1999 VOL. X(XVIII)

## MECHANICS

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# WAVES OF DEFORMATIONS IN NETS

**Abstract**

*Characteristic equations of space motion of the net in local parameters are obtained. Existence of three type waves is shown.*

Many investigations [1-4] were devoted to the threads and threads systems. In the article [2] for the first time the net motion equations were given. Generally the investigations have been devoted to the onedimensional or the plane motion of the net. Here in contradistinction to the earlier obtained characteristic equations due to relative parameters the last ones have been obtained due to local parameters.

### 1. Equation of motion.

Write the equation of motion of the net elements in space [2] in the following form:

$$\sum_{i=1}^{2}\frac{\partial}{\partial s_i}\sigma_i l_i^j = \rho\frac{\partial^2 x_j}{\partial t^2} \quad (j=1,2,3), \tag{1}$$

where $t$ is time, $S_i$ are lagrange coordinates of particles of the net, $x_j$ are displacement components; $\sigma_i$ is strain; $l_i^j$ are directing cosines; $\rho$ is a mass of the unit area of the net at primary condition.

The geometric conditions [1] have a view:

$$(1+e_i)l_i^j = \frac{\partial x_j}{\partial s_i} \ . \tag{2}$$

Moreover

$$\sum_{j=1}^{3} l_i^j l_i^j = 1 \ . \tag{3}$$

Adding the deformation law

$$\sigma_i = \Phi_i(e_i), \tag{4}$$

where $\Phi_i$ is the given functional we come to the system of thirteen equations (1), (2), (3), (4) which is used for determination of thirteen unknown values $\sigma_i, l_i, l_i^j, x_j$.

Further for investigation it will be convenient to transform the pointed out equations system.

Having differentiate by $S_i$ by (2) we have

$$l_i^j\frac{\partial e_i}{\partial s_i} + (1+e_i)\frac{\partial l_i^j}{\partial s_i} = \frac{\partial^2 x_j}{\partial s_i^2} \ . \tag{5}$$

Multiplying the last equations by $l_i^j$ and adding up on $j$ we have

$$\frac{\partial e_i}{\partial s_i}\sum_{j=1}^{3} l_i^j l_i^j + (1+e_i)\sum_{j=1}^{3} l_i^j \frac{\partial l_i^j}{\partial s_i} = \sum_{j=1}^{3} l_i^j \frac{\partial^2 x_j}{\partial s_i^2}. \tag{6}$$

Having differentiate in (3) by $s_i$, that is

$$\sum_{j=1}^{3} l_i^j \frac{\partial l_i^j}{\partial s_i} = 0 \tag{7}$$

and taking into account (3) and (7) in (6) it can be obtained

$$\frac{\partial e_i}{\partial s_i} = \sum_{j=1}^{3} l_i^j \frac{\partial^2 x_j}{\partial s_i^2} \tag{8}$$

or substituting into (5) we obtain

$$(1+e_i)\frac{\partial l_i^j}{\partial s_i} = \frac{\partial^2 x_j}{\partial s_i^2} - l_i^j \sum_{k=1}^{3} l_i^k \frac{\partial^2 x_k}{\partial s_i^2}. \tag{9}$$

Having differentiate in (1) the product we have

$$\sum_{i=1}^{2}\left( l_i^j \frac{\partial \sigma_i}{\partial s_i} + \sigma_i \frac{\partial l_i^j}{\partial s_i}\right) = \rho \frac{\partial^2 x_j}{\partial t^2}. \tag{10}$$

If the consider the material of the threads be nonlinear elastic, that is, $\sigma_i = f(e_i)$, where $f(e_i)$ is some one-to-one function and introducing the denotations $f'(e_i)/\rho = a$, $\sigma_i/\rho(1+e_i) = b_i$ the equations (10) using (8), (9) can be reduced to the following

$$\sum_{i=1}^{2}\left[ a^2 l_i^j \sum_{k=1}^{3} l_i^k \frac{\partial^2 x_k}{\partial s_i^2} + b_i^2\left( \frac{\partial^2 x_j}{\partial s_i^2} - l_i^j \sum_{k=1}^{3} l_i^k \frac{\partial^2 x_k}{\partial s_i^2}\right)\right] = \frac{\partial^2 x_j}{\partial t^2}$$

or

$$\sum_{i=1}^{2}\left[ \left(a^2 - b_i^2\right) l_i^j \sum_{k=1}^{3} l_i^k \frac{\partial^2 x_k}{\partial s_i^2} + b_i^2 \frac{\partial^2 x_j}{\partial s_i^2}\right] = \frac{\partial^2 x_j}{\partial t^2} \tag{11}$$

Transforming (11) from the independent variables $s_i$ to $\xi$ and $\eta$ we apply the indefinition condition of the determinant of the coefficients for the pointed out derivatives is equated with zero, that is

$$\begin{vmatrix} \left(m_1^2 l_{11}^2 + b_1^2\right)\xi_1^2 + \left(m_2^2 l_{21}^2 + b_2^2\right)\xi_2^2 - \xi_t^2 & m_1^2 l_{11} l_{12}\xi_1^2 + m_2^2 l_{21} l_{22}\xi_2^2 & m_1^2 l_{11} l_{13}\xi_1^2 + m_2^2 l_{21} l_{23}\xi_2^2 \\ m_1^2 l_{12} l_{11}\xi_1^2 + m_2^2 l_{21} l_{22}\xi_2^2 & \left(m_1^2 l_{12}^2 + b_1^2\right)\xi_1^2 + \left(m_2^2 l_{22}^2 + b_2^2\right)\xi_2^2 - \xi_t^2 & m_1^2 l_{12} l_{13}\xi_1^2 + m_2^2 l_{22} l_{23}\xi_2^2 \\ m_1^2 l_{11} l_{13}\xi_1^2 + m_2^2 l_{21} l_{23}\xi_2^2 & m_1^2 l_{12} l_{13}\xi_1^2 + m_2^2 l_{22} l_{23}\xi_2^2 & \left(m_1^2 l_{13}^2 + b_1^2\right)\xi_1^2 + \left(m_2^2 l_{23}^2 + b_2^2\right)\xi_2^2 - \xi_t^2 \end{vmatrix} = 0$$

where $a^2 - b_i^2 = m_i^2$; $l_i^j = l_{ij}$.

Performing some operations we obtain the characteristic equation in the form:

$$a^2 b_1^4 \xi_1^6 + a^2 b_2^4 \xi_2^6 + b_1^2\left(a^4 + b_1^2 b_2^2 + a^2 b_2^2\right)\xi_1^4 \xi_2^2 +$$
$$+ b_2^2\left(a^4 + b_1^2 b_2^2 + a^2 b_1^2\right)\xi_1^2 \xi_2^4 - b_1^2\left(2a^2 + b_1^2\right)\xi_1^4 \xi_t^2 -$$

$$
-b_2^2\left(2a^2+b_2^2\right)\xi_2^4\xi_t^2-\left(a^4+a^2b_1^2+a^2b_2^2+3b_1^2b_2^2\right)\xi_1^2\xi_2^2\xi_t^2+
+\left(a^2+2b_1^2\right)\xi_1^2\xi_t^4+\left(a^2+2b_2^2\right)\xi_2^2\xi_t^4-\xi_t^6-\left(a^2-b_1^2\right)\left(a^2-b_2^2\right)\times
\times\left(b_1^2+b_2^2\right)l^2\left(\xi_1^2+\xi_2^2\right)\xi_1^2\xi_2^2+\left(a^2-b_1^2\right)\left(a^2-b_2^2\right)l^2\xi_1^2\xi_2^2\xi_t^2=0
$$

Where $l$ a cosine of the angle between the threads

$$
l=l_{11}l_{21}+l_{12}l_{22}+l_{13}l_{23}.
$$

The obtained characteristic equation can be represented as

$$
\left(a^2\xi_1^2+b_2^2\xi_2^2-\xi_t^2\right)\left(a^2\xi_2^2+b_1^2\xi_1^2-\xi_t^2\right)\left(b_1^2\xi_1^2+b_2^2\xi_2^2-\xi_t^2\right)-
-\left(a^2-b_1^2\right)\left(a^2-b_2^2\right)\left[\left(b_1^2+b_2^2\right)\left(\xi_1^2+\xi_2^2\right)+\xi_t^2\right]l^2\xi_1^2\xi_2^2=0 \tag{12}
$$

In case of insignificant a distortion of net's cells that is $l\approx 0$, characteristic equation (12) is disintegrated to three

$$
a^2\xi_1^2+b_2^2\xi_2^2-\xi_t^2=0
$$

$$
a^2\xi_2^2+b_1^2\xi_1^2-\xi_t^2=0
$$

$$
b_1^2\xi_1^2+b_2^2\xi_2^2-\xi_t^2=0
$$

that corresponds to the three wave types: longitudinal, distortion and transverse. However the pointed out waves don't arise in pure form that requires special investigation.

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Received May 19, 1999; Revised July 9, 1999.

Translated by Soltanova S.M.