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CYCLIC TORSION OF THE BAR FABRICATED FROM THE VISCOUS-ELASTIC-PLASTIC MATERIAL. DEFORMATION AND DURABLE FAILURE

Abstract

The stress-strain of the circle bar of the viscous-elastic-plastic material for cyclic torsion is determined. The times of origin of damages in the body and the time before failure are calculated considering the action of stress- state.

Let the linear bar of circle cross-section with radius R is subjected to cyclic torsion with torsion moments $M_k(t)$ $(k = 1,2,...)$. Time of action of each moment $M_k(t)$ is $t_k - t_{k-1}$ $(t_0 = 0)$. Assume the properties of the material of the bar as for the first torsion from natural state as for loading-off and alternating sign torsion are described by Moscwitin V.V. correlations [1]. The determining correlations of the material of the bar for torsion from natural state are:

$$\frac{S_{ij}^{(1)}}{2G_0} = \varphi^{(1)}\left(\varepsilon_+^{(1)}\right)e_{ij}^{(1)} - \int_0^t R(t-\tau)\varphi^{(1)}\left(\varepsilon_+^{(1)}\right)e_{ij}^{(1)}d\tau\,, \tag{1}$$

$$\theta^{(1)} = 0\,. \tag{2}$$

Here $S_{ij}^{(1)} = \sigma_{ij}^{(1)} - \sigma_{cp}^{(1)}\delta_{ij}$; $e_{ij}^{(1)} = \varepsilon_{ij}^{(1)} - \varepsilon_{cp}^{(1)}\delta_{ij}$ are deviators of stress tensors $\sigma_{ij}^{(1)}$ and deformation tensors $\varepsilon_{ij}^{(1)}$; $\sigma_{cp}^{(1)} = \sigma_{ij}^{(1)}\delta_{cp}/3$; $\varepsilon_{cp}^{(1)} = \varepsilon_{ij}^{(1)}\delta_{ij}/3$, δ_{ij} are Kronecker's symbols, $\theta^{(1)} = 3\varepsilon_{cp}^{(1)}$, $\varepsilon_+^{(1)} = \left(\frac{2}{3}e_{ij}e_{ij}\right)^{1/2}$ is deformation intensively. Function $\varphi^{(1)}\left(\varepsilon_+^{(1)}\right)$ characterizes degree of physical non-linearity of the bar material for the first torsion and the function $R(t)$ is determined by hereditary properties of the material. These functions are found forever material by experiment using the corresponding method. As it follows from (2), the material is considered mechanical non-compressible.

Use as usual the hypotheses of plane cross- sections and the assumption on that the radial fibers remain linear with unchangeable angles between them, which we will consider, fulfilled during all cyclic torsion process. Displacements for initial torsion are:

$$u_r^{(1)} = u_z^{(1)} = 0\,;\quad u_\varphi^{(1)} = 2\theta_1(t)rz\,. \tag{3}$$

Hence

$$\varepsilon_r^{(1)} = \varepsilon_\varphi^{(1)} = \varepsilon_z^{(1)} = \varepsilon_{r\varphi}^{(1)} = \varepsilon_{rz}^{(1)} = 0\,;\quad \varepsilon_{z\varphi}^{(1)} = \theta_1(t)r\,. \tag{4}$$

From six equations (1), (2) taking into account (4) remain only one equation

$$\frac{\sigma_{z\varphi}^{(1)}}{2G_0} = \varphi^{(1)}\left(\frac{2\varepsilon_{z\varphi}^{(1)}}{\sqrt{3}}\right)\varepsilon_{z\varphi}^{(1)} - \int_0^t R(t-\tau)\varphi^{(1)}\left(\frac{2\varepsilon_{z\varphi}^{(1)}}{\sqrt{3}}\right)\varepsilon_{z\varphi}^{(1)}d\tau\,. \tag{5}$$

As it is seen, the problem is reduced to determination of the function $\theta_1(t)$. Taking approximations $\varphi^{(1)}\left(\varepsilon_+^{(1)}\right) = Q_1\left(\varepsilon_+^{(1)}\right)^{\gamma_1}$ valid, where $Q_1 = const$, $\gamma_1 = const$ we will obtain the expression for torsion of the bar $\theta_1(t)$:

$$\theta_1^{1+\gamma_1}(t) = \frac{\gamma_1 + 4}{4\pi\left(\frac{2}{\sqrt{3}}\right)^{\gamma_1} G_0 Q_1 R^{\gamma_1+4}} \left(M_1(t) + \int_0^t \Gamma(t-\tau) M_1(\tau) d\tau \right).$$

Here $\Gamma(t-\tau)$ is a resolvent of the nuclear $R(t-\tau)$. After determination of $Q_1(t)$ displacement $u_\varphi^{(1)}$ is found by formula (3), deformation $\varepsilon_{Z\varphi}^{(1)}$ is found by (4) and stress $\sigma_{Z\varphi}^{(1)}$ by (5). Write out the expression of stress $\sigma_{Z\varphi}^{(1)}$ in case $M_1(t) = M_0 H(t)$, where $M_0 = const$, $H(t)$ is unit Heavyside function:

$$\sigma_{Z\varphi}^{(1)} == \frac{4+\gamma_1}{2\pi R^3}\left(\frac{r}{R}\right)^{1=\gamma_1} M_0 \ . \tag{6}$$

Now let loading happen beginning from moment $t = t_k \ (k = 1,2,\ldots)$ and the further valuable torsion with torsion moments $M_k(t) = (-1)^{k-1} M_0 H(t - t_{k-1})$. For k-th loading stresses $\sigma_{Z\varphi}^{(k)}$ and deformations $\varepsilon_{Z\varphi}^{(k)}$ arise in the bar. Following [1] introduce the differences

$$\begin{aligned} \sigma_{Z\varphi}^{*(k)} &= (-1)^k \left(\sigma_{Z\varphi}^{(k-1)} - \sigma_{Z\varphi}^{(k)}\right); \\ \varepsilon_{Z\varphi}^{*(k)} &= (-1)^k \left(\varepsilon_{Z\varphi}^{(k-1)} - \varepsilon_{Z\varphi}^{(k)}\right), \end{aligned} \tag{7}$$

Take the determining equations of the bar material for its k-th torsion in view [1]:

$$\frac{\sigma_{Z\varphi}^{*(k)}}{2G_0} = \varphi^{(k)}\left(\frac{2\varepsilon_{Z\varphi}^{*(k)}}{\sqrt{3}}\right)\varepsilon_{Z\varphi}^{*(k)} - \int_0^{t-t_{k-1}} R(t - t_{k-1} - \tau)\varphi^{(k)}\left(\frac{2\varepsilon_{Z\varphi}^{*(k)}}{\sqrt{3}}\right)\varepsilon_{Z\varphi}^{*(k)} d\tau \ . \tag{8}$$

If we take $\varphi^{(k)}(x) = Q_k x^{\gamma_k}$, then according to the theorem on alternative loading by Moscwitin V.V. [1] $\sigma_{Z\varphi}^{*(k)}$, $\varepsilon_{Z\varphi}^{*(k)}$ is solution $\sigma_{Z\varphi}^{(1)}$ and $\varepsilon_{Z\varphi}^{(1)}$ where it should make substitutions: M_0 by $2M_0$, Q_1 by Q_k, γ_1 by γ_k, t by $t - t_{k-1}$. After determination of $\sigma_{Z\varphi}^{*(k)}$, $\varepsilon_{Z\varphi}^{*(k)}$ the desired stress $\sigma_{Z\varphi}^{(k)}$ and deformation $\varepsilon_{Z\varphi}^{(k)}$ are found by (7). Finally, for $\sigma_{Z\varphi}^{(k)}$ we obtain the following

$$\sigma_{Z\varphi}^{(k)} = \frac{M_0}{2\pi R^3} \cdot \frac{r}{R}\left[(4+\gamma_1)\left(\frac{r}{R}\right)^{\gamma_1} - 2\sum_{i=1}^{k}(-1)^i (4+\gamma_i)\left(\frac{r}{R}\right)^{\gamma_i}\right]. \tag{9}$$

Now let's consider the problem on durable cyclic failure of the investigated bar under action stress (9) in it. Use the criterion of cyclic strength of viscous-elastic-plastic bodies for non-stationary cyclic loading [2] considering the important experimental fact of existence of incubation of damage accumulation. For damaging condition in case of isothermal loading from [2] we have:

$$(1+m_1)\sum_{k=1}^{N'} \int_{t_{k-1}}^{t_k} \frac{(t'-\tau)^{m_1} d\tau}{t_{0N}^{1+m_1}\left(\sigma_k^*\right)} + D(1+m_2)\int_0^{t'} \frac{(t'-\tau)^{m_2} d\tau}{t_0^{1+m_2}(\sigma)} = L \ . \tag{10}$$

And the condition of cyclic strength, which also follows from [2], has a view:

$$(1+m_1)\sum_{k=1}^{N_*} \int_{t_{k-1}}^{t_k} \frac{(t_*-\tau)^{m_1} d\tau}{t_{0N}^{1+m_1}\left(\sigma_k^*\right)} + D(1+m_2)\int_0^{t_*} \frac{(t_*-\tau)^{m_2} d\tau}{t_0^{1+m_2}(\sigma(\tau))} = L + D\left(1 - A^{1+m_2}\right). \tag{11}$$

In (10) and (11) the quantities m_1, m_2, D, α, A are material constants. Functions $t_0(\sigma)$, $t_{0N}\left(\sigma_k^*\right)$ are material functions, correspondingly the times before failure of the pattern of bar material for ordinary creeping $\sigma = \sigma_0 = const$ and for symmetric cyclic by stress $\sigma_k^* = const$; $\sigma(t) = \left(S_{ij}S_{ij}\right)^{1/2}$; $\sigma_k^* = (-1)^k\left(\sigma_{k-1}(t_{k-1}) - \sigma_k(t)\right)$; σ_k is modulus of σ on the k- part $t_{k-1} \le t \le t_k$ $(t_0 = 0)$ of monotone changing of this modulus. Quantities t' and t^* are correspondingly time of origin of damages in the body and time failure for alternating $\sigma = \sigma(t)$, $\sigma_k^* = \sigma_k^*(t)$. These times are determined by the damaging condition (10) and the condition of cyclic strength (11), which follows from [2] for the conditions

$$\frac{t_1(\sigma)}{t_0(\sigma)} \approx \frac{t_{1N}\left(\sigma_k^*\right)}{t_{0N}\left(\sigma_k^*\right)} \approx A = const,$$

$$\frac{t_{1N}\left(\sigma_k^*\right)}{t_1\left(\dfrac{\sigma_k^*}{2}\right)} \approx \frac{t_{0N}\left(\sigma_k^*\right)}{t_0\left(\dfrac{\sigma_k^*}{2}\right)} \approx B = const.$$

In these conditions $t_1 = t_1(\sigma)$ and $t_{1N} = t_{1N}\left(\sigma_k^*\right)$ are material functions, they are times of origin of damages in the body for constant $\sigma = \sigma_0 = const$, $\sigma_k^* = \sigma_{k0}^* = const$. Unknown times t' and t_* are connected with unknown number of loading before appearance of damages N' and with unknown number of loading before failure N_* by correlations:

$$t' = \sum_{k=1}^{N'}\left(t_k - t_{k-1}\right);\quad t_* = \sum_{k=1}^{N_*}\left(t_k - t_{k-1}\right),\quad (t_0 = 0). \tag{12}$$

In (10) and (11) the constants D and L are expressed by A, B, m_1, m_2:

$$D = \left(1 - A^{1+m_1}\right) \Big/ \left[\left(1 - A^{1+m_2}\right)\left(1 - B^{1+m_2}\right)\right];$$

$$L = A^{1+m_1} + D(AB)^{1+m_2}.$$

The material functions $t_0 = t_0(\sigma)$ and $t_{0N} = t_{0N}\left(\sigma_k^*\right)$ are approximated by the following correlations [3]:

$$t_0 = t_0(\sigma) = t_{0S}\exp\left[\beta_0\left(1 - \frac{\sigma}{\sigma_S}\right)\right], \tag{13}$$

$$t_{0N} = t_{0N}\left(\sigma_k^*\right) = t_{0NS}\exp\left[\beta_{0N}\left(1 - \frac{\sigma_k^*}{\sigma_S^*}\right)\right]. \tag{14}$$

Here σ_S and σ_S^* are corresponding reduction stresses: $t_{0S} = t_0(\sigma_S)$, $t_{0NS} = t_{0N}\left(\sigma_S^*\right)$.

In our case it is not difficult to determine:

$$\sigma = 2\sigma_{Z\varphi}^{(k)}, \tag{15}$$

$$\sigma_k^* = \sqrt{2}\,\frac{M_0}{\pi R^3}\left(4 + \gamma_k\right)\left(\frac{r}{R}\right)^{1+\gamma_k}. \tag{16}$$

Now damaging condition (10) and the condition of cyclic strength (11) using (13)-(16) pass to correlations:

$$\frac{D}{S_0} e^{F_{\kappa p y z}(r,N',\gamma_{N'})}\left(t'\right)^{1+m_2} - \frac{1}{S_N}\sum_{k=1}^{N'} e^{\Phi_{\kappa p y z}(r,\gamma_k)}\left[\left(t'-t_k\right)^{1+m_1} - \left(t'-t_{k-1}\right)^{1+m_1}\right] = L, \quad (17)$$

$$\frac{D}{S_0} e^{F_{\kappa p y z}(r,N_*,\alpha_{N_*})} t_*^{1+m_2} - \frac{1}{S_N}\sum_{k=1}^{N_*} e^{\Phi_{\kappa p y z}(r,\gamma_k)}\left[\left(t_*-t_k\right)^{1+m_1} - \left(t_*-t_{k-1}\right)^{1+m_1}\right] =$$

$$= L + D\left(1 - A^{1+m_2}\right). \quad (18)$$

Here

$$S_0 = t_{0S}^{1+m_2} e^{(1+m_2)\beta_0}; \quad S_N = t_{0NS} 1 + m_1 e^{(1+m_1)\beta_{0N}};$$

$$F_{\kappa p y z}(r,k,\gamma_k) = (1+m_2)\beta_0 \frac{\sqrt{2}}{\sigma_S}\cdot\frac{M_0}{2\pi R^3}\frac{r}{R}\times$$

$$\times\left[(4+\gamma_2)\left(\frac{r}{R}\right)^{\gamma_1} - 2\sum_{i=2}^{k}(-1)^i(4+\gamma_i)\left(\frac{r}{R}\right)^{\gamma_i}\right];$$

$$\Phi_{\hbar p-\iota}(r,\gamma_k) = (1+m_1)\beta_{0N}\frac{\sqrt{2}}{\sigma_S^*}\cdot\frac{M_0}{\pi R^3}(4+\gamma_k)\left(\frac{r}{R}\right)^{1+\gamma_k}.$$

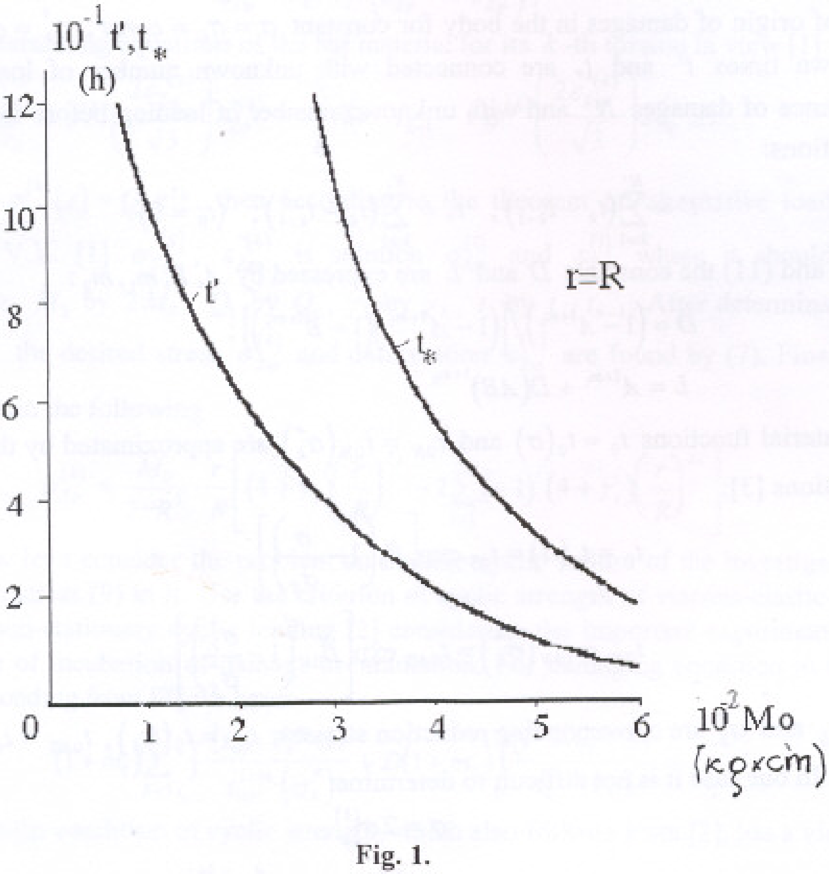

Fig. 1.

Using (17) and (18) the graphs of dependencies $t' \sim M_0$, $t_* \sim M_0$ on $r = R$ (fig.1) and also $t' \sim r$, $t_* \sim r$ for the fixed $M_0 = 400\,\text{kg}\cdot\text{sm}$ have been constructed (fig. 2). The data corresponding to nickel alloy ЭИ 867 for temperature 1123^0 K [1, 3]: $t_k - t_{k-1} = \tau = 1\,\text{sec}$; taking into account (12) $-t' = N'\tau$, $t_* = N_*\tau$, $t_k = k\tau$; $\beta_0 = 1{,}6$; $\beta_{0N} = 13.8$; $m_1 = 0{,}2$; $m_2 = 0{,}8$; $t_{0S} = 100\ \tau$; $t_{0NS} = 83{,}3\ \tau$; $\sigma_S = 125$ MPa; $\sigma_S^* = 600$ MPa; $\beta = 0{,}38$; $A = 0{,}54$; $R = 1\,\text{dm}$; $\gamma_1 = -0{,}8$; $\gamma_2 = -1$; $\gamma_k = \gamma_2(k-1)^\delta$ $(k \ge 2)$; $\delta = 0{,}6$.

The represented graphs for the given data let determine durability of the viscous-elastic-plastic bar working on cyclic torsion. Analogous result can be obtained also in case of other numerical data.

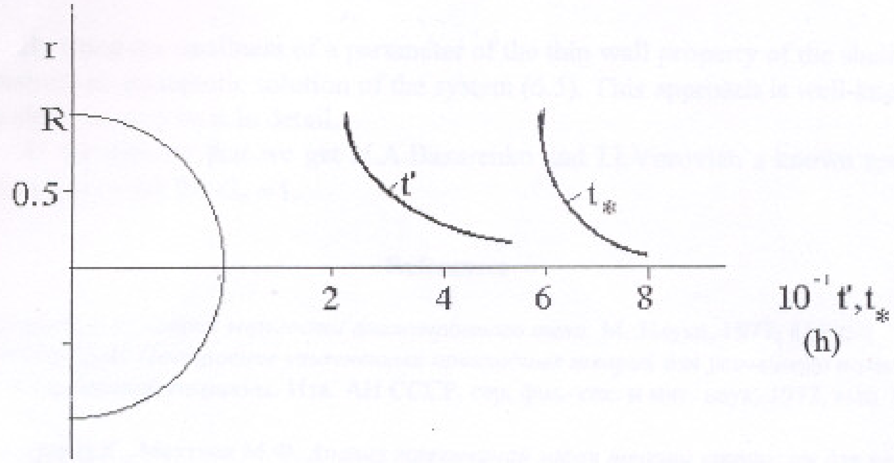

Fig. 2.

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