Veli M. KURBANOV, Yasemen I. HUSEYNOVA

ON CONVERGENCE OF SPECTRAL EXPANSION OF ABSOLUTELY CONTINUOUS VECTOR-FUNCTION IN EIGEN VECTOR-FUNCTIONS OF FOURTH ORDER DIFFERENTIAL OPERATOR

Abstract

In the paper, a fourth order ordinary differential operator with matrix coefficients is considered, absolute and uniform convergence of orthogonal expansion of an absolutely continuous vector-function in eigen vector-functions of the given operator is studied, and the rate of uniform convergence of this expansion is established.

Consider on the interval $G = (0, 1)$ the operator

$$L\psi = \psi^{(4)} + U_2(x)\psi^{(2)} + U_3(x)\psi^{(1)} + U_4(x)\psi$$

with matrix coefficients $U_\ell(x) = (u_{\ell ij}(x))_{i,j=1}^m$, $\ell = 2, 4$, where $u_{\ell ij}(x) \in L_1(G)$ are real functions $u_{\ell ij}(x) = u_{\ell ji}(x)$.

Denote by $D(G)$ the class of $m$-component vector-functions absolutely continuous together with own derivatives to the third order inclusively on the closed interval $G = [0, 1]$ ($D(G) = W_{1,1,0}(G)$).

Under the eigen vector-function of the operator $L$ corresponding to the eigen value $\lambda$ we’ll understand any vector-function $\psi(x) = (\psi_1(x), \psi_2(x), ..., \psi_m(x))^T \in D(G)$ identically not equal to zero and satisfying almost everywhere in $G$ the equation (see [1])

$$L\psi + \lambda\psi = 0.$$

Let $L^m_p(G)$, $p \geq 1$, be the space of $m$-component vector-functions $f(x) = (f_1(x), f_2(x), ..., f_m(x))^T$ with the norm

$$\|f\|_{p,m} = \left\{ \int_G |f(x)|^p \, dx \right\}^{1/p} = \left\{ \int_G \left( \sum_{i=1}^m |f_i(x)|^2 \right)^{p/2} \, dx \right\}^{1/p}.$$

Suppose that $\{\psi_k(x)\}_{k=1}^\infty$ is a complete, orthonormalized system in $L^m_2(G)$ consisting of eigen-functions of the operator $L$. Denote by $\{\lambda_k\}_{k=1}^\infty$, $\lambda_k \leq 0$ the appropriate system of eigen values.

Denoting $\mu_k = \sqrt{-\lambda_k}$ introduce into consideration the partial sum of the orthogonal expansion of the vector-function $f(x) \in W_{1,1,m}(G)$ in the system $\{\psi_k(x)\}_{k=1}^\infty$

$$\sigma_\nu(x,f) = \sum_{\mu_k \leq \nu} f_k\psi_k(x), \quad \nu > 0,$$

where

$$f_k = (f, \psi_k) = \int_0^1 (f(x), \psi_k(x)) \, dx = \int_0^1 \sum_{j=1}^m f_j(x)\psi_{kj}(x) \, dx,$$