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ON DYNAMICS OF RECTANGULAR BARS

Abstract

The problems involving the dynamics of the elastic semi-infinite rectangular bar are investigated. Closed form solutions are obtained for different boundary conditions and for one special case it was extended to numerical calculations and graphs.

§1. Prism under action of axial loads.

The half-infinite rectangle prism is considered which occupies on the chosen Cartesian coordinate system the part of spaces: $-a \le x \le a;\quad -b \le y \le b;\quad z \ge 0$, where $2a$ and $2b$ are sizes of cross section.

At moment $t = 0$, the linear elastic prism which is in nondeformed state, is subjected to action of suddenly applied axial forces distributed, at the end section $z = 0$. It is obviously, the process is described by three-dimensional Lame equations system, which has a following view in vector form:

$$\rho\frac{\partial^2 \overline{U}}{\partial t^2} = (\lambda + \mu) grad div \vec{U} + \mu\Delta\vec{U} \qquad (1)$$
$$\vec{U} = \vec{U}(u, v, w)$$

where $\vec{U}$ is a displacement vector, ρ is a density of material.

As it is well known from [1], equation system (1) under boundary conditions:

$$\left.\begin{aligned} &\sigma_{zz} = \sigma_0(x,y)f(t) \\ &u = 0 \\ &v = 0 \end{aligned}\right\} \quad \text{under } z = 0 \qquad (2)$$

and under zero initial data, is reduced to the more simplified system:

$$\begin{aligned} &\mu q H_2\psi_2 = (\lambda + 2\mu)H_1\varphi \\ &H_2\psi_1 = 0 \\ &H_0 H_2\psi_2 = -\frac{\sigma(x,y)}{\mu} f(p), \end{aligned} \qquad (3)$$

where

$$H_1 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \left(\frac{p^2}{c_i^2} + q^2\right), \quad i = 1,2$$

$$H_0 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - q^2$$

are Helmholtz operators, $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$; $c_2 = \sqrt{\mu/\rho}$; and functions φ, ψ_1 and ψ_2 are connected with two-fold integral transformations (Laplace on t, and Fourier on z) functions of displacements by the following formulas:

$$\overline{u}_s = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi_1}{\partial y} - q\frac{\partial \psi_2}{\partial x};$$

$$\overline{v}_s = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi_1}{\partial x} - q\frac{\partial \psi_2}{\partial y}; \tag{4}$$

$$\overline{w}_c = q\varphi - \frac{\partial^2 \psi_2}{\partial x^2} - \frac{\partial^2 \psi_2}{\partial y^2}$$

Here indexes s and c point to sin and cos of Fourier transformations, and p and q, accordingly, are parameters of transformations by Laplace and Fourier.

System (3) need to integrate in context of lateral conditions, which are chosen in this case in following variants:

$$\text{a)}\ \left.\begin{matrix}\sigma_{xx}=0\\ \sigma_{xy}=0\\ \sigma_{xz}=0\end{matrix}\right\} \text{ under } x=\pm a \qquad \left.\begin{matrix}\sigma_{yy}=0\\ u=w=0\end{matrix}\right\} \text{ under } y=\pm b$$

$$\text{b)}\ \left.\begin{matrix}\sigma_{xx}=0\\ \sigma_{xy}=0\\ \sigma_{xz}=0\end{matrix}\right\} \text{ under } x=\pm a \qquad \left.\begin{matrix}\sigma_{yx}=0\\ \sigma_{yz}=0\\ v=0\end{matrix}\right\} \text{ under } y=\pm b \tag{5}$$

that is two opposite surfaces $x=\pm a$ are free from forces, but the rest two: $y=\pm b$ in the case of a) covered nondeformed by flexible membranes: but in the case of b) they are contacted with the slither hard surface (sliding contact).

We search the solution of system (1.3) for symmetrical $\sigma_0(x,y)$ in a view:

$$\varphi = \sum_k f_k(x)\cos\beta_k y \tag{6}$$

$$\psi_1 = \sum_k l_k(x)\sin\beta_k y \tag{7}$$

$$\psi_2 = \sum_k g_k(x)\cos\beta_k y, \tag{8}$$

where parameter β_k in different variants takes the following values:

a) $\beta_k = \left(\frac{1}{2}+k\right)\frac{\pi}{b};$

b) $\beta_k = \frac{\pi k}{b};\quad k=0,1,\ldots$

Choice of solution in a view (6) promotes automatic satisfaction the conditions on lateral surfaces $y=\pm b$.

Factors functions can be defined from the system (3). Substituting (8) into the third equation of the system (3) we get the differential equation $g_k(x)$;

$$g_k^{IV}(x) - g''(x)\left[2\beta_k^2 + \frac{p^2}{c_2^2} + 2q^2\right] + g_k(x)\left[\beta_k^4 + \beta_k^2 q^2 + \left(\frac{p^2}{c_2^2}+q^2\right)\left(\beta_k^2+q^2\right)\right] = \frac{\theta\sigma_k(x)f(p)}{\mu} \tag{9}$$

where $\sigma_k(x) = \frac{1}{2b}\int_{-b}^{b}\sigma_0(x,y)\cos\beta_k y\,dy$.

Denote one quotient solution, corresponding to the right-hand side of equation (9) by $\Omega_{2k}(x)$. Here we can notice, that $\sigma_0(x,y)=\sigma_0(y)$ or $\sigma_0(x,y)=\sigma_0=const$ this quotient solution will be the following:

$$\Omega_{2k}=-\frac{\sigma_k f(p)}{\mu v_{2k}^2\left(\beta_k^2+q^2\right)}, \tag{10}$$

where σ_k are constant numbers.

It is not difficult to perceive that the general solution of equation (9) is represented in a view:

$$g_k(x)=A_k ch v_{2k}x+B_k ch\sqrt{\beta_k^2+q^2}x+\Omega_{2k}(k) \tag{11}$$

From first two equations system (3) we can define $f_k(x)$ and $l_k(x)$:

$$f_k(x)=c_k ch v_{1k}x+qB_k ch\sqrt{\beta_k^2+q^2}x+\Omega_{1k}(k) \tag{12}$$

$$l_k(x)=l_k sh v_{2k}x \tag{13}$$

Analogously for the simplest variants $\sigma_0(x,y)$

$$\Omega_{1k}=-\frac{\sigma_k f(p)q}{(\lambda+2\mu)v_{1k}^2\left(\beta_k^2+q^2\right)} \tag{14}$$

denotations were accepted here: $v_{ik}=\sqrt{\dfrac{p}{c_1^2}+q^2+\beta_k}$; $i=1,2$; $k=\overline{0,\infty}$.

It is easy to see that a field of displacements, corresponding to members, contains $ch\sqrt{\beta_k^2+q^2}$, is equal to zero, as formulas (4) show it. Therefore, these members in solutions (11) and (12) must be fallen off and the remained three ensembles of constants may be correctly defined from conditions on surfaces $x=\pm a$. However, note that only for the case $\sigma_0(x,y)=\sigma_0=const$ and when on verges $y=\pm b$ the sliding contact conditions take place (i.e. variant b), all Ω_{ik} except Ω_{i0}, are equal to zero. In this case condition $\sigma_{xy}\big|_{x=\pm a}=0$ is satisfied automatically. A_0 and B_0 are completely defined from remained two conditions under $x=\pm a$; then solution of the second of system (3) is trivial and all $l_k=0$ $\left(k=\overline{0,\infty}\right)$.

But in other cases for free lateral surfaces it is impossible not to take a torsional motion into account, described by ψ_1.

In general case, constants A_k, l_k, c_k are determined from the following system:

$$(\lambda+2\mu)\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\psi_1}{\partial x\partial y}-q\frac{\partial^2\psi_2}{\partial x^2}\right)+$$

$$+\lambda\left(\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\psi_1}{\partial x\partial y}-q^2\varphi+q\frac{\partial^2\psi_2}{\partial x^2}\right)=0$$

$$2\frac{\partial^2\varphi}{\partial x\partial y}-2q\frac{\partial^2\psi_2}{\partial x\partial y}+\frac{\partial^2\psi_1}{\partial y^2}-\frac{\partial^2\psi_1}{\partial x^2}=0$$

$$2q\frac{\partial\varphi}{\partial x}+q\frac{\partial\psi_1}{\partial y}-q^2\frac{\partial\psi_2}{\partial x}-\frac{\partial^3\psi_2}{\partial x^3}-\frac{\partial^3\psi_2}{\partial x\partial y^2}=0$$

under $x=\pm a$, which has been received by presentations of formulas (4) in the transformed conditions on verges $x=\pm a$.

Substituting the solutions of (6)-(8) type taking into account (11)-(13) into (15) for determination of constants A_k, l_k, c_k we obtain the following linear system of algebraic equations:

$$\begin{Bmatrix} c_k \\ A_k \\ l_k \end{Bmatrix} \{D\}_k = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}, \tag{15}$$

where the matrix of the third rank $\{D\}$ has the following components:

$$\{D\}_k = \left\{ \begin{matrix} \left[v_{1k}^2(\lambda + 2\mu) - \lambda\left(\beta_k^2 + q^2\right) \right] chv_{1k}a & -2\mu q v_{2k}^2 chv_{2k}a & 2\mu\beta_k v_{2k} chv_{2k}a \\ -2\beta_k v_{1k} shv_{1k}a & -2\beta_k q v_{2k} shv_{2k}a & -\left(\beta_k^2 + v_{2k}^2\right) shv_{2k}a \\ 2q v_{1k} shv_{1k}a & -v_{2k}\left(q^2 + v_{2k}^2 - \beta_k^2\right) & q\beta_k shv_{2k}a \end{matrix} \right\} \tag{16}$$

$$\begin{aligned} d_1 &= \lambda\Omega_{1k}\left(\beta_k^2 + q^2\right) \\ d_2 &= 0 \qquad \text{(for constants } \Omega_{ik}\text{)} \\ d_3 &= 0 \end{aligned} \tag{17}$$

In such case the final expressions of functions $f_k(x), g_k(x)$ and $l_k(x)$ are presented in a view:

$$\begin{aligned} f_k(x) &= \frac{|D_1|_k}{|D|_k} chv_{1k}x - \frac{\sigma_k f(p)}{(\lambda + 2\mu)v_{1k}^2\left(\beta_k^2 + q^2\right)} \\ g_k(x) &= \frac{|D_2|_k}{|D|_k} chv_{2k}x - \frac{\sigma_k f(p)}{\mu v_{2k}^2\left(\beta_k^2 + q^2\right)} \\ l_k(x) &= \frac{|D_3|_k}{|D|_k} shv_{2k}x, \end{aligned} \tag{18}$$

where the following designations were accepted:

$$\begin{aligned} |D|_k = {} & 2\mu v_{2k}^2 v_{1k}\left[2q^2\left(\beta_k^2 + v_{2k}^2\right) + q^2\beta_k^2 + \beta_k^2\left(q^2 + v_{2k}^2 - \beta_k^2\right)\right] \times \\ & \times chv_{2k}a \cdot shv_{2k}a \cdot shv_{1k}a - \left[\left(\beta_k^2 + v_{2k}^2\right)\left(q^2 + v_{2k}^2 - \beta_k^2\right) + q^2\beta_k^2\right] \times \\ & \times \left[v_{1k}^2(\lambda + 2\mu) - \lambda\left(\beta_k^2 + q^2\right)\right] v_{2k} sh^2 v_{2k}a \cdot chv_{1k}a \end{aligned} \tag{19}$$

$$|D_1|_k = +\frac{\lambda\sigma_k f(p)}{(\lambda + 2\mu)v_{1k}^2} q v_{2k}\left[\left(\beta_k^2 + v_{2k}^2\right)\left(q^2 + v_{2k}^2 - \beta_k^2\right) + q^2\beta_k^2\right] sh^2 v_{2k}a \tag{20}$$

$$|D_2|_k = \frac{\lambda\sigma_k f(p)}{(\lambda + 2\mu)v_{1k}} q^2\left(\beta_k^2 + v_{2k}^2\right) shv_{1k}a \cdot shv_{2k}a \tag{21}$$

$$|D_3|_k = -\frac{\lambda\sigma_k f(p)}{(\lambda + 2\mu)v_{1k}} q\beta_k v_{2k}\left(3q^2 + v_{2k}^2 - \beta_k^2\right) shv_{1k}a \cdot shv_{2k}a. \tag{22}$$

As it is already known, the general solutions have a view of infinite series (6)-(8), and considering the expression (18)-(22), we can confirm that their convergence up to the derivatives of third order is obvious.

As it was expressed above, at case $\sigma_0(x,y) = \sigma_0 = const$, when sides $y = \pm b$ contact with expressed an ideal smooth surface, (takes place variant b)) solution takes a simple type:

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$$= \begin{Bmatrix} \left[v_{1k}^2(\lambda+2\mu)-\lambda\left(\beta_k^2+q^2\right)\right]chv_{1k}a & -2\mu q v_{2k}^2 chv_{2k}a & 2\mu\beta_k v_{2k} chv_{2k}a \\ -2\beta_k v_{1k} shv_{1k}a & -2\beta_k q v_{2k} shv_{2k}a & -\left(\beta_k^2+v_{2k}^2\right)shv_{2k}a \\ 2q v_{1k} shv_{1k}a & -v_{2k}\left(q^2+v_{2k}^2-\beta_k^2\right) & q\beta_k shv_{2k}a \end{Bmatrix} \qquad (16)$$

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where the following designations were accepted:

$$\begin{aligned} |D|_k &= 2\mu v_{2k}^2 v_{1k}\left[2q^2\left(\beta_k^2+v_{2k}^2\right)+q^2\beta_k^2+\beta_k^2\left(q^2+v_{2k}^2-\beta_k^2\right)\right]\times \\ &\times chv_{2k}a\cdot shv_{2k}a\cdot shv_{1k}a-\left[\left(\beta_k^2+v_{2k}^2\right)\left(q^2+v_{2k}^2-\beta_k^2\right)+q^2\beta_k^2\right]\times \\ &\times\left[v_{1k}^2(\lambda+2\mu)-\lambda\left(\beta_k^2+q^2\right)\right]v_{2k}sh^2v_{2k}a\cdot chv_{1k}a \end{aligned} \qquad (19)$$

$$|D_1|_k = +\frac{\lambda\sigma_k f(p)}{(\lambda+2\mu)v_{1k}^2} q v_{2k}\left[\left(\beta_k^2+v_{2k}^2\right)\left(q^2+v_{2k}^2-\beta_k^2\right)+q^2\beta_k^2\right]sh^2v_{2k}a \qquad (20)$$

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$$\frac{v_{20}^4}{D_0^*} \approx \frac{1}{\mu}\left(\frac{1}{v_{20}} - \frac{eq^2}{v_{20}^3}\right), \text{ where } e = 1 - \frac{c_1 - c_2}{2c_2} + \frac{\lambda}{\lambda + 2\mu}.$$

Finally we can give a view to expression (1.26):

$$\dot{W} \approx -q^2 \frac{\lambda\sigma_0}{\lambda + 2\mu}\left[\frac{q^2 + v_{20}^2}{v_{20}^3}\frac{shv_{20}a}{chv_{20}a}\frac{1}{v_{20}^2}\frac{chv_{10}x}{chv_{10}a} - \right.$$
$$\left. -2\frac{chv_{20}x}{v_{20}^2 chv_{20}a}\frac{shv_{10}a}{v_{10}chx_{10}a}\right]\left(\frac{1}{v_{20}} - \frac{eq^2}{v_{20}^3}\right) - \frac{\sigma_0}{(\lambda + 2\mu)}\frac{1}{v_{20}^2} \qquad (28)$$

Expression in the square bracket, as it is seen, represents the meromorphic function in hall-planes $\operatorname{Re} p > 0$, and disappears at infinity $|p| = R_n \to \infty$ and also has single poles in points of axis $\operatorname{Re} p = 0$.

So we can use the second theorem on expansions [2], by which

$$F(t) = \sum_{p_k} res_{p_k} F^*(p) e^{pt},$$

where function $F^*(p) = L[F(t)]$ has above pointed properties.

First note, that for small values of t, the last member in expression (28) dominates: $-\frac{\sigma_0}{\lambda + 2\mu}\frac{1}{v_{10}^2} = -\frac{\pi}{2}\frac{\sigma_0 c_1}{\lambda + 2\mu}H\left(t - \frac{z}{c_1}\right)$, so as the first member, by rough estimations, is the smallest value for two degrees by t or: $\dot{W} \approx t^2 O(t) + O(t)$ for small values of t.

However, give the inverse functions of expressions, coming into expressions (28), for construction of necessary graphs:

1) $$\frac{q^2 + v_{20}^2}{v_{20}^3}\frac{shv_{20}a}{chv_{20}a} \div c_2 aq\sin(qc_2 t) - $$
$$-2\sum_k \frac{c_2}{a}\frac{(q^2 - a_k^2)\sin\left(c_2 t\sqrt{q^2 + \alpha_k^2}\right)}{\alpha_k^2\sqrt{q^2 + \alpha_k^2}}$$

2) $$\frac{1}{v_{i0}^2}\frac{chv_{i0}x}{chv_{i0}a} \div \frac{c_i}{q}\sin(qc_i t) - $$
$$-2\sum_m \frac{c_i}{a}\frac{(-1)^m\cos(\alpha_m x)\cdot\sin\left(c_i t\sqrt{q^2 + \alpha_m^2}\right)}{\alpha_m^2\sqrt{q^2 + \alpha_m^2}}, \quad i = 1,2$$

3) $$\frac{shv_{10}a}{v_{10}chv_{10}a} \div \frac{2c_1}{a}\sum_k \frac{\sin\left(c_1 t\sqrt{q^2 + \alpha_k^2}\right)}{\sqrt{q^2 + \alpha_k^2}}$$

4) $$\frac{1}{v_{20}}\left[1 - \frac{eq^2}{v_{20}^2}\right] \div c_2 J_0(c_2 qt) - \frac{e}{q}\frac{\sqrt{\pi}c_2^2 t}{\Gamma(3/2)}J_1(c_2 qt)$$

$$\frac{v_{20}^4}{D_0^*} \approx \frac{1}{\mu}\left(\frac{1}{v_{20}} - \frac{eq^2}{v_{20}^3}\right), \text{ where } e = 1 - \frac{c_1 - c_2}{2c_2} + \frac{\lambda}{\lambda + 2\mu}.$$

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3) $$\frac{shv_{10}a}{v_{10}chv_{10}a} \div \frac{2c_1}{a}\sum_k \frac{\sin\left(c_1 t\sqrt{q^2 + \alpha_k^2}\right)}{\sqrt{q^2 + \alpha_k^2}}$$

4) $$\frac{1}{v_{20}}\left[1 - \frac{eq^2}{v_{20}^2}\right] \div c_2 J_0(c_2 qt) - \frac{e}{q}\frac{\sqrt{\pi}c_2^2 tq}{\Gamma(3/2)}J_1(c_2 qt)$$

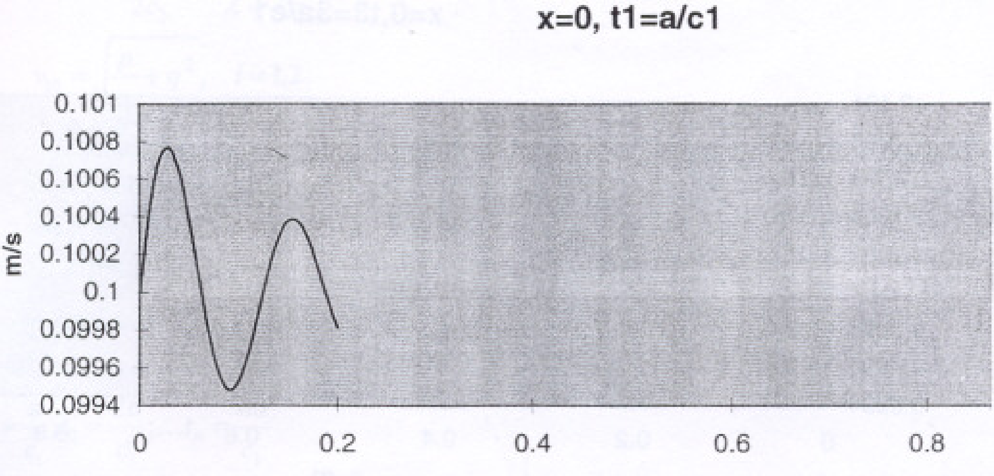


Figure 1. Graph of $\dot{W}$ at time $t_1 = \dfrac{a}{c_1}$

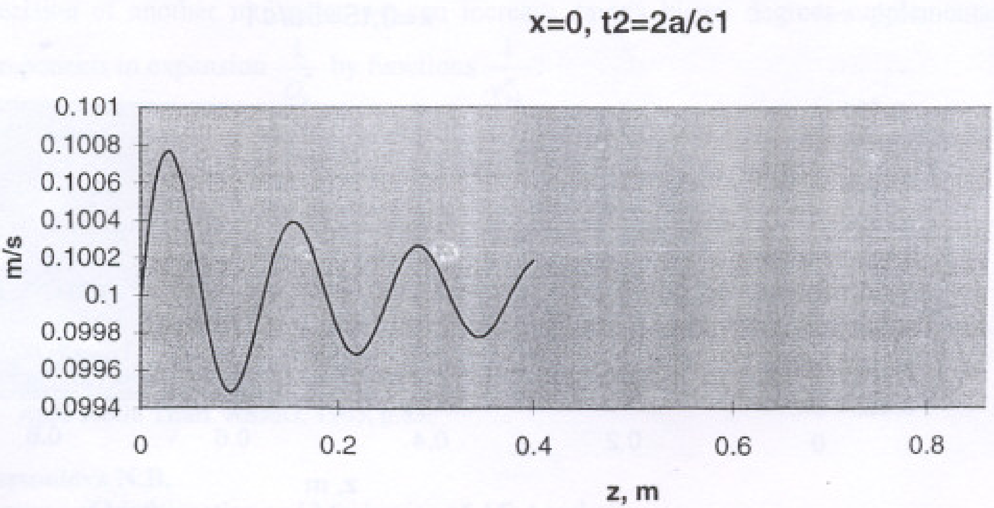
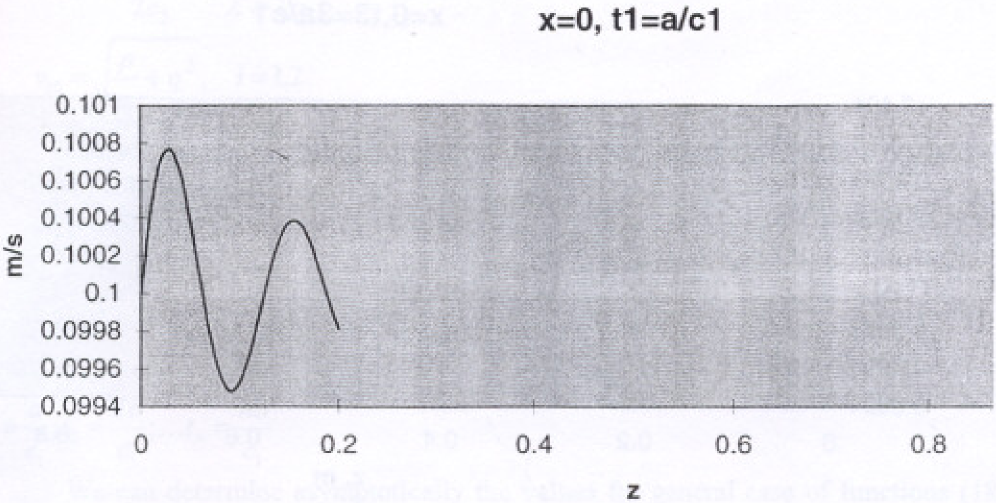


Figure 2. Graph of $\dot{W}$ at time $t_2 = \dfrac{2a}{c_1}$



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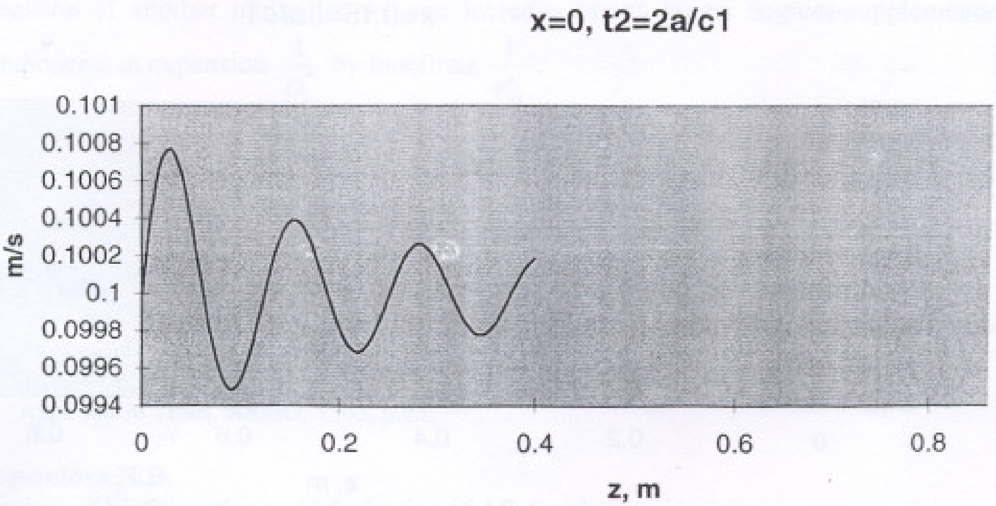

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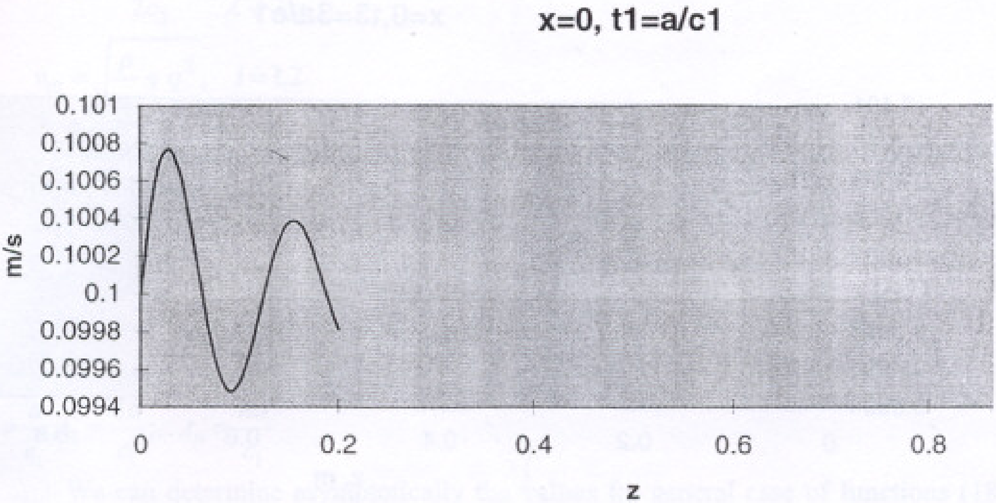

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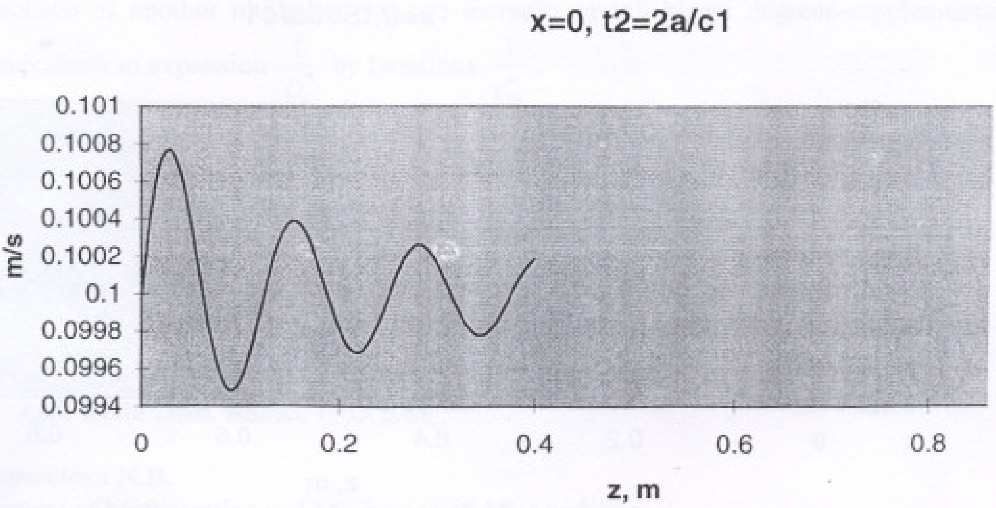

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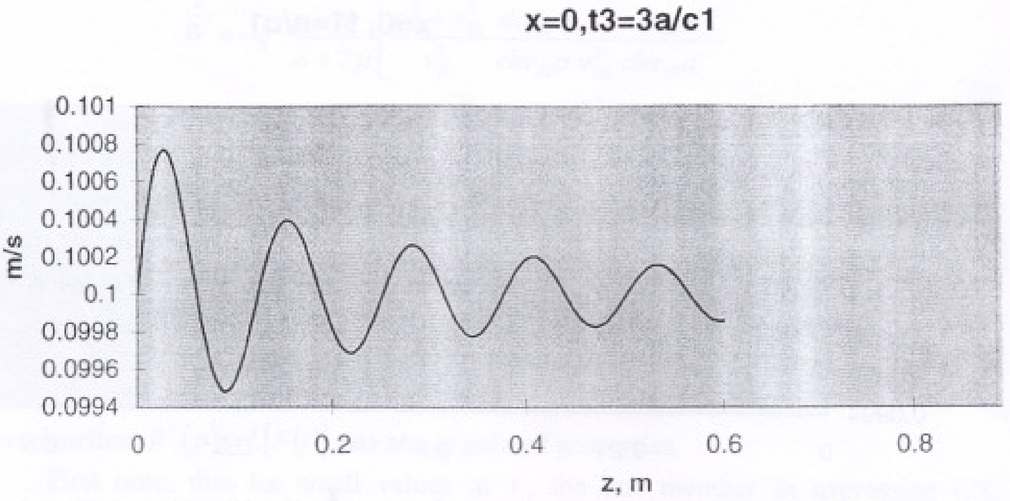

Figure 3.Graph of $\dot{W}$ at time $t_3 = \dfrac{3a}{c_1}$

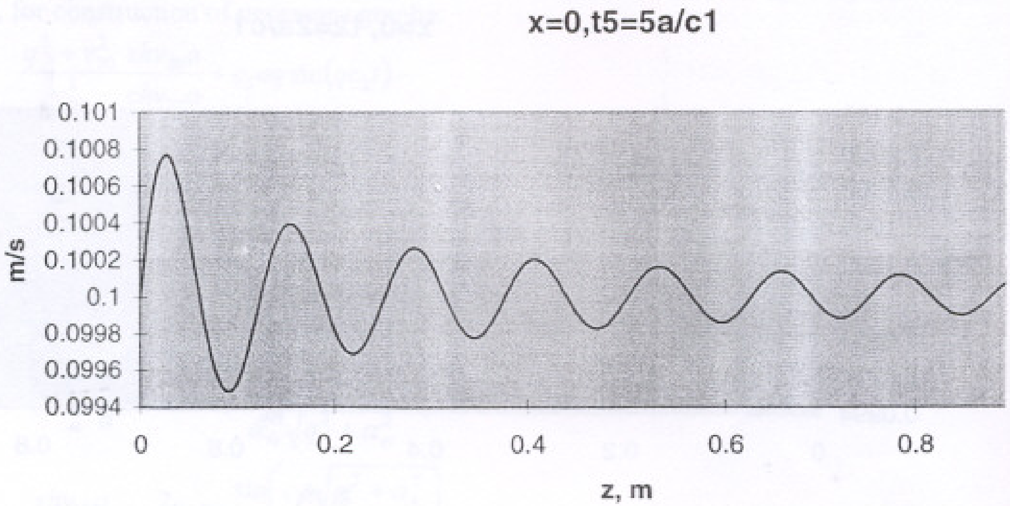

Figure 4.Graph of $\dot{W}$ at time $t_5 = \dfrac{5a}{c_1}$

$$\alpha_k = \left(\frac{\pi}{2} + \pi k\right)\frac{1}{a}$$

$$e = 1 - \frac{c_1 - c_2}{2c_2} + \frac{\lambda}{\lambda + 2\mu}$$

$$v_{i0} = \sqrt{\frac{p}{c_i} + q^2}, \quad i = 1,2$$

Calculation of $\dot{W}$ has been carried out for the following values of parameters and for the following intervals of time

$c_1 = 2c_2 = 2000 m/c$

$\lambda = 2\mu = 2000000 kg/cm^2$

$\sigma_0 = -200 kg/cm^2$

$a = 0.2m$

$t_1 = \frac{a}{c_1}; t_2 = \frac{2a}{c_1}; \ldots t_n = \frac{na}{c_1}$

We can determine asymptotically the values for general case of functions (18)-(22) by analogous method. For this it is necessary to separate the rational, meromorphic part of functions with ordinary poles, but to the remained multiplier of $\frac{1}{D_k^*}$ type we can use the method described above for case $D_k^* = D_0^*$.

The inverse functions of the rational part are also determined for any time. Precision of another multiplier we can increase saving bigger degrees-supplementary components in expansion $\frac{1}{D_k^*}$ by functions $\frac{1}{v_{2k}^n}$.

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