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ON SPECIAL SUB- AND SUPERSOLUTIONS OF NON-DIVERGENT PARABOLIC EQUATIONS OF THE SECOND ORDER

Abstract

At the present paper the class of parabolic equations of the second order with continuous coefficients, which satisfy to Dini condition with respect to space variables is considered. The existence of the sub- and supersolutions which have the same singularity as fundamental solution of the heat equation has been proved.

Let R_{n+1} be $(n+1)$-dimensional Euclidean space of points $(x,t)=(x_1,...,x_n,t)$, D be bounded domain in half space $t<0$, $\Gamma(D)$ be parabolic boundary of D (see [1]), $(0,0)\in\Gamma(D)$. We say that D as R-domain at the neighborhood of point $(0,0)$, if there exists $\delta>0$ such that for any $\tau\in(-\delta,0),(y,\tau)\in\Gamma(D)$ segment $\{(z,t): z=y, \tau\le t\le 0\}$ does not intersect D. The simplest example of R-domain is figure of rotation $\{(x,t): |x|^2<\alpha(-t),\ -\delta<t<0\}$, where $\alpha(z)$ is non-decreasing continuous on $[0,\delta]$ function.

Consider in D the parabolic equation of the form

$$Lu=\sum_{i,j=1}^{n} a_{ij}(x,t)u_{ij}-u_t=0, \tag{1}$$

where $\|a_{ij}(x,t)\|$ is real symmetric matrix, $u_i=\frac{\partial u}{\partial x_i}, u_{ij}=\frac{\partial^2 u}{\partial x_i \partial x_j}; i,j=1,...,n$. Denote by O_δ the cylinder $\{(x,t): |x|<\delta, -\delta<t<0\}$. We will suppose that relative to coefficients of operator L following conditions hold

$$\mu|\xi|^2\le\sum_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j\le\mu^{-1}|\xi|^2;\ \mu\in(0,1] \tag{2}$$

for all $(x,t)\in\overline{O_\delta\cap D}$ and any n-dimensional vector ξ,

$$|a_{ij}(x,t)-a_{ij}(y,t)|\le\varphi(|x-y|),(i,j=1,...,n),\ \int_0^\delta\frac{\varphi(z)}{z}dz<\infty \tag{3}$$

for $(x,t),(y,t)\in\overline{O_\delta\cap D}$ and some non-decreasing on $[0,2\delta]$ function $\varphi(z)$,

$$a_{ij}(x,t)\in C\left(\overline{O_\delta\cap D}\right),\quad (i,j=1,..,n) \tag{4}$$

$$a_{ij}(y,\tau)=\delta_{ij}=\begin{cases}1, & \text{if } i=j\\ 0 & \text{if } i\ne j\end{cases};\ (y,\tau)\in\overline{O_\delta\cap\Gamma(D)}. \tag{5}$$

Function $u(x,t)\in C^{2,1}(D)$ is called subsolution of equation (1) in D, if $Lu(x,t)\ge 0$, $(x,t)\in D$. In this case function $-u(x,t)$ is called supersolution of equation (1) in D.

Let

$$G(x,t)=\begin{cases} t^{-n/2}\exp\left[-\frac{|x|^2}{4t}\right], & \text{if } t>0,\\ 0, & \text{if } t\le 0.\end{cases}$$

It is well known from [2-3], that if coefficients of operator L satisfy to uniform Dini condition by aggregate of variables (x,t) at some neighborhood of point $(0,0)$, and more over the conditions (2) and (5) hold, then at this neighborhood there exist sub- and supersolutions of equation (1) D, which have the same singularity as function G. In present paper it was shown that in case of R-domain for validity of above-mentioned fact we can slack uniform Dini condition by time variable up to condition of uniform continuity. Further this result could be useful for obtaining the criterion of regularity for boundary point of Wiener's type with respect to first boundary value problem for equation (1) (see [4-8]).

Everywhere below we will suppose that coefficients a_{ij} of operator L are expanded by Cronecker's symbol $\delta_{ij}(i,j=1,...,n)$ in $CD\cap\{(x,t):-\delta\le t\le 0\}$. By C we will denote positive constants, which depend only on n,μ,δ and function φ.

Let $(x,t)\in D\cap O_\delta$, $(y,\tau)\in\Gamma(D)\cap O_\delta, t>\tau, A(y,t,\tau)=\left\|\int_\tau^t a_{ij}(y,v)dv\right\|$, $(i,j=1,...,n)$, $A^{-1}(y;t,\tau)$ is matrix inverse to $A(y;t,\tau)$, $\rho(x,y;t,\tau)=\frac{n+1}{2n}\ln\frac{1}{\det A(y;t,\tau)}-$ $-\frac{1}{4}\left(x-y,A^{-1}(y;t,\tau)(x-y)\right)$.

Lemma 1. *Let relative to coefficients of operator L conditions (2)-(4) are satisfied. Then there exist constants C_1 and C_2, such that if*

$$\psi(\rho)=\begin{cases} C_1\varphi\left(C_2e^{-\rho/n+1}\right), & \text{if } C_2e^{-\rho/n+1}\le\delta,\\ C_1\varphi(\delta), & \text{if } C_2e^{-\rho/n+1}>\delta;\end{cases}$$

$$\Phi^-(x,y;t,\tau)=\left(\det A(y;t,\tau)\right)^{\frac{1}{2n}}\int_{-\infty}^{\rho(x,y;t,\tau)}\exp\left[\int_0^u\frac{dv}{1+\psi(v)}\right]du;$$

then

$$L_{(x,t)}\Phi^-\le 0,\quad (x,t)\in O_\delta\cap D\cap\{(z,v):v>\tau\}. \tag{6}$$

Proof. At first, note that

$$\det A\ge\mu^n(t-\tau)^n. \tag{7}$$

Really, if $\lambda_1,...,\lambda_n$ are eigenvalues of matrix A,

$$\lambda_{\min}=\min\{\lambda_1,...,\lambda_n\},\quad \lambda_{\max}=\max\{\lambda_1,...,\lambda_n\},$$

then

$$\det A=\prod_{i=1}^n\lambda_i\ge(\lambda_{\min})^n\ge\left[\int_\tau^t\inf_{|\xi|=1,(y,v)\in O_\delta}\sum_{i,j=1}^n a_{ij}(y,v)\xi_i\xi_j dv\right]^n\ge\mu^n(t-\tau)^n.$$

By the similar way we can show that

$$\lambda_{\max}\le\mu^{-1}(t-\tau),$$

therefore for any n-dimensional vector ξ

$$\left(A^{-1}\xi,\xi\right)\geq\frac{1}{\lambda_{\max}}|\xi|^2\geq\frac{\mu}{t-\tau}|\xi|^2.$$

From here it follows that

$$\left(x-y,A^{-1}(x-y)\right)\geq\frac{\mu}{t-\tau}|x-y|^2,$$

and then

$$|x-y|^2\leq\frac{t-\tau}{\mu}\left(x-y,A^{-1}(x-y)\right),$$

$$|x-y|^{n+1}\leq\frac{(t-\tau)^{\frac{n+1}{2}}}{\mu^{\frac{n+1}{2}}}\left[\left(x-y,A^{-1}(x-y)\right)\right]^{\frac{n+1}{2}}. \tag{8}$$

For all positive z inequality $\frac{z^{\frac{n+1}{2}}}{e^{z/4}}\leq C_3$ is valid, therefore from (8) we obtain

$$|x-y|^{n+1}\leq C_4(t-\tau)^{\frac{n+1}{2}}\exp\left[\frac{\left(x-y,A^{-1}(x-y)\right)}{4}\right]$$

or,

$$|x-y|\leq C_5(t-\tau)^{\frac{1}{2}}\exp\left[\frac{\left(x-y,A^{-1}(x-y)\right)}{4(n+1)}\right]. \tag{9}$$

From the other hand, using (7), we have

$$e^{-\rho/n+1}=(\det A)^{\frac{1}{2n}}\exp\left[\frac{\left(x-y,A^{-1}(x-y)\right)}{4(n+1)}\right]\geq$$

$$\geq\mu^{\frac{1}{2}}(t-\tau)^{\frac{1}{2}}\exp\left[\frac{\left(x-y,A^{-1}(x-y)\right)}{4(n+1)}\right].$$

Supposing $C_2=C_5\mu^{\frac{-1}{2}}$ from the last estimate and (9) we conclude

$$|x-y|\leq C_2\exp\left[-\frac{\rho(x,y;t,\tau)}{n+1}\right]. \tag{10}$$

Let $a=\|a_{ij}\|,\ A^{-1}=\|b_{ij}\|,\ A=\|c_{ij}\|\ (i,j=1,...,n)$. We will show that

$$\left\|\left(b_{ij}\right)_t\right\|=-A^{-1}aA^{-1},\ (i,j=1,...,n). \tag{11}$$

Really, if $I=\|\delta_{ij}\|$ is unit matrix, 0 is zero-matrix, then

$$A\left\|\left(b_{ij}\right)_t\right\|+\left\|\left(c_{ij}\right)_t\right\|A^{-1}=\left\|\left(\delta_{ij}\right)_t\right\|=0.$$

Therefore, taking into account $\left\|\left(c_{ij}\right)_t\right\|=a$ we obtain

$$A\left\|\left(b_{ij}\right)_t\right\|=-aA^{-1},$$

and required equality (11) is proved.

From the other hand, if Γ_{ij} is algebraic complement of element c_{ij} of matrix A $(i,j=1,...,n)$, then

$$(\det A)_t=\sum_{i=1}^{n}\sum_{j=1}^{n}\left(c_{ij}\right)_t\Gamma_{ij}=\sum_{i,j=1}^{n}a_{ij}\Gamma_{ij}. \tag{12}$$

Using (11) and (12) we obtain for $i,j=1,...,n$

$$\Phi_{ij}^{-}=(\det A)^{\frac{1}{2n}}V(\rho)\frac{1}{4(1+\psi(\rho))}\sum_{m=1}^{n}b_{mi}(x_m-y_m)\sum_{p=1}^{n}b_{pj}(x_p-y_p)- \\ -\frac{1}{2}(\det A)^{\frac{1}{2n}}V(\rho)b_{ij}, \tag{13}$$

$$\Phi_t^{-}=\frac{1}{2n}(\det A)^{\frac{1}{2n}-1}\sum_{m=1}^{n}a_{ij}(y,t)\Gamma_{ij}\int_{-\infty}^{\rho}V(u)du+ \\ +(\det A)^{\frac{1}{2n}}V(\rho)\left[-\frac{n+1}{2n\det A}\sum_{i,j=1}^{n}a_{ij}(y,t)\Gamma_{ij}+\frac{1}{4}\left(x-y,A^{-1}a(y,t)A^{-1}(x-y)\right)\right], \tag{14}$$

where $V(\rho)=\exp\left[\int_0^{\rho}\frac{dv}{1+\psi(v)}\right]$.

Now we will show that

$$\int_{-\infty}^{\rho}V(u)du\geq(1+\psi(\rho))V(\rho). \tag{15}$$

Really, taking into account that function $\psi(\rho)$ is non-increasing by ρ, we have

$$\int_{-\infty}^{\rho}V(u)du=\int_{-\infty}^{\rho}(1+\psi(u))V(u)\frac{1}{1+\psi(u)}du\geq(1+\psi(\rho))\int_{-\infty}^{\rho}dV(u).$$

Now it is enough to note that $V(-\infty)=0$ and (15) is proved. Using (15) and the fact that

$$\sum_{i,j=1}^{n}a_{ij}(y,t)\Gamma_{ij}\geq\frac{1}{\det A}\sum_{i,j=1}^{n}a_{ij}(y,t)b_{ij}>0;\left(x-y,A^{-1}a(y,t)A^{-1}(x-y)\right)= \\ =\left(A^{-1}(x-y),a(y,t)A^{-1}(x-y)\right),$$

from (14) we obtain

$$\Phi_t^{-}\geq\frac{\psi(\rho)-n}{2n}(\det A)^{\frac{1}{2n}}V(\rho)\sum_{i,j=1}^{n}a_{ij}(y,t)b_{ij}+(\det A)^{\frac{1}{2n}}V(\rho)\times \\ \times\frac{1}{4}\left(A^{-1}(x-y),a(y,t)A^{-1}(x-y)\right). \tag{16}$$

But from the other hand

$$\left(A^{-1}(x-y),a(y,t)A^{-1}(x-y)\right)=\sum_{i,j=1}^{n}a_{ij}(y,t)\sum_{m=1}^{n}b_{mi}(x_m-y_m)\sum_{p=1}^{n}b_{pi}(x_p-y_p).$$

Thus from (13), (16) and last equality we obtain

$$L\Phi^{-}\leq\frac{(\det A)^{\frac{1}{2n}}V(\rho)}{4}\left[\frac{1}{1+\psi(\rho)}\sum_{i,j=1}^{n}\left|a_{ij}(x,t)-a_{ij}(y,t)\right|\left|\sum_{m=1}^{n}b_{mi}(x_m-y_m)\right|\times\right. \\ \left.\times\left|\sum_{m=1}^{n}b_{pj}(x_p-y_p)\right|-\frac{\psi(\rho)}{1+\psi(\rho)}\sum_{i,j=1}^{n}a_{ij}(y,t)\sum_{m=1}^{n}b_{mi}(x_m-y_m)\sum_{p=1}^{n}b_{pj}(x_p-y_p)\right]+ \\ +\frac{(\det A)^{\frac{1}{2n}}V(\rho)}{2}\left[\sum_{i,j=1}^{n}\left|a_{ij}(y,t)-a_{ij}(x,t)\right|\left|b_{ij}\right|-\frac{\psi(\rho)}{2n}\sum_{i,j=1}^{n}a_{ij}(y,t)b_{ij}\right]=i_1+i_2. \tag{17}$$

Further we have

$$i_1 \le \frac{(\det A)^{\frac{1}{2n}} V(\rho)|x-y|^2}{4(t-\tau)^2(1+\psi(\rho))}\left[C_6\varphi(|x-y|)-C_7\psi(\rho)\right]. \quad (18)$$

Moreover,

$$\sum_{i,j=1}^{n} a_{ij}(y,t)b_{ij} = Tr\left(aA^{-1}\right),$$

and if γ is eigenvalue of matrix aA^{-1}, and z is corresponding eigenvector, then

$$\gamma = \frac{(az,z)}{(Az,z)} \ge \frac{\mu^2}{t-\tau}.$$

Therefore

$$i_2 \le \frac{(\det A)^{\frac{1}{2n}} V(\rho)}{2(t-\tau)}\left[C_8\varphi(|x-y|)-C_9\psi(\rho)\right]. \quad (19)$$

Without loosing of generality, we could take $\varphi(2\delta) \le C_{10}\varphi(\delta)$. If $\rho < (n+1)\ln\frac{C_2}{\delta}$, then

$$C_6\varphi(|x-y|)-C_7\psi(\rho) \le (C_6C_{10}-C_7C_1)\varphi(\delta)$$

i.e. according to (18) for $C_1 \ge \frac{C_6C_{10}}{C_7}$ $i_1 \le 0$. If $\rho \ge (n+1)\ln\frac{C_2}{\delta}$, then taking into account (10) we have

$$C_6\varphi(|x-y|)-C_7\psi(\rho) \le \left(\frac{C_6}{C_1}-C_7\right)\psi(\rho),$$

and according to (18) for $C_1 \ge \frac{C_6}{C_7}$ $i_1 \le 0$. By the similar way from (19) we can conclude that for $C_1 \ge \frac{C_8C_{10}}{C_9}$ $i_2 \le 0$. We choose now $C_1 = \max\left\{\frac{C_6C_{10}}{C_7}, \frac{C_8C_{10}}{C_9}\right\}$. Then $i_1 \le 0, i_2 \le 0$ and from (17) the required estimate (6) follows. Lemma is proved.

Lemma 2. *If relative to coefficients of operator L conditions (2)-(4) are satisfied and*

$$\Phi^+(x,y;t,\tau) = (\det A(y;t,\tau))^{\frac{1}{2n}} \int_{-\infty}^{\rho(x,y;t,\tau)} \exp\left[-\int_0^u \frac{dv}{1+\psi(v)}\right]du,$$

then

$$L_{(x,t)}\Phi^+ \ge 0, \quad (x,t) \in O_\delta \cap D \cap \{(z,v): v > \tau\}.$$

This Lemma could be proved by the same scheme as it was done for previous one.

Theorem 1. *Let $(x,t) \in D \cap O_\delta$, $(y,\tau) \in \Gamma(D) \cap O_\delta, t > \tau$ and relative to coefficients of L operator conditions (2)-(5) are satisfied. Then*

$$C_{11}G(x-y,t-\tau) \le \Phi^-(x,y;t,\tau) \le$$

$$\le \begin{cases} C_{12}(t-\tau)^{1/2}, & \text{if } |x-y|^2 > 2(n+1)(t-\tau)\ln\frac{1}{t-\tau} \\ C_{13}G(x-y,t-\tau), & \text{if } |x-y|^2 \le 2(n+1)(t-\tau)\ln\frac{1}{t-\tau}. \end{cases} \quad (20)$$

Proof. Analogous to inequality (7) it can be shown that

$$\det A \le \mu^{-n}(t-\tau)^{-n}.$$

Therefore taking into account (15) we obtain

$$C_{14}(t-\tau)^{1/2}V(\rho) \le \Phi^- \le C_{15}(t-\tau)^{1/2}\int_{-\infty}^{\rho} V(u)du. \tag{21}$$

But from the other hand

$$\int_{-\infty}^{\rho} V(u)du \le C_{16}\int_{-\infty}^{\rho}\frac{1}{1+\psi(u)}V(u)du = C_{16}\int_{-\infty}^{\rho} dV(u) = C_{16}V(\rho).$$

With (21) it gives following inequality

$$C_{14}(t-\tau)^{1/2}V(\rho) \le \Phi^- \le C_{16}(t-\tau)^{1/2}V(\rho). \tag{22}$$

Further we have

$$V(\rho) \le 1, \quad \text{if} \quad \rho < 0; \tag{23}$$

$$V(\rho) = \exp[\rho]\exp\left[-\int_0^{\rho}\frac{\psi(v)}{1+\psi(v)}dv\right] = \frac{1}{(\det A)^{\frac{n+1}{2n}}}\times$$

$$\times\exp\left[-\frac{1}{4}\left(x-y, A^{-1}(x-y)\right)\right]\exp\left[-\int_0^{\rho}\frac{\psi(v)}{1+\psi(v)}dv\right] = j_1 j_2 j_3;$$

and so as

$$C_{17}(t-\tau)^{-\frac{n+1}{2}} \le j_1 \le C_{18}(t-\tau)^{-\frac{n+1}{2}},$$

then, supposing $C_{12} = C_{16}$, from (22)-(23) we obtain

$$\left.\begin{array}{ll} C_{19}(t-\tau)^{-n/2} j_2, & \text{if} \quad \rho < 0 \\ C_{19}(t-\tau)^{-n/2} j_2 j_3, & \text{if} \quad \rho \ge 0 \end{array}\right\} \le \Phi^- \le \left\{\begin{array}{ll} C_{12}(t-\tau)^{1/2} j_2, & \text{if} \quad \rho < 0 \\ C_{20}(t-\tau)^{-n/2} j_2 j_3, & \text{if} \quad \rho \ge 0. \end{array}\right. \tag{24}$$

Here we use the fact that $j_3 \ge 1$ for $\rho < 0$, $j \le 1$ for $\rho \ge 0$.

Now taking into account that D is R-domain at the neighborhood of point $(0,0)$ for $i,j = 1,\dots,n$, $v \in [\tau, t]$, we have $a_{ij}(y,v) = a_{ij}(y,\tau) = \delta_{ij}$. Therefore $A(y;t,\tau) = (t-\tau)I$, $A^{-1}(y;t,\tau) = (t-\tau)I$ and

$$j_2 = \exp\left[-\frac{|x-y|^2}{4(t-\tau)}\right]. \tag{25}$$

Supposing $C = C_{20}$, from (24) we obtain

$$\left.\begin{array}{ll} C_{19}G(x-y,t-\tau), & \text{if} \quad \rho < 0 \\ C_{19}G(x-y,t-\tau)j_3, & \text{if} \quad \rho \ge 0 \end{array}\right\} \le \Phi^- \le \left\{\begin{array}{ll} C_{12}(t-\tau)^{1/2}, & \text{if} \quad \rho < 0 \\ C_{13}G(x-y,t-\tau), & \text{if} \quad \rho \ge 0. \end{array}\right. \tag{26}$$

But for $\rho \ge 0$

$$\int_0^{\rho}\frac{\psi(v)}{1+\psi(v)}dv = \int_0^{\delta_1}\frac{\psi(v)}{1+\psi(v)}dv + \int_{\delta_1}^{\rho}\frac{\psi(v)}{1+\psi(v)}dv,$$

where $\delta_1 = (n+1)\ln\dfrac{C_2}{\delta}$. If $\rho \le \delta_1$, then

$$\int_0^{\rho}\frac{\psi(v)}{1+\psi(v)}dv \le C_{21}. \tag{27}$$

But if $\rho > \delta_1$, then

$$\int_{\delta_1}^{\rho}\frac{\psi(v)}{1+\psi(v)}dv \le C_{22}\int_{\delta_1}^{\rho}\varphi\left(C_2 e^{-\frac{v}{n+1}}\right)dv \le C_{23}\int_{0}^{C_2 e^{-\frac{\delta_1}{n+1}}}\frac{\varphi(z)}{z}dz = C_{24}\,. \qquad (28)$$

Supposing now $C_{11} = C_{19}\exp[-(C_{21}+C_{24})]$, from (26)-(28) we obtain

$$C_{11}G(x-y,t-\tau) \le \Phi^- \le \begin{cases} C_{12}(t-\tau)^{1/2}, & \text{if } \rho<0 \\ C_{13}G(x-y,t-\tau), & \text{if } \rho\ge 0\,. \end{cases} \qquad (29)$$

Now it is enough to note that according to (29) the sets $\{(x,t):\rho(x,y;t,\tau)<0\}$ and $\{(x,t):\rho(x,y;t,\tau)\ge 0\}$ coincide with sets $\left\{(x,t):|x-y|^2 > 2(n+1)(t-\tau)\ln\frac{1}{t-\tau}\right\}$ and $\left\{(x,t):|x-y|^2 \le 2(n+1)(t-\tau)\ln\frac{1}{t-\tau}\right\}$ correspondingly, therefore from (29) follows the required estimate (20). Theorem is proved.

Theorem 2. *If* $(x,t)\in D\cap O_\delta$, $(y,\tau)\in\Gamma(D)\cap O_\delta$, $t>\tau$ *and relative to coefficients of operator* L *conditions (2-5) are satisfied, then*

$$\left.\begin{array}{l} C_{25}(t-\tau)^{\frac{1}{2}}, \quad \text{if } |x-y|^2 > 2(n+1)(t-\tau)\ln\dfrac{1}{t-\tau} \\ C_{26}G(x-y,t-\tau), \quad \text{if } |x-y|^2 \le 2(n+1)(t-\tau)\ln\dfrac{1}{t-\tau} \end{array}\right\} \le \Phi^+(x,y;t,\tau) \le C_{27}G(x-y,t-\tau)\,.$$

This theorem could be proved by the similar way as it was done for previous one.

References

[1]. Ландис Е.М. *Уравнения второго порядка эллиптического и параболического типов.* М., «Наука» , 1971, 288 с.

[2]. Новрузов А.А. *О некоторых критериях регулярности граничных точек для линейных и квазилинейных параболических уравнений.* Докл. АН СССР, 1973, т. 209, № 4, с. 785-787.

[3]. Мамедов И.Т. *О суб- и суперрешениях параболического оператора.* Уч. зап. АзИНЕФТЕХИМ, 1974, № 6, с. 63-71.

[4]. Мамедов И.Т. *О регулярности граничных точек для линейных уравнений параболического типа.* Матем. заметки, 1976, т. 20, с. 717-723.

[5]. Evans L.C., Gariepy R.F. *Wiener's criterion for the heat equation.* Arch. Rath. Mech. Ann., 1982, v. 78, № 4, p. 293-314.

[6]. Ландис Е.М. *О регулярности граничной точки для уравнения теплопроводности.* В сб. «Качественная теория краевых задач математической физики», вып. I, Баку, «Элм», 1991, с. 69-96.

[7]. Мамедов И.Т. *Оценка модуля непрерывности решения первой краевой задачи для уравнения теплопроводности в граничной точке.* Труды ИММ АН Азербайджана, 1996, т. V(XIII), с. 30-50.

[8]. Carofalo N., Lanconelli E. *Wiener's criterion for parabolic equations with variable coefficients and its consequences.* Trans. of Amer. Math. Soc., 1988, v. 308, № 2, p. 811-836.

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