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# ASYMPTOTIC EXPANSION FOR THE DISTRIBUTION OF NONLINEAR RENEWAL PROCESS

Abstract

*In present paper the asymptotic expansion of high order for distribution of process of nonlinear reconstruction.*

**I. Introduction.** Let $\xi_n, n \geq 1$ be independent identically distributed random variables determined on some probability space $(\Omega, \mathcal{F}, P)$ and let $fa(t), t > 0, a > 0$ be a family of nonlinear (non-random) Borellian functions.

Put:

$$S_0 = 0, \quad S_n = \sum_{k=1}^{n} \xi_k, n \geq 1 \quad \text{is the}$$

$$\tau_a = \inf\{n \geq 1 : S_n > fa(n)\}$$

is the first exit moment beyond the nonlinear boundary $f_a(t)$.

Asymptotic expansion for the distribution of markovian moment $\tau_a$ has been studied in paper [1,2]. The first two terms for the asymptotic expansions on the boundary $f_a(t)$, and on the distributions of the random variables $\xi_1$ have been obtained in [2]. In this paper, the highest order asymptotic expansion for the distribution of $\tau_a$ is studied.

**2. Conditions and notations.** Assume, that for each $a > 0$ the function $fa(t) > 0$ and it is continuously differentiable for $t > 0$, where

$$\sup_{a,t} f'_a(t) < M = const. \tag{1}$$

We can easily verify that for the family of boundaries $fa(t) = K(t+a)^\beta + c$, where $K > 0, c \geq 0, 0 \leq \beta < 1$, the condition (1) is fulfilled with $M = K \cdot \beta$. Further, assume that:

$$P(\xi_1 \geq M) = 1$$

and

$$\rho_m = E|\xi - M\xi|^m < \infty, \quad m = \overline{3, r}. \tag{2}$$

Denote:

$$C_n = \frac{fa(n) - n\mu}{\sigma\sqrt{n}},$$

where $\mu = E\xi_1$ and $\sigma^2 = D\xi_1$.

To derive the highest order asymptotic expansions we assume that the characteristic function $\psi_\xi(\lambda)$ of r.v. $\xi_1$ must satisfy the following condition:

$$\overline{\lim_{|\lambda| \to \infty}} |\psi_\xi(\lambda)| < 1. \tag{3}$$

Remark that the condition (3) is fulfilled for any nonsingular distribution of r.v. $\xi_1$ [3], and also for any strongly non-latticed distribution [4].

We also denote:

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{u^2}{2}}du$$

$$\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}.$$

**3. The basic result.** Let the above enumerated conditions (1), (2) and (3) be fulfilled. Besides, let $C_n=O(1)$ for $n=n(a)\to\infty$.

Then:

$$P(\tau_a\le n)=\Phi(-C_n)-\varphi(C_n)\cdot$$
$$\cdot\sum_{k=3}^{r}n^{-\frac{k}{2}+1}R_k(C_n)+0\left(n^{-\frac{r}{2}+1}\right),$$

where $R_k=R_n(x), x\in(R)$ is a polynomial with real coefficients depending only on $\rho_1,\rho_2,...,\rho_r$, but not on $n$ and $z$ (or on other characteristics of the distribution $\xi_1$). See [3, p.607] for the construction of the polynomial $R_k(x)$.

**4. Proof of the basic result.**

We need the following

**Lemma.** *Let the conditions (1) and (2) be fulfilled. Then*

$$P(\tau_a\le n)=P(S_n>fa(n))$$

*holds.*

**Proof.** It is clear that

$$\{\omega:\tau_a>n\}=\{\omega:\max_{k\le n}(S_k-fa(k))\le 0\}. \qquad (4)$$

We have:

$$\max_{1\le k\le n}(S_k-fa(k))=\max_{1\le k\le n}\left[\sum_{i=1}^{k}(\xi_i-fa(i)-fa(i-1))+fa(0)\right]. \qquad (5)$$

It follows from the condition (2),that for each $i$ with probability 1

$$\xi_i-(fa(i)-fa(i-1))\ge 0.$$

Therefore, it follows from (5), that

$$\max_{1\le k\le n}(S_k-f_a(k))=\sum_{i=1}^{k}(\xi_i-f_a(i)-f_a(i-1))+f_a(0)=S_n-f_a(n).$$

Now the statement of the lemma follows from (4) and (5).

Now prove the statement of the theorem.

Denote

$$F_n(x)=P\left(\frac{S_n-n\mu}{\sigma\sqrt{n}}\le x\right).$$

By virtue of the lemma we have

$$P(\tau\le n)=1-F_n(C_n). \qquad (6)$$

Under the assumptions concerning the finiteness of the moment $p_r<\infty$ for some $r$, it holds the Edgeworts expression [3].

$$F_n(x)=\Phi(x)+\varphi(x)\sum_{i=3}^{r}n^{-\frac{i}{2}+1}R_i(x)+o\left(n^{-\frac{r}{2}+1}\right) \qquad (7)$$

uniformly in $x$.

Considering that $\Phi(x)+\Phi(-x)=1$, we obtain from (6) and (7) the affirmation of the theorem.

Note that the statement of the theorem for $r=3$ is contained in paper [2], where random variables with values of different signs have been considered.

## References

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