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OSCILLATIONS OF NON-HOMOGENEOUS RECTANGLE PLATE

Abstract

In the paper natural oscillations of the rectangle non-homogeneous plate on viscous-elastic foundation are investigated. It is supposed that elasticity modulus of the material depends on coordinates of points of the material. In general case the motion equation with respect to deflection of the plate is obtained.

Consider the rectangle plate fabricated of the continuous non-homogeneous material, which is on the resistant base.

The coordinate system is chosen so that the axes OX and OY are arranged in the midplane of the plate, axis OZ is directed perpendicularly them.

It is supposed that elasticity of the plane material depends on coordinates of points of the plate so: $E = E_1(x,y)f(z)$, and Poisson's coefficient ν is considered constant $\left(\nu = \frac{1}{2}\right)$.

Connection between the stress and deformation components has the following view:

$$\sigma_{11} = \frac{4}{3}E_1(x,y)f(z)\left(\varepsilon_{11} + \frac{1}{2}\varepsilon_{22}\right);$$

$$\sigma_{22} = \frac{4}{3}E_1(x,y)f(z)\left(\varepsilon_{22} + \frac{1}{2}\varepsilon_{11}\right); \qquad \sigma_{12} = \frac{2}{3}E_1(x,y)f(z)\varepsilon_{12}. \tag{1}$$

Use Kirkhgoff- Lyav hypotheses:

$$\varepsilon_{11} = l_{11} - z\chi_{11}; \quad \varepsilon_{22} = l_{22} - z\chi_{22}; \quad \varepsilon_{12} = l_{12} - z\chi_{12}. \tag{2}$$

Here l_{11}, l_{22}, l_{12} are infinite small changes of deformation of midplane; χ_{11}, χ_{22}, χ_{12} are infinite small changes of curvature, moreover:

$$\chi_{11} = \frac{\partial^2 W}{\partial x^2}; \quad \chi_{22} = \frac{\partial^2 W}{\partial y^2}; \quad \chi_{12} = \frac{\partial^2 W}{\partial x \partial y}, \tag{3}$$

where W is a deflection of the midplane of the plane. Forces and moments components are calculated by formulas:

$$T_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} dz; \quad M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z dz. \tag{4}$$

Taking into account (1), (2) from (4) we obtain

$$T_{11} = \frac{4}{3}E_1(x,y)\left[a_0\left(l_{11} + \frac{1}{2}l_{22}\right) - a_1\left(\chi_{11} + \frac{1}{2}\chi_{22}\right)\right],$$

$$T_{22} = \frac{4}{3}E_1(x,y)\left[a_0\left(l_{22} + \frac{1}{2}l_{11}\right) - a_1\left(\chi_{22} + \frac{1}{2}\chi_{11}\right)\right], \tag{5}$$

$$T_{12} = \frac{2}{3}E_1(x,y)\left(a_0 l_{12} - a_1\chi_{12}\right).$$

By analogy for components of moments we find

$$M_{11}=\frac{4}{3}E_1(x,y)\left[a_1\left(l_{11}+\frac{1}{2}l_{22}\right)-a_2\left(\chi_{11}+\frac{1}{2}\chi_{22}\right)\right];$$
$$M_{22}=\frac{4}{3}E_1(x,y)\left[a_1\left(l_{22}+\frac{1}{2}l_{11}\right)-a_2\left(\chi_{22}+\frac{1}{2}\chi_{11}\right)\right]; \quad (6)$$
$$M_{12}=\frac{2}{3}E_1(x,y)(a_1l_{12}-a_2\chi_{12}).$$

In these formulas the following denotations have been introduced:

$$a_i=\int_{-h/2}^{h/2}f(z)z^i dz. \quad (7)$$

As it is known [1, 2], motion equations of rectangle plates consist of:

$$\frac{\partial T_{11}}{\partial x}+\frac{\partial T_{12}}{\partial y}+\rho h\frac{\partial^2 u}{\partial t^2}=0;\quad \frac{\partial T_{12}}{\partial x}+\frac{\partial T_{22}}{\partial y}+\rho h\frac{\partial^2 \vartheta}{\partial t^2}=0; \quad (8)$$

$$\frac{\partial^2 M_{11}}{\partial x^2}+2\frac{\partial^2 M_{12}}{\partial x\partial y}+\frac{\partial^2 M_{22}}{\partial y^2}+T_{11}\frac{\partial^2 W}{\partial x^2}+2T_{12}\frac{\partial^2 W}{\partial x\partial y}+T_{22}\frac{\partial^2 W}{\partial y^2}+a_0W+$$
$$+a_1\frac{\partial W}{\partial t}+\rho h\frac{\partial^2 w}{\partial t^2}=0. \quad (9)$$

Here ρ is a density of the material; h is a thickness of the plate; a_0, a_1 are constant quantities characterizing the properties of the medium.

It be noted that deformations of the midplane satisfy the equation of deformations compatibility:

$$\frac{\partial^2 l_{22}}{\partial x^2}+\frac{\partial^2 l_{11}}{\partial y^2}-2\frac{\partial^2 l_{12}}{\partial x\partial y}=0. \quad (10)$$

In general case taking into account (5) and (6) from (8) and (9) the systems of motion equations of non-homogeneous rectangle plates in displacements are obtained.

Now consider the approximate formulation of the problem. Suppose that in the system (8) we can neglect the inertial terms, that is $\left(\rho h\frac{\partial^2 u}{\partial t^2}\to 0 \text{ and } \rho h\frac{\partial^2 \vartheta}{\partial t^2}\to 0\right)$, moreover we will consider that

$$T_{11}\approx T_{22}\approx T_{12}\approx 0.$$

Taking into account (5) from (6) we obtain:

$$M_{11}=D_1E_1(x,y)\left(\chi_{11}+\frac{1}{2}\chi_{22}\right);$$
$$M_{22}=D_1E_1(x,y)\left(\chi_{22}+\frac{1}{2}\chi_{11}\right); \quad (11)$$
$$M_{12}=\frac{D_1}{2}E(x,y)\chi_{12},$$

where

$$D_1=\frac{4}{3}\left(\frac{a_1^2}{a_0}-a_2\right).$$

Substituting (11) into (9) and taking into account (3) after some transformations we will obtain the following final motion equation with respect to deflection of the plate:

$$D_1 E_1(x,y)\Delta\Delta W + 2D_1\frac{\partial E_1}{\partial x}\frac{\partial}{\partial x}\Delta W + 2D_1\frac{\partial E_1}{\partial y}\frac{\partial}{\partial y}\Delta W + D_1\frac{\partial^2 E_1}{\partial x\partial y}\cdot\frac{\partial^2 W}{\partial x\partial y} +$$

$$+ D_1\frac{\partial^2 E_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2} + \frac{1}{2}\frac{\partial^2 W}{\partial y^2}\right) + D_1\frac{\partial E_1}{\partial y^2}\left(\frac{\partial W}{\partial y^2} + \frac{1}{2}\frac{\partial W}{\partial x^2}\right) + a_0 W + a_1\frac{\partial W}{\partial t}\rho h\frac{\partial^2 W}{\partial t^2} = 0, \quad (12)$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Let consider for concreteness the following view of non-homogeneity:

$$E_1(x,y) = E_0\left(1 + \frac{\varepsilon x}{a}\right). \quad (13)$$

Taking into account (13) equation (12) is transformed to the view:

$$D_0\left(1 + \frac{\varepsilon}{a}x\right)\Delta\Delta W + 2D_0\frac{\varepsilon}{a}\frac{\partial}{\partial x}\Delta W + a_0 W + a_1\frac{\partial W}{\partial t} + \rho h\frac{\partial^2 W}{\partial t^2} = 0 \quad (14)$$

$$(D_0 = E_0 D_1).$$

We will seek solution of (14) by the method of separation of variables in the form:

$$W(x,y,t) = \varphi(x,y)T(t). \quad (15)$$

Substituting (15) into (14) we will obtain

$$D_0\left(1 + \frac{\varepsilon}{a}x\right)\Delta\Delta\varphi\cdot T + 2D_0\frac{\varepsilon}{a}\frac{\partial}{\partial x}\Delta\varphi\cdot T + a_0\varphi\cdot T + a_1\varphi\frac{\partial T}{dt} + \rho h\varphi\frac{d^2 T}{dt^2} = 0. \quad (16)$$

Represent (16) in the form:

$$\frac{\frac{D_0}{\rho h}\left(1 + \frac{\varepsilon}{a}x\right)\Delta\Delta\varphi + 2\frac{D_0}{\rho h}\frac{\varepsilon}{a}\frac{\partial}{\partial x}\Delta\varphi + \frac{a_0}{\rho h}\varphi}{\varphi} = -\frac{\frac{a_1}{\rho h}\frac{dT}{dt} + \frac{d^2 T}{dt^2}}{T} = \omega_0^2. \quad (17)$$

From (17) we will obtain the following system of equations:

$$\left(1 + \frac{\varepsilon x}{a}\right)\Delta\Delta\varphi + 2\frac{\varepsilon}{a}\frac{\partial}{\partial x}\Delta\varphi + \tilde{a}_0\varphi - \omega^2\varphi = 0, \quad (18)$$

$$\frac{d^2 T}{dt^2} + \frac{a_1}{\rho h}\cdot\frac{dT}{dt} + \omega_0^2 T = 0, \quad (19)$$

where the denotations have been taken:

$$\omega^2 = \omega_0^2\frac{\rho h}{D_0}; \quad \tilde{a}_0 = \frac{a_0}{D_0}. \quad (20)$$

Here ω_0 is a frequency of natural oscillations of the plate.

Therefore, frequency of natural oscillations of non-homogeneous plate is determined from (18). Solution of (18) is constructed by Bubnov- Galerkin method:

$$\omega^2 = \frac{\int_0^a\int_0^b\left\{\left(1 + \frac{\varepsilon}{a}x\right)\Delta\Delta\varphi + 2\frac{\varepsilon}{a}\frac{\partial}{\partial x}\Delta\varphi + \tilde{a}_0\varphi\right\}\varphi dxdy}{\int_0^a\int_0^b\varphi^2 dxdy}. \quad (21)$$

Natural forms of oscillations of the beared rectangle plate with the sides a and b can be sought in the form:

$$\varphi(x,y)=W_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \tag{22}$$

Here m, n are the numbers of halfwaves along the corresponding sides.

Taking into account (22) from (21) it is not difficult to obtain:

$$\omega^2=\frac{A_1I_1+A_2I_2-A_3I_3}{I_1}. \tag{23}$$

Here the following denotations have been introduced:

$$I_1=\int_0^a\sin^2\frac{m\pi x}{a}dx\int_0^b\sin^2\frac{n\pi y}{b}dy;$$

$$I_2=\int_0^a x\sin^2\frac{m\pi x}{a}dx\int_0^b\sin^2\frac{n\pi y}{b}dy;$$

$$I_3=\int_0^a\sin\frac{m\pi x}{a}\cos\frac{m\pi x}{a}dx\int_0^b\sin^2\frac{n\pi y}{b}dy.$$

$$A_1=\tilde{a}_0+\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2; \tag{24}$$

$$A_2=\frac{\varepsilon}{a}\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2;$$

$$A_3=2\frac{\varepsilon}{a}\left(\frac{m\pi}{a}\right)\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right].$$

Opening the integrals (24) from (23) for determination of natural frequency of non-homogeneous plate we will obtain the following:

$$\omega^2=\left(1+\frac{\varepsilon}{2}\right)\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2+\tilde{a}_0. \tag{25}$$

As it is seen from (25) in limiting cases hence the well- known classic solutions [1, 5] are obtained.

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