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ON CALCULATION FOR CARRIED CAPABILITY OF DAMAGING BODIES AT NON-HOMOGENEOUS STATE

Abstract

The approaches to study time process of failure in the bodies with non-homogeneous stress state are discussed.

Comparative distincts of same investigations for isotropic and anisotropic bodies are analyzed. As an example the solution of the problem on scattered failure of the cylindrical anisotropic tube under action of internal pressure has been given.

The main problem, which we must solve at projecting, is to avoid failure of the projected elements of constructions and objects during the expected exploitation period. Most of constructions operate in conditions of complex stress state. Estimation of durability in these cases is difficult and when the material of the construction has viscous properties then it is problematic. First it is connected with difference of failure time of different parts of the construction. Expansion of the failed parts changes the bound of separation of the failed parts and unfailed ones. With the last the concept of failure front is connected introduced for the first time by L.M. Kachanov [1]. The same situation arises for the non-homogeneous stress state of constructions. The existing theories of strength are not fit for investigation of failure of such bodies. The element of the material is considered for which the failure criterion is fulfilled and description of the further behavior of this element under continuous loading in the frame of theory of strength is absent. Here the theories of damaging or the theories of scattered failure give great opportunities.

One of the ways of analysis of failure of the body which is at non- homogeneous stress state is the way based on concept of failure front. In addition to the determining equations and the failure criterion the supplementary suggestions are required which do not follow from the model of deformation and failure.

As far as at non-homogeneous stress state the stress levels are different in different points then in according to this the damaging degrees of these points also differ. The equations connected stresses with deformations- the determining equations- will be valid in each point until the corresponding failure criterion is fulfilled for it. Since that time the given particle of the material is not able to fulfill its function to carry some loading and is failed. Because of that predistribution of stresses happens in the body, which reduces to failure of the neighbour particles of the material further. During some time the failed part of the body grows until the whole construction losses its carrier capability.

Therefore, there are two stages of the scattered failure. The first stage called the stage of latent failure or incubation propagates till time t_0 when in the body for the first time the failed area forms which can consist of one point of the body as less. Further this area of the body grows. Motion of the failure front which characterizes growing of the failed area happens till the time t_p when the construction losses its carrier capability and completely fail. This period of time from t till t_p is called the stage of failure propagation. Determination of time t_p needs supplementary assumptions. Thus, for example, the hypotheses of transform of the motion velocity of the failure front into infinity are possible. However, this hypothesis is not always acceptable so as for some constructions the motion velocity of the failure front during all stage of failure propagation remains finite.

The motion equation of failure front is determined by failure criterion.

As a base model of the damaging body it is convenient the model [2] considering failure as some finite stage of deformation of the material. Convenient is in that one and the same operator characterizing the damages accumulation process comes as in deformation correlations as in failure criterion. In general case investigation of failure process is connected with determination of the stress state for the current moment of time with further check of performing of failure criterion for all unfilled part of the body. The same thing is usually managed to do by only numerical way, but it also gives mathematical difficulties. For dynamic problems in comparison with static ones the difficulties increase much more. But also for static problem even for the cases of continuous loading it is not managed to avoid mathematical troubles. Though even in those cases taking into account the damaging operator behaves as an ordinary operator of viscous flow, the stress state is managed to determine analytically, for the motion equation of failure front the non-linear integral Volterr equation of the second order. But even it takes place for isotropic bodies when stresses represent the rational functions of damaging operator whose interpretation is made on the base of correlations of algebra of resolving operators [3]. For anisotropic bodies the same thing doesn't take place. Only for materials for which anisotropy of mechanical properties develops via their mechanical characteristics it is managed to obtain the observed formulas and correlations.

Demonstrate the mentioned on calculation example for durable strength of circle tube subjected to internal pressure and having the property of cylindrical anisotropy. For this kind of loading the damaging operator is continuous and stress state can be determined using the correspondence principle by Volterr-Rabotnov. The solution of the elastic problem was given in [4]. Its analysis shows that the prevailing one is the circle stress σ_θ, which will determine failure process. For it the formula takes place:

$$\sigma_\theta = kp\frac{c^{k+1}}{1-c^{2k}}\left(\rho^{k-1}+\frac{1}{\rho^{k+1}}\right), \tag{1}$$

where p is the internal pressure, $\rho = r/b$, $c = a/b$, a is the inner radius, b is the external radius of the tube, r is the current radius of the point of inner area of the tube. We have

$$k = \sqrt{\frac{E_\theta}{E_r}\frac{1-\nu_{zr}\nu_{rz}}{1-\nu_{z\theta}\nu_{\theta z}}}. \tag{2}$$

The experimental investigations [5] showed, that for a wide class of polymer and composite materials the rerhologic properties are isotropic, because of that for constant Poisson's coefficients substitution of elasticity modulus in circle direction E_θ and in radial direction E_r by the corresponding operators it doesn't change a view of formula (2) remaining it as a scalar.

Maximum σ_0 by (1) reaches on the inner contour of the circle area of cross-section of the tube (plane deformation state). So for the first time failure will happen namely there occupying new layers during some time. For setting up of the motion equation of the failure front by [1] we take $c = \beta(\tau)$, $\rho = \beta(t)$, $0 \le \tau \le t$. Then formula

[1] will be expressed via the unique function $\beta(t)$ determining the radial coordinate of the failure front:

$$\sigma_\theta(t,\tau)=kp\frac{\beta^{k+1}(\tau)}{1-\beta^{2k}(\tau)}\left\{\beta^{k-1}(t)+\beta^{-k-1}(t)\right\}. \qquad (3)$$

The failure criterion [2] is represented in a view

$$\sigma_\theta(t,t)+\int_0^t M(t,\tau)\sigma_\theta(t,\tau)d\tau=\sigma_0\,, \qquad (4)$$

where $M(t,\tau)$ is a nuclear of the damaging operator, σ_0 is a limit of strength of the defectless material and taking into account (3) in it we will obtain the following equation of motion of the failure front:

$$p(t)\frac{1+\beta^{2k}(t)}{1-\beta^{2k}(t)}+\frac{1+\beta^{2k}(t)}{\beta^{k+1}(t)}\int_0^t M(t,\tau)\frac{\beta^{k+1}(\tau)p(\tau)}{1-\beta^{2k}(\tau)}d\tau=\frac{\sigma_0}{k}. \qquad (5)$$

Investigate the process for the case $p=p_0=const$, $M(t,\tau)=m=const$. Then introduce the denotations:

$$Z(t)=\frac{1+\beta^{2k}(t)}{1-\beta^{2k}(t)};\quad g=\frac{\sigma_0}{p} \qquad (6)$$

and the integral equation (5) will be:

$$Z(t)+\frac{\lambda Z(t)}{\left(Z(t)-1\right)^{\frac{k+1}{2k}}\left(Z(t)+1\right)^{\frac{k-1}{2k}}}\int_0^t\left(Z(\tau)+1\right)^{\frac{k-1}{2k}}\left(Z(\tau)-1\right)^{\frac{k+1}{2k}}d\tau=\frac{g}{k} \qquad (7)$$

excluding the integral term by repeated differentiation we will obtain the following differential equation with respect to $Z(t)$:

$$\frac{dZ}{dt}=\frac{\lambda k^2Z^2\left(Z^2-1\right)}{g(Z+k)-kZ^2(1+kZ)}. \qquad (8)$$

The initial condition for this equation is determined by incubation to value which we will obtain from the integral equation (6), moreover when $0\le\tau$; $t<t_0$ there is not failure and when $\beta(\tau)=\beta(t)=\beta_0=a/b$, that is $Z(\tau)=Z(t)=Z_0=\left(1+\beta_0^{2k}\right)/\left(1-\beta_0^{2k}\right)$. We have for it

$$\lambda t_0=\frac{g}{Z_0}-1. \qquad (9)$$

For $t\ge t_0$ therefore we have Cauchy problem (8), (9). Numerical calculation has been carried out for $g=4$ and $\beta_0=0.5$. The calculated values for $k=0.5$ $\left(E_0<E_r\right)$ and $k=2$ $\left(E_0>E_r\right)$ are given in tables:

$k=0.5$ Table 1

λt	0.33	0.34	0.35	0.36
β	0.5	0.51	0.52	0.53

$k=2$

Table 2

λt	2.5	2.6	2.7	2.8
β	0.5	0.52	0.56	0.61

Analysis of calculations shows that if rigidity in radial direction is more than rigidity in transverse direction then failure begins earlier, but further it goes slowly almost with constant velocity. If rigidity in radial direction is less than rigidity in transverse direction, then failure begins much later, but after it happens very quickly with increasing velocity. For isotropy, $k=1$, the well-known results are obtained [2].

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