma_{2k}(\rho)\, m_k(\xi_j) \text{ where } j = 1, 2 \tag{4.8}$$

The solution of this problem are the "k"-th addends in sums of formulae (4.6). The main vector which corresponds to stress state of problem (4.8) in the section $\xi = const$ is reduced to the following form:

$$P_k = 2\pi r_0^2 \varepsilon \left(b_{1k} m_k(\xi) + b_{2k} m_k'(\xi) \right) \tag{4.9}$$

According to the solvability condition of problem of elasticity theory the main vector P_k shouldn't depend on the variable ξ. However in relation (4.9) the right side depends on ξ. Hence, it follows that $P_k = 0$, i.e., $b_{1k} m_k(\xi) + b_{2k} m_k'(\xi) = 0$. By virtue of linear independence $m_k(\xi)$ and $m_k'(\xi)$ we have $b_{1k} = b_{2k} = 0$.

Thus $w_k = 0 \ \ (k = 1, 2, ...)$. For the main vector we obtain:

$$P = 2\pi r_0^2 \varepsilon C_0 w_0 \tag{4.10}$$

The stress state corresponding to the third group of solutions is self-balanced in each section $\xi = const$.

Solution (3.2) corresponding to the first asymptotic process determines the internal stress-strain state of a shell. The first members of its expansion in ε determine the momentless stress state. The third asymptotic process determines solution (3.5) that have the character of boundary layer. The first members of (3.5) are completely equivalent to the boundary Saint-Venant effect of inhomogeneous plate [2;3].

5. Assume the following boundary condition are given on the edges of the cylinder:

$$u_\xi = q_1^{\pm}(\rho), \quad \sigma_{\rho\xi} = q_2^{\pm}(\rho), \quad \text{at } \xi = \pm l \tag{5.1}$$

Here $q_1^{\pm}(\rho)$, $q_2^{\pm}(\rho)$ are sufficiently smooth functions and satisfy equilibrium conditions.

As it is shown not-self balanced part (5.1) we can take off by means of penetrating solution (3.2), moreover connection of the constant C_0 with the main vector P is given by equality (4.8).

Further we'll assume that $P = 0$. By virtue of accepted assumption $C_0 = 0$.

We'll seek the solution in the form (4.3). For finding unknown constants C_k and D_k we use Betti theorem [5]. Let $u_\xi^{(i)}, u_\rho^{(i)}, \sigma_{\xi\xi}^{(i)}, \sigma_{\rho\xi}^{(i)} \quad (i = 1, 2)$ be permutations and stresses of the first and second state. Then according to Betti theorem on the section $\xi = const$ the equality

$$\int_{-1}^{1} \left(u_\xi^{(1)} \sigma_{\xi\xi}^{(2)} + \sigma_{\rho\xi}^{(2)} u_\rho^{(1)} \right) e^{2\varepsilon\rho} d\rho = \int_{-1}^{1} \left(u_\xi^{(2)} \sigma_{\xi\xi}^{(1)} + u_\rho^{(2)} \sigma_{\rho\xi}^{(1)} \right) e^{2\varepsilon\rho} d\rho \tag{5.2}$$

is true.

As the first state we'll take the "k"-th elementary solution, and as the second one the "n"-th elementary solution. Substituting (4.3), (4.6) into (5.2) we obtain:

$$m_k(\xi)\, m_n'(\xi) \int_{-1}^{1} (w_k \sigma_{1n} - u_n \sigma_{2k})\, e^{2\varepsilon\rho} d\rho +$$

$$+ m_n(\xi)\, m_k'(\xi) \int_{-1}^{1} (\sigma_{2n} u_k - w_n \sigma_{1k})\, e^{2\varepsilon\rho} d\rho = 0$$

Since this equality is true at any ξ, then we obtain conditions of generalized orthogonality:

$$\int_{-1}^{1} (w_k\sigma_{1n} - u_n\sigma_{2k})\, e^{2\varepsilon\rho} d\rho = 0 \qquad k \neq n \tag{5.3}$$

Satisfying boundary conditions (5.1) by means of (5.3) we find unknown constants D_k and C_k:

$$D_k = \frac{z^+ e^{\mu_k l} - z^- e^{-\mu_k l}}{2sh\,(2\mu_k l)}; \quad C_k = \frac{z^- e^{\mu_k l} - z^+ e^{-\mu_k l}}{2sh\,(2\mu_k l)}$$

where

$$z^{\pm} = \frac{\int_{-1}^{1} \left(q_1^{\pm}\sigma_{1k} - q_2^{\pm} u_k\right) e^{2\varepsilon\rho} d\rho}{\int_{-1}^{1} (w_k\sigma_{1k} - \sigma_{2k} u_k)\, e^{2\varepsilon\rho} d\rho}.$$

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