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ESTIMATION OF THE SOLUTION TO CAUCHY PROBLEM FOR A CORRECT BY PETROVSKY EQUATION

Abstract

Behavior of solution to the Cauchy problem for a correct by Petrovsky equation for large values of time is studied.

Introduction. While studying propagation of perturbations in liquid gas, when there is no viscosity there arises an equation

$$\left(\frac{\partial^2}{\partial t^2}-\omega\frac{\partial}{\partial t}\Delta_3-a^2\Delta_3\right)u\left(x,t\right)=f\left(x,t\right),\quad x\in R_3,\quad t>0,\tag{0.1}$$

where $\Delta_3\dfrac{\partial^2}{\partial x_1^2}+\dfrac{\partial^2}{\partial x_2^2}+\dfrac{\partial^2}{\partial x_3^2}$, a is sound velocity in gas, $\omega=\dfrac{4}{3}v, v$ is kinematic coefficient of viscosity [1]. In the paper [2], as the result of investigation of the solution of the Cauchy problem for an operator-differential equation, a sufficient condition of uniform stabilization for large values of time of the Cauchy problem solution for equation (0.1) with periodic initial data is given. In the paper [3] behavior of the Cauchy problem solution is studied for large values of time for the equation

$$\left(\frac{\partial^2}{\partial t^2}-\omega\frac{\partial}{\partial t}\Delta_n-a^2\Delta_n\right)u\left(x,t\right)=f\left(x,t\right),\tag{0.2}$$

where Δ_n is a Laplace operator with respect to $x\left(x_1,x_2,...,x_n\right)$.

1. Representation of the solution to the Cauchy problem and auxiliary statements

We'll study behavior of the Cauchy problem solution for large values of time for an equation more general than equation (0.2).

Consider the following Cauchy problem

$$\left(\frac{\partial^2}{\partial t^2}-\omega\frac{\partial}{\partial t}\Delta_{n,m}-a^2\Delta_n\right)u\left(x,t\right)=f\left(x,t\right)\tag{1}$$

$$u\left(x,t\right)\big|_{t=0}=\varphi_0\left(x\right),\quad \left.\frac{\partial u\left(x,t\right)}{\partial t}\right|_{t=0}=\varphi_1\left(x\right),\tag{2}$$

where n,m are non-negative integers Δ_{n+m} is a Laplace operator with respect to $x\left(x_1,x_2,...,x_{n+m}\right)\in R_{n+m}$, $\varphi_0\left(x\right),\ \varphi_1\left(x\right),\ f\left(x,t\right)$ are the given functions whose conditions will be given below,

$$\Delta_n=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\cdots+\frac{\partial^2}{\partial x_n^2},$$

$$\Delta_{n,m} = \frac{\partial^2}{\partial x_1^2} + \dots + \frac{\partial^2}{\partial x_n^2} + \frac{\partial^2}{\partial x_{n+1}^2} + \dots + \frac{\partial^2}{\partial x_{n+m}^2}.$$

Equation (1) is a correct by Petrovsky equation.Existence and uniqueness of the Cauchy problem solution in the calss of distributions are studied in [4] (p. 124-183).

Introduce a space of functions $v(x) \in H^{(\alpha)}(R_{n+m}, \rho(x))$, whose elements have derivatives in Sobolev Sl;obodetskiy sense with respect to x up to order l for which

$$\int\limits_{R_{n+m}} \rho(x) \sum_{|\alpha|=0}^{l} \left(D^{(\alpha)} v(x)\right)^2 \le C,$$

where $\rho(x) \ge 0$ is a measurable function decrasing at infinity as $|x|^{-(n+m+1)}$, $C > 0$ is some constant.

By $D(R_n)$ we denote a space of finite infinitely differentiable functions.

Definition. *Under the solution of problem (1)-(2) we'll understand a twice continuously differentiable with respect to t function satisfying equation (2) in distributions and for $t = 0$ initial conditions.*

Assuming $u(x,t)$ a distribution over the space $D(R_n)$ and applying the Fourier transformation to problem (1),(2) we get a dual problem with parameter $s = (s_1, ..., s_n, s_{n+1}, ..., s_{n+m})$

$$\left(\frac{\partial^2}{\partial t^2} + \omega |s|^2 \frac{\partial}{\partial t} + a^2 |\overline{s}|^2\right) V(s,t) = \widetilde{f}(s,t), \tag{3}$$

$$V(s,t)|_{t=0} = \widetilde{\varphi}_0(s), \quad \frac{\partial V(s,t)}{\partial t} = \widetilde{\varphi}_1(s), \tag{4}$$

where the sign $\sim$ on the function means its Fourier transformation with respect to x,

$$|s| = \left(s_1^2 + s_2^2 + \dots + s_{n+m}^2\right)^{\frac{1}{2}}, \quad |\overline{s}| = \left(s_1^2 + s_2^2 + \dots + s_n^2\right)^{\frac{1}{2}},$$

$$\left|\overline{\overline{s}}\right| = \left(s_{n+1}^2 + s_{n+2}^2 + \dots + s_{n+m}^2\right)^{\frac{1}{2}}.$$

Solving problem (3),(4) we get:

$$V(s,t) = \frac{\lambda_2(s) e^{t\lambda_1(s)} - \lambda_1(s) e^{t\lambda_2(s)}}{\lambda_2(s) - \lambda_1(s)} \widetilde{\varphi}_0(s) + \frac{e^{t\lambda_2(s)} - e^{t\lambda_1(s)}}{\lambda_2(s) - \lambda_1(s)} \widetilde{\varphi}_1(s) +$$

$$+ \int\limits_0^t \left[\frac{e^{(t-\tau)\lambda_2(s)}}{\lambda_2(s) - \lambda_1(s)} - \frac{e^{(t-\tau)\lambda_1(s)}}{\lambda_2(s) - \lambda_1(s)}\right] \widetilde{f}(s,\tau)\, d\tau, \tag{5}$$

where

$$\lambda_{1,2}(s) = -\frac{|s|^2 \omega}{2} \pm \sqrt{\frac{\omega^2 |s|^4}{4} - a^2 |\overline{s}|^2}$$

are the roots of the characteristic equation $\lambda^2 + |s|^2 \omega\lambda + a^2 |\overline{s}|^2 = 0$. The solution of the Cauchy problem (1),(2) is determined as the Fourier inverse transformation from the function $V(s,t)$. By the formula of Fourier transformation, from (5) we get

$$u(x,t)=\int\limits_{R_{n+m}} G_0(x-\xi,t)\,\varphi_0(\xi)\,d\xi+\int\limits_{R_{n+m}} G_1(x-\xi,t)\,\varphi_1(\xi)\,d\xi+$$

$$+\int\limits_0^t d\tau\int\limits_{R_{n+m}} G_1(x-\xi,t-\tau)\,f(\xi,\tau)\,d\xi\equiv u_0(x,t)+u_1(x,t)+u_f(x,t)\,, \qquad (6)$$

where

$$G_0(x,t)\equiv\frac{1}{(2\pi)^{n+m}}\int\limits_{R_{n+m}} Q_0(s,t)\,e^{-i(x,s)}ds\equiv\frac{1}{(2\pi)^{n+m}}\times$$

$$\times\int\limits_{R_{n+m}}\frac{\lambda_2(s)\,e^{t\lambda_1(s)}-\lambda_1(s)\,e^{t\lambda_2(s)}}{\lambda_2(s)-\lambda_1(s)}e^{-i(x,s)}ds \qquad (7)$$

$$G_1(x,t)\equiv\frac{1}{(2\pi)^{n+m}}\int\limits_{R_{n+m}} Q_1(s,t)\,e^{-i(x,s)}ds\equiv$$

$$\equiv\frac{1}{(2\pi)^{n+m}}\int\limits_{R_{n+m}}\frac{e^{t\lambda_2(s)}-e^{t\lambda_1(s)}}{\lambda_2(s)-\lambda_1(s)}e^{-i(x,s)}ds\equiv$$

$$\equiv\frac{1}{(2\pi)^{n+m}}\left[G_{1,1}(x,t)+G_{1,2}(x,t)\right], \qquad (8)$$

where $G_1(x,t)$ is a Green function of the Cauchy problem for equation (1) with initial data

$$G_1(x,0)=0,\ G_t'(x,0)=\delta(x)\,,$$

here $\delta(x)$ is Dirac's function. It is easy to see that $\operatorname{Re}\lambda_1(s)\le 0$, and $\operatorname{Re}\lambda_2(s)\le -\dfrac{\omega|s|^2}{2}$ for $s\in R_{n+m}$. Therefore, we'll understand the convergence of integrals in (7) and (8) in distributions [5] (p. 125-130). Since for sufficiently smooth function $\varphi(x)$

$$\widetilde{\varphi}(x)=(-1)^{\mu}\left(1+\rho^2\right)^{-\mu}(1-\Delta_{n+m})^{\mu}\,\varphi(x)\,,\quad \rho\,|s|\,, \qquad (9)$$

where μ is a natural number that will be chosen later. Besides, in sequal, we'll assume that at infinity, $\varphi(x)$ decreases very rapidly. Using the theory of Fourier transformation of distrbutions [5] (pp. 152-179) and (9) we can represent the integrals in (7)-(8) in the following form

$$u_j(x,t)=(-1)^{\mu}\int\limits_{R_{n+m}} G_j^{(*)}(x-\xi,t)\,(1-\Delta_{n+m})^{\mu-j}\,\varphi_j(\xi)\,d\xi, \qquad (10)$$

where

$$G_j^{(*)}(x,t)=\frac{1}{(2\pi)^{n+m}}\int\limits_{R_{n+m}} Q_j(s,t)\left(1+|s|^2\right)^{-\mu+j}e^{-i(x,s)}ds,\quad j=0,1. \qquad (11)$$

The number μ is determined in the following way

$$2\mu \begin{cases} n+m+1, & \text{if } n+m \text{ is odd,} \\ \\ n+m+2, & \text{if } n+m \text{ is even,} \end{cases}$$

2. Asymptotics of functions $G_0^*(x,t)$ and $G_1^*(x,t)$ for large values of time

For studying asymptotics of the functions $G_j^*(x,t)\,,\quad j=0,1$ for large values of time we pass to spherical coordinates

$$\begin{gathered} s_1=\rho\cos\varphi_1 \\ s_2=\rho\sin\varphi_1\cos\varphi_2 \\ s_3=\rho\sin\varphi_1\sin\varphi_2\cos\varphi_3 \\ \cdots\ \cdots\ \cdots\cdots\ \cdots\cdots\ \cdots\cdots\ \cdot\cdot \\ s_n=\rho\sin\varphi_1\sin\varphi_2...\sin\varphi_{n-1}\cos\varphi_n \\ s_{n+1}=\rho\sin\varphi_1\sin\varphi_2...\sin\varphi_n\cos\varphi_{n+1} \\ \cdots\ \cdots\ \cdots\cdots\ \cdots\cdots\ \cdots\cdots\ \cdots\ \ \cdots\ \ \cdots\ \ \cdot\cdot \\ s_{n+m-1}=\rho\sin\varphi_1\sin\varphi_2...\sin\varphi_{n+m-2}\cos\varphi_{n+m-1} \\ s_{n+m}=\rho\sin\varphi_1\sin\varphi_2...\sin\varphi_{n+m}\cos\varphi_{n+m-1} \end{gathered} \tag{12}$$

Here $|s|=\left(s_1^2+s_2^2+\cdots s_{n+m}^2\right)^{\frac{1}{2}}=\rho$, where $0\le\varphi_j\le\pi$, $j=1,2,...,n+m-2$, $0\le\varphi_{n+m-1}\le 2\pi$. We'll denote $\overline{\varphi}=(\varphi_1,...,\varphi_n)\,,\ \overline{\overline{\varphi}}=\left(\varphi_{n+1},...,\varphi_{n+m-1}\right)$.

It follows from formula (12) that

$$|s|^2=s_1^2+\cdots+s_n^2=\rho^2\left(1-\sin^2\varphi_1...\sin^2\varphi_n\right)\equiv\rho^2T_1(\varphi) \tag{13}$$

Using (12),(13) and passing in (11) to polar coordinates, we get

$$G_1^{(*)}(x,t)=\frac{1}{(2\pi)^{n+m}}\int\limits_0^\infty\rho^{n+m-1}\left(1+\rho^2\right)^{-\mu+1}\times$$

$$\times\left[\int\limits_0^\pi\cdots\int\limits_0^\pi\sin^{n+m-2}\varphi_1d\varphi_1\sin^{n+m-3}\varphi_2d\varphi_2\cdots\sin^{m-1}\varphi_nd\varphi_nQ\left(\rho,\overline{\varphi},t\right)\times\right.$$

$$\left.\times\int\limits_0^\pi\sin^{m-2}\varphi_{n+1}d\varphi_{n+1}\cdots\int\limits_0^\pi\sin\varphi_{n+m-2}d\varphi_{n+m-2}\int\limits_0^{2\pi}e^{i(x,\rho\delta(\varphi))}d\varphi_{n+m-1}\right]d\rho\equiv$$

$$\equiv G_{1,1}^{(*)}(x,t)+G_{1,2}^{(*)}(x,t)\,,$$

where $\delta=(\delta_1,\delta_2,...,\delta_{n+m-1})$, $\delta_j=\sin\varphi_1\cdots\sin\varphi_{j-1}\cos\varphi_j$, $j=1,...,n+m-1$; $\delta_{n+m}=\sin\varphi_1\cdots\sin\varphi_{n+m-1}$.

Denote internal integral in (16) by $J(x,\rho,\overline{\varphi})$

$$J(x,\rho,\overline{\varphi})=\int_0^{\pi}\sin^{m-2}\varphi_{n+1}d\varphi_{n+1}\cdots\int_0^{\pi}\sin\varphi_{n+m-2}d\varphi_{n+m-2}\int_0^{2\pi}e^{i(x,\rho\delta(\varphi))}d\varphi_{n+m-1}.$$

In sequel, we'll need the following lemma that is easily proved.

Lemma 1. *For $s\in R_{n+m}$ for $\lambda_1(s)$ and $\lambda_2(s)$ the following estimations take place:*

$$\operatorname{Re}\lambda_1(s)\leq-\frac{\omega^2}{2}\rho^2,\ \text{for}\ \rho\leq\frac{2a}{\omega},\tag{14}$$

$$\lambda_1(s)<-\frac{a^2T_1^2(\overline{\varphi})}{\omega},\ \text{for}\ \rho>\frac{2a}{\omega}.\tag{15}$$

Hold for all $s\in R_{n+m}$

$$\lambda_2(s)\leq-\frac{\omega^2}{2}\rho^2,\tag{16}$$

where $\overline{\varphi}=(\varphi_1,\varphi_2,...,\varphi_n)$, $0\leq\varphi_j\leq\pi$, $j=1,2,...,n$.

Consider $G_1(x,t)$. Represent it in the form:

$$G_1^*(x,t)=\frac{1}{(2\pi)^{n+m}}\left\{\int_0^{\frac{2a}{\omega}}+\int_{\frac{2a}{\omega}}^{0}\right\}\int_0^{\pi}\cdots\int_0^{\pi}\rho^{n+m-1}\left(1+\rho^2\right)^{-\mu+1}T_2(\overline{\varphi})\times$$

$$\times J(x,\rho,\overline{\varphi})\frac{e^{t\lambda_1(\rho,\overline{\varphi})}-e^{t\lambda_2(\rho,\overline{\varphi})}}{\lambda_2(\rho,\overline{\varphi})-\lambda_1(\rho,\overline{\varphi})}d\overline{\varphi}d\rho\equiv G_{1,1}^{(*)}(x,t)+G_{1,2}^{(*)}(x,t),\tag{17}$$

where $T_2(\overline{\varphi})=\sin^{n+m-2}\varphi_1\cdots\sin^{m-1}\varphi_n$.

For estimation $G_{1,1}^{(*)}(x,t)$ represent an integrand expression in $G_{1,1}^{(*)}(x,t)$ in the form

$$J(t,\rho,\overline{\varphi})=\frac{e^{t\lambda_1(\rho,\overline{\varphi})}-e^{t\lambda_2(\rho,\overline{\varphi})}}{\lambda_2(\rho,\overline{\varphi})-\lambda_1(\rho,\overline{\varphi})}=-e^{t\lambda_1(\rho,\overline{\varphi})}\int_0^{t}e^{\tau(\lambda_2-\lambda_1)(\rho,\overline{\varphi})}d\tau.$$

Using lemma 1 for $J(t,\rho,\overline{\varphi})$, we get the following estimates

$$|J(t,\rho,\overline{\varphi})|\leq te^{-\frac{\omega^2}{2}t\rho^2}\ \text{for}\ \rho\leq\frac{2a}{\omega},\tag{18}$$

$$|J(t,\rho,\overline{\varphi})|\leq te^{-\frac{a^2T_1^2(\overline{\varphi})t}{\omega}}\ \text{for}\ \rho>\frac{2a}{\omega}.\tag{19}$$

Using estimation (18), for $G_{1,1}^{(*)}(x,t)$ we get

$$\left|G_{1,1}^{(*)}(x,t)\right|\leq C_0(n,m)\,t\int_0^{\frac{2a}{\omega}}\rho^{n+m-1}e^{-\frac{\omega^2}{2}t\rho^2}d\rho.\tag{20}$$

Applying Watson's lemma [6] (p.57) to integral (20) as $t\to+\infty$ we get

$$G_{1,1}^{(*)}(x,t)=O\left(t^{1-\frac{n+m}{2}}\right)\tag{21}$$

uniformly with respect to $x \in R_{n+m}$. Consider $G_{1,2}^{(*)}(x,t)$. Using the estimation (19), we have

$$\left|G_{1,2}^{(*)}(x,t)\right| \leq C_1(n,m)\, t \int_0^{\pi} \cdots \int_0^{\pi} e^{-\frac{a^2 T_1^2(\overline{\varphi})}{\omega}} d\overline{\varphi} \tag{22}$$

For finding the asymptotics of the integral (22) as $t \to +\infty$ we use the saddle point method. The point $\overline{\varphi}_0 = \left(\frac{\pi}{2}, \frac{\pi}{2}, ..., \frac{\pi}{2}\right)$ is a saddle point.Really,

$$\frac{\partial^2}{\partial \varphi_j^2} T_1^2(\overline{\varphi}) = -2\sin\varphi_j \cos\varphi_j \sin^2\varphi_1 \widehat{\cdots} \sin^2\varphi_n$$

The sign $\wedge$ means that in this series there is no multiplier with index j. Hence we get

$$\left.\frac{\partial^2}{\partial \varphi_j^2} T_1^2(\overline{\varphi})\right|_{\overline{\varphi}=\overline{\varphi}_0} = 2, \quad j = 1, 2, ..., n$$

and

$$\left.\frac{\partial^2}{\partial \varphi_j \partial \varphi_\mu} T_1^2(\overline{\varphi})\right|_{\overline{\varphi}=\overline{\varphi}_0} = 0, \quad j \neq \mu, \quad j, \mu = 1, 2, ..., n.$$

Consequently,

$$\det\left\|\frac{\partial^2 T_1^2(\overline{\varphi}_0)}{\partial \varphi_j \partial \varphi_\mu}\right\| = 2^n,$$

i.e. the point $\overline{\varphi} = \overline{\varphi}_0$ is a non-degenerate saddle point. Applying the saddle-point method [6] (p. 408-420) as $t \to +\infty$ to the integral (22),we get

$$G_{1,2}^{(*)}(x,t) = O\left(t^{1-\frac{n}{2}}\right), \tag{23}$$

uniformly with respect to $x \in R_{n+m}$. It follows from (21) and (23) that as $t \to +\infty$

$$G_1^{(*)}(x,t) = O\left(t^{1-\frac{n}{2}}\right), \tag{24}$$

uniformly with respect to $x \in R_{n+m}$.

Now, let's consider $G_0^*(x,t)$. Represent $Q_0(s,t)$ in the form:

$$Q_0(s,t) = e^{t\lambda_1(s)} - \frac{\lambda_1(s)\left(e^{t\lambda_2(s)} - e^{t\lambda_1(s)}\right)}{\lambda_2(s) - \lambda_1(s)} \equiv Q_{01}(s,t) + Q_{02}(s,t)$$

Then

$$G_0^*(x,t) \equiv G_{01}^*(x,t) + G_{02}^*(x,t), \tag{25}$$

respectively, where $G_0^*(x,t)$, $G_{01}^*(x,t)$, $G_{02}^*(x,t)$ are determined by formula (11). Consider $G_{01}^*(x,t)$

$$G_{01}^*(x,t) = \frac{1}{(2\pi)^{n+m}} \left\{\int_0^{\frac{2a}{\omega}} + \int_{\frac{2a}{\omega}}^{0}\right\} \int_0^{\pi} \cdots \int_0^{\pi} \rho^{n+m-1}\left(1+\rho^2\right)^{-\mu} T_2(\overline{\varphi}) \times$$

$$\times J(x,\rho,\overline{\varphi})\, e^{\lambda_1(\rho,\overline{\varphi})} d\overline{\varphi} d\rho \equiv G^{(*)}_{011}(x,t) + G^{(*)}_{012}(x,t)$$

By lemma 1, for $G^{(*)}_{011}(x,t)$ we have

$$\left|G^{(*)}_{011}(x,t)\right| \le C_2(n,m) \int\limits_0^{\frac{2a}{\omega}} \rho^{n+m-1}\left(1+\rho^2\right)^{-\mu} e^{-\frac{\omega^2}{2}t\rho^2} d\rho. \tag{26}$$

Applying the Watson lemma [6] (p.57) to the integral in the right hand side of (26), as $t \to +\infty$ we get

$$G^{(*)}_{011}(x,t) = O\left(t^{-\frac{n+m}{2}}\right), \tag{27}$$

uniformly with respect to $x \in R_{n+m}$. For $G^{(*)}_{012}(x,t)$, by lemma 1, we have

$$\left|G^{(*)}_{012}(x,t)\right| \le C_3(n,m) \int\limits_{\frac{2a}{\omega}}^{\infty} \rho^{n+m-1}\left(1+\rho^2\right)^{-\mu} d\rho \int\limits_0^{\pi} \cdots \int\limits_0^{\pi} e^{-\frac{a^2T_1^2(\overline{\varphi})t}{\omega}} d\overline{\varphi} =$$

$$= C_4(n,m) \int\limits_0^{\pi} \cdots \int\limits_0^{\pi} e^{-\frac{a^2T_1^2(\overline{\varphi})t}{\omega}} d\overline{\varphi}. \tag{28}$$

As in estimation of $G^{(*)}_{12}(x,t)$, applying the saddle-point method to the integral in the right hand side of (28), we get

$$G^{(*)}_{012}(x,t) = O\left(t^{-\frac{n}{2}}\right),$$

uniformly with respect to $x \in R_{n+m}$. It follows from estimations (27) and (28) that as $t \to +\infty$

$$G^{(*)}_{01}(x,t) = O\left(t^{-\frac{n}{2}}\right) \tag{29}$$

uniformly with respect to $x \in R_{n+m}$. Estimation of $G^{(*)}_{02}(x,t)$ is conducted in the same way as for $G^{(*)}_{1}(x,t)$. Therefore as $t \to +\infty$, we have

$$G^{(*)}_{02}(x,t) = O\left(t^{1-\frac{n}{2}}\right) \tag{30}$$

uniformly with respect to $x \in R_{n+m}$. It follows from estimations (29) and (30) that as $t \to +\infty$

$$G^{(*)}_{0}(x,t) = O\left(t^{1-\frac{n}{2}}\right) \tag{31}$$

uniformly with respect to $x \in R_{n+m}$.The following theorem follows from the estimations (24) and (31).

Theorem 1. *For the functions* $G^{(*)}_{j}(x,t)$, $j=0,1$ *as* $t \to +\infty$ *the following asymptotic estimation*

$$G^{(*)}_{j}(x,t) = O\left(t^{1-\frac{n}{2}}\right), \quad j=0,1 \tag{32}$$

holds uniformly with respect to $x \in R_{n+m}$.

3. Asymptotics of the solution to Cauchy problem (1)-(2)

Using estimation (32) we'll prove a theorem on estimation of solution to the Cauchy problem (1)-(2). As a preliminary, we cite the following lemma from the paper [3].

Lemma 2. *Let*

$$B(t)=\int_0^t \frac{\left(1+\tau^{\beta}\right)d\tau}{(1+t-\tau)^{\frac{n}{2}-1}},$$

where β is any real number, $n\geq 1$. Then as $t\to+\infty$ it holds the following estimation

$$B(t)=O\left(t^{1+\gamma}\right),$$

where $\gamma=\max\left\{-\dfrac{n}{2}+1,\beta\right\}$.

Using lemma 2, we prove the following theorem.

Theorem 2. *Let*

$$\left\|\varphi_j(\xi)\right\|^{2(\mu-j)}_{H(\rho(\xi),R_{n+m})}\leq C,\quad j=0,\ 1$$

and

$$\|f(\xi,\tau)\|_{L_2(\rho(\xi),R_{n+m})}\leq C\,|\tau|^{\beta_1}, \tag{33}$$

where $\rho(\xi)=\left(1+|\xi|^2\right)^{\frac{n+m+1}{2}}$, β_1 is any real number. *Then as $t\to+\infty$, for the solution of Cauchy problem (1)-(2) it holds the estimation*

$$\sup_{x\in K}|u_j(x,t)|\leq Ct^{1-\frac{n}{2}}\left\|\varphi_j(\xi)\right\|^{2(\mu-j)}_{H(\rho(\xi),R_{n+m})},\quad j=0,1,$$

$$\sup_{x\in K}|u_f(x,t)|\,Ct^{\gamma_1},$$

where $\gamma_1=\max\left\{-\left(\dfrac{n}{2}-1\right),\beta_1\right\}$, *$K\subset R_{n+m}$ is any compact.*

Proof. Using estimation (32) we estimate each of the terms in (6). Then

$$|u_0(x,t)|\leq\int_{R_{n+m}}\left|G_0^{(*)}(x-\xi,t)\right|\left|(1-\Delta_{n+m})^{\mu}\varphi_0(\xi)\right|d\xi\leq$$

$$\leq Ct^{1-\frac{n}{2}}\int_{R_{n+m}}\left|(1-\Delta_{n+m})^{\mu}\varphi_0(\xi)\right|d\xi$$

Applying the Cauchy-Bunyakowskii inequality, we get

$$|u_0(x,t)|\leq Ct^{1-\frac{n}{2}}\left(\int_{R_{n+m}}\left(1+|\xi|^2\right)^{\frac{n+m+1}{2}}\left|(1-\Delta)^{\mu}\varphi_0(\xi)\right|^2d\xi\right)^{\frac{1}{2}}$$

Hence

$$\sup_{x\in K_{n+m}}|u_0(x,t)|\leq Ct^{1-\frac{n}{2}}\left\|\varphi_0(\xi)\right\|^{2\mu}_{H(\rho(\xi),R_{n+m})}$$

in any compact $K_{n+m} \subset R_{n+m}$.

In the similar way we get

$$\sup_{x\in K_{n+m}} |u_1(x,t)| \le Ct^{1-\frac{n}{2}} \|\varphi_1(\xi)\|^{2(\mu-1)}_{H(\rho(\xi),R_{n+m})}$$

in any compact $K_{n+m} \subset R_{n+m}$.

Now, let's estimate $u_f(x,t)$. Applying the Cauchy Bunyakowskii inequality, we get

$$|u_f(x,t)| \le C\int_0^t (1+|t-\tau|)^{1-\frac{n}{2}} \|f(\xi,\tau)\|_{L_2(\rho(\xi),R_{n+m})}$$

Considering condition (33), from the last inequality we get

$$|u_f(x,t)| \le C\int_0^t \frac{|\tau|^{\beta}\,d\tau}{[1+|t-\tau|]^{\frac{n}{2}-1}}. \tag{34}$$

Applying lemma 2 in (34) as $t\to+\infty$, we get

$$\sup_{x\in K_{n+m}} |u_f(x,t)| \le Ct^{\gamma_1}$$

in any compact $K_{n+m} \subset R_{n+m}$, where $\gamma_1 = \max\left\{-\left(\frac{n}{2}-1\right),\beta_1\right\}$. The theorem is proved.

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