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LATERAL OSCILLATIONS OF A BEAM MADE OF MULTI-MODULUS MATERIAL LYING ON INHOMOGENEOUS VISCO-ELASTIC FOUNDATION

Abstract

In the paper we consider a problem of free lateral oscillations of a beam inhomogeneous in length and made of multi-modulus material, subject to external visco-elastic inhomogeneous resistance. By using the separation of variables and Bubnov-Galerkin methods, a formula for determining the values of angular frequency was found. The numerical analysis was carried out, the results were represented in the form of tables and characteristic graphs.

As is known the structural elements whose properties essentially depend on the form of stress state [1,3] are often used in construction of engineering structures, machine building and in many fields of engineering. Some sorts of pig iron, filled polymers, composite materials, rocks, etc. are related to these elements.

In these materials, the hydrostatic pressure essentially influences on the dependence stress-strain: At different forms of stress state these materials display not identical mechanical properties (fig. 1)

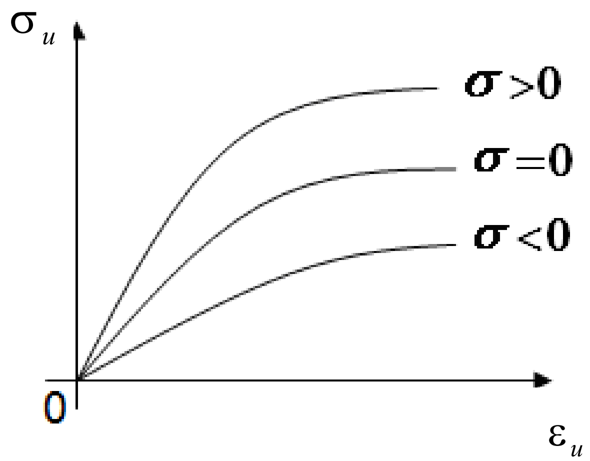


Fig.1.

Here σ_u and ε_u are stress and strain intensities, respectively, σ is a hydrostatic pressure.

Note that account of multimodulus and real property of the foundation complicates much the solution of the problems of lateral oscillations, and its ignorance may lead to essential errors.

In the present paper we solve a problem of eigen oscillation of symmetric cross-section of a non-homogeneous bar with regard to resistance of a non-homogeneous visco-elastic medium, whose reaction of the foundation is connected with the flexure by the following relation [5]:

$$q = K_1(x)W + K_2(x)\frac{\partial^2 W}{\partial t^2}. \tag{1}$$

Here $K_1(x)$ and $K_2(x)$ are the characteristics of the foundation and are determined by means of the experiments, W is flexure, t is time.

The stress in section is distributed as follows:

$$\sigma^+ = E^+(\ell + y\wp) \quad for \quad y \in S_1$$

$$\sigma^- = E^-(\ell + y\wp) \quad for \quad y \in S_2. \tag{2}$$

Here ℓ and $\wp$ are deformation and curvature of the central line, respectively, S_1 and S_2 are the domains of stretching and compressed areas.

The boundary of the neutral line y_0 is determined from the following condition:

$$\ell + y_0\wp = 0. \tag{3}$$

The quantity ℓ is connected with the curvature with no axial force condition and has the following form:

$$\ell = -\wp\frac{\int\limits_{S_1} yb(y)\,dy + \alpha\int\limits_{S_2} yb(y)\,dy}{\int\limits_{S_1} b(y)\,dy + n\int\limits_{S_2} b(y)\,dy}. \tag{4}$$

Here $b(y)$ is the width of the bar $n = \frac{E^-}{E^+}$.

It is easy to establish that the bending moment may be represented in the following form:

$$M = M_O K \cdot f(x), \tag{5}$$

where $M_0 = \frac{2}{3}E^+J\wp$

$$K = \frac{1}{J}\left[\int\limits_{F_1}\rho^2 b(\rho)\,d\rho + \alpha\int\limits_{F_2}\rho^2 b(\rho)\,d\rho - \frac{\left[\int\limits_{F_1}\rho b(\rho) + \alpha\int\limits_{F_2}\rho b(\rho)\,d\rho\right]^2}{\int\limits_{F_1} b(\rho)\,d\rho + \alpha\int\limits_{F_2} b(\rho)\,d\rho}\right]. \tag{6}$$

For a beam of rectangular cross section the equation of motion with regard to (1) and (5) is written in the form

$$\frac{\partial^2}{\partial x^2}\left[f(x)\frac{\partial^2 W}{\partial x^2}\right] + \overline{K}_1(x)W + \overline{K}_2(x)\frac{\partial^2 W}{\partial t^2} + \overline{\rho}\psi(x)\frac{\partial^2 W}{\partial t^2} = 0, \tag{7}$$

where the following de notations are accepted: $\overline{K}_1(x) = K_1(x)\cdot(E^{+}JK)^{-1}$, $\overline{K}_2(x) = K_2(x)\cdot(E^{+}JK)^{-1}$, $\overline{\rho} = \rho_0(E^{+}J_0K)^{-1}$.

The solution of equation (7) will be realized by the combined approximately analytic methods. At the first stage we use the method of separation of variables and look for W as follows:

$$W(x,t) = V(x)\exp(imt). \tag{8}$$

Here ω is an angular velocity, the function $V(x)$ should satisfy the respective boundary conditions.

Substituting (8) in (7), we get:

$$\frac{d^2}{dx^2}\left[f(x)\frac{\partial^2 W}{\partial x^2}\right] + \overline{K}_1(x)V - \omega^2\overline{K}_2(x)V + \omega^2\overline{\rho}\psi(x)V(x) = 0. \tag{9}$$

We determine the value of ω^2 by means of Bubnov-Galerkin's orthogonalization method, and accept the function $V(x)$ in the form:

$$V(x) = \sum_{i=1}^{n} C_i\theta_i(x), \tag{10}$$

where C_i are unknown constants, and each $\theta_i(x)$ should satisfy the boundary conditions.

Allowing for (9) and (10), the error function will take the form:

$$\eta_i(x) = \sum_{i=1}^{n} C_i\left\{\frac{d^2}{dx^2}\left[f(x)\frac{\partial^2\theta_i}{\partial x^2}\right] + \overline{K}_1\theta(x) - \omega^2\left[(K_2(x) + \rho\psi(x))\right]\theta_i\right\} \neq 0. \tag{11}$$

On the basis of the orthogonalization method we can write:

$$\int_0^{\ell}\eta_i(x)\theta_q(x)dx = 0 \quad q = (1,2,...) \tag{12}$$

In the general form, ω^2 is determined from the system of linear homogeneous algebraic equations constituted from the coefficients C_i. For the existence of non-trivial solutions, the principal determinant of this system should equal zero

$$\left\|\omega^2\right\| = 0. \tag{13}$$

However, for engineering calculations the first approximation is usually neglected. Then the principal frequency tone is determined from the orthogonalization condition:

$$\int_0^{\ell}\eta_i(x)\theta_i(x)dx = 0$$

or

$$\omega^2 = \frac{\int_0^{\ell}\left[\frac{d^2\theta_i}{dx^2}\left(f(x)\frac{d^2\theta_i}{dx^2}\right) + K_1(x)\theta_i(x)\right]\theta_i^2 dx}{\int_0^{\ell}\left[\overline{K}_2(x) + \rho\psi(x)\right]\theta_i^2 dx}. \tag{14}$$

As an example, consider the case of a right cross-section bar whose ends are hingely supported, and the characteristic functions change by the following rules

$$\theta_i(x) = \sin m\overline{x}; \quad f(x) = 1 + \varepsilon\overline{x}; \quad K_1(x) = K_{10}(1 + \mu_1\overline{x});$$
$$K_2(x) = K_{20}(1 + \mu_2\overline{x});$$

here

$$\varepsilon \in [0,1], \quad \mu_i \in [0,1] \quad \psi(x) = 1 + \mu_3\overline{x}; \quad \left(\overline{x} = x \cdot l^{-1}\right). \tag{15}$$

Allowing for (15), formula (14) has the following form:

$$\omega^2 = \frac{\int_0^1 \left[(1+\varepsilon\overline{x}) \ \frac{m\pi}{\ell} \sin m\pi\overline{x} - 2\varepsilon\ell^{-1}\left(\frac{m\pi}{\ell}\right)^3 \cos\frac{m\pi}{x} + K_{10}(1+\mu_1\overline{x}) \sin m\pi\overline{x} \ \right] \sin m\pi\overline{x} dx}{\int_0^1 [K_{20}(1+\mu_2\overline{x}) + \rho(1+\mu_3\overline{x})] \sin^2 m\pi\overline{x} dx}. \tag{16}$$

Taking into account

$$\int_0^1 \sin^2 m\pi\overline{x} dx = \frac{1}{2}; \quad \int_0^1 \overline{x} \sin^2 m\pi\overline{x} dx = \frac{1}{4}; \quad \int_0^1 \sin 2m\pi\overline{x} dx = 0,$$

from (16) we can get:

$$\omega^2 = \frac{\left(\frac{m\pi}{l}\right)^4 (1 + 0,5\varepsilon) + \overline{K}_{10}(1 + 0,5\mu_1)}{\overline{K}_{20}(1 + 0,5\mu_2) + \overline{\rho}(1 + 0,5\mu_3)}. \tag{17}$$

For linear homogeneous viscoelastic resistance from (17) we get:

$$\omega_n^2 = \frac{\left(\frac{m\pi}{l}\right)^4 (1 + 0,5\varepsilon) + \overline{K}_{10}}{\overline{K}_{20} + \overline{\rho}(1 + 0,5\mu_3)}. \tag{18}$$

For the homogeneous case it holds:

$$\omega_0^2 = \frac{\left(\frac{m\pi}{l}\right)^4 + \overline{K}_{10}(1 + 0,5\mu_1)}{\overline{K}_{20}(1 + 0,5\mu_2) + \overline{\rho}}. \tag{19}$$

For resistancless external medium, for a nonhomogeneous bar we get:

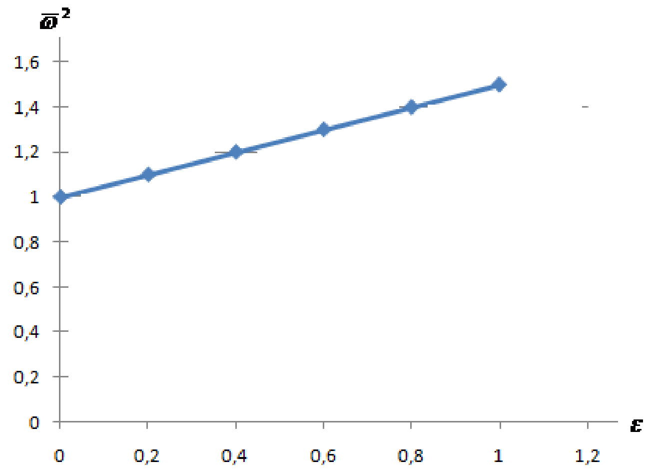
$$\omega_n^2 = \frac{\left(\frac{m\pi}{l}\right)^4 (1 + 0,5\varepsilon)}{\overline{\rho}(1 + 0,5\mu_3)}. \tag{20}$$

For a resistancless homogeneous bar we should take $\varepsilon = 0$, $\mu_3 = 0$.
From (20) we get

$$\omega_{n'}^2 = \frac{\left(\frac{m\pi}{l}\right)^4}{\overline{\rho}}$$

$$\overline{\omega}^2 = \left(\frac{\omega_n}{\omega_{n'}}\right)^2 = \frac{1 + 0,5\varepsilon}{1 + 0,5\mu_3}.$$

The results of calculations are in tables 1.2 and fig. 3.4

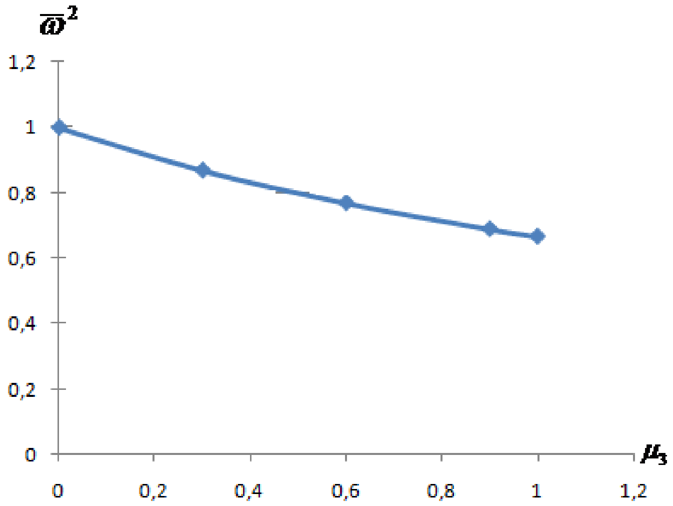


$\mu_3 = 0$

ε	$\bar{\omega}^2$
0	1
0,2	1,1
0,4	1,2
0,6	1,3
0,8	1,4
1	1,5

tab.1.

Fig.2.



$\varepsilon = 0$

μ_3	$\bar{\omega}^2$
0	1
0,3	0,869
0,6	0,769
0,9	0,689
1	0,666

tab. 2.

Fig.3.

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