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ON NATURAL OSCILLATIONS OF THE RECTANGLE PLATE OF COMPOSITE MATERIALS WITH LOCALLY CURVED STRUCTURES

Abstract

Natural oscillation of the rectangle plate fabricated from the composite material with local curvatures in structure is considered. For solving the problem the half-analytic version of Finite Elements Method is used in the frame of Hamilton and Ostrogradsky variation principal.

Curvatures in structure of composite materials as it were expressed in [4, 7] can emerge in two cases: either as a result of technological processes making these materials or as a result of constructive requires to these materials. The pointed curvatures can be classified as local and periodic forms of curvatures. For studying of mechanical problems of elements of constructions made of composite materials with above mentioned curved structures in [1] the continual theory was suggested. Using the variation principle by Hamilton and Ostrogradsky and in the frame of theory [1] in [2] the approach was developed for investigation of natural oscillation of elements of constructions of composite materials with curved structures. However, for the present time a few concrete investigations on dynamics of composites with curved structures have been carried out. Taking into account the above mentioned in this paper the attempts are made in this direction and natural oscillations of the rectangle thick plate of composite material with local curved structures is investigated. With the help of above pointed variation principle the considered problem is investigated by Finite Elements Method. On the basis of the obtained numerical results influence of curvature parameters on values of natural oscillations of the considered plate is analyzed. Note that in case, when curvatures in structure of the plate material are periodic the analogous problem was investigated in [6].

Let consider the plates of many- layer composite material with local curvatures in structure and suppose that the plate takes area $\{0 \le x_1 \le l_1; 0 \le x_2 \le h; 0 \le x_3 \le l_3\}$. Take that the reinforcing layers are arranged on plane Ox_1x_3 and the structure parameters of the plate material satisfy the restrictions of theory [1]. Moreover, it is assumed, that the reinforcing layers of the plate have curvatures only in direction of axis Ox_1 (that is, the direction of axis Ox_3 are absent) and the form of this curvature in coordinate system $Ox_1x_2x_3$ is represented by the following function

$$F(x_1) = l_1 \begin{cases} \varepsilon\lambda^3 (x_1/l_1 - c)^2 (x_1/l_1 - d)^2 \exp\left[-\lambda^{2n}(x_1/l_1 - l_0)^{2n}\right] \times \\ \times \cos\left[m\pi\lambda(x_1/l_1 - l_0)\right], \ \ if \ \ x_1/l_1 \in [c,d]; \\ 0, \ \ if \ \ x_1/l_1 \in [0,c) \cup (d,1]. \end{cases} \qquad (1)$$

In (1) the following designations were taken: $c = c_1/l_1$, $d = d_1/l_1$, $\Lambda_1 = d_1 - c_1$, $\lambda = 1/(d-c)$, $\varepsilon = H'/\Lambda_1$, $l_0 = \mathsf{L}/l_1$, where H' is a maximal length of rise of the local curvature, L is a distance from the origin of coordinate system to the point which rise of local curvature gets the maximal value. Parameters m and n characterize the forms of local curvature.

It is taken that in case $\varepsilon = 0$ (that is structure of the plate material has not got any curvatures) material of the considered plate is homogeneous and orthotrop with the reduced mechanical properties $\mu^0_{ij\alpha\beta}$ and the main axes of elastic symmetry along Ox_1, Ox_2, Ox_3. Consequently, we can write for $\mu^0_{ij\alpha\beta}$

$$\mu^0_{ij\alpha\beta} = \delta_i^j \delta_\alpha^\beta A^0_{i\beta} + \left(1-\delta_i^j\right)\left(\delta_i^\alpha \delta_j^\beta + \delta_i^\beta \delta_j^\alpha\right) G^0_{ij};$$
$$A^0_{44} = G^0_{13}; A^0_{55} = G^0_{23}; A^0_{66} = G^0_{12}, \tag{2}$$

where δ_i^j are Kronecker's symbols, $A^0_{i\beta}$ and G^0_{ij} are well-known denotations for elastic constants of orthotrop materials. Note that in case when $\varepsilon \neq 0$ according to continual approach [1] material of the considered plate can be considered as a continuous non-homogenous material with the reduced mechanical properties $\mu_{ij\alpha\beta}(x_1)$ which are determined from the following expressions

$$\mu_{1111}(x_1) = A^0_{11}\phi_1^4(x_1) + \left(2A^0_{12} + 4G^0_{12}\right)\phi_1^2(x_1)\phi_2^2(x_1) + A^0_{22}\phi_2^4(x_1);$$
$$\mu_{2222}(x_1) = A^0_{11}\phi_2^4(x_1) + \left(2A^0_{12} + 4G^0_{12}\right)\phi_1^2(x_1)\phi_2^2(x_1) + A^0_{22}\phi_1^4(x_1);$$
$$\mu_{3333}(x_1) = A^0_{33}; \quad \mu_{1313}(x_1) = G^0_{13}\phi_1^2(x_1) + G^0_{23}\phi_2^2(x_1);$$
$$\mu_{2323}(x_1) = G^0_{23}\phi_1^2(x_1) + G^0_{13}\phi_2^2(x_1)$$
$$\mu_{1212}(x_1) = \left(A^0_{11} - 2A^0_{12} + A^0_{22} + G^0_{12}\right)\phi_1^2(x_1)\phi_2^2(x_1) + G^0_{12}\left[\phi_1^4(x_1) + \phi_2^4(x_1)\right];$$
$$\mu_{1122}(x_1) = \left(A^0_{11} + A^0_{22} - 4G^0_{12}\right)\phi_1^2(x_1)\phi_2^2(x_1) + A^0_{12}\left[\phi_1^4(x_1) + \phi_2^4(x_1)\right];$$
$$\mu_{1133}(x_1) = A^0_{13}\phi_1^2(x_1) + A^0_{23}\phi_2^2(x_1);$$
$$\mu_{1112}(x_1) = \left(A^0_{12} - A^0_{11} + 2G^0_{12}\right)\phi_1^3(x_1)\phi_2(x_1) + \left(A^0_{22} - A^0_{12} - 2G^0_{12}\right)\phi_1(x_1)\phi_2^3(x_1);$$
$$\mu_{2233}(x_1) = A^0_{13}\phi_2^2(x_1) + A^0_{23}\phi_1^2(x_1);$$
$$\mu_{2212}(x_1) = \left(A^0_{12} - A^0_{11} + 2G^0_{12}\right)\phi_1(x_1)\phi_2^3(x_1) + \left(A^0_{22} - A^0_{12} - 2G^0_{12}\right)\phi_1^3(x_1)\phi_2(x_1);$$
$$\mu_{3312}(x_1) = \left(A^0_{23} - A^0_{13}\right)\phi_2(x_1)\phi_1(x_1);$$
$$\mu_{1323}(x_1) = \left(G^0_{13} - G^0_{23}\right)\phi_2(x_1)\phi_1(x_1). \tag{3}$$

Here the following denotations were taken:

$$\phi_1(x_1) = \left[1 + \left(\frac{dF(x_1)}{dx_1}\right)^2\right]^{-\frac{1}{2}};$$

$$\phi_2(x_1) = \frac{dF(x_1)}{dx_1} \cdot \phi_1(x_1).$$

Therefore, we can write the complete equations system for investigation of the considered problem

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{d^2 u_i}{dt^2}; \quad \sigma_{ij} = \mu_{ij\alpha\beta}(x_1)\varepsilon_{\alpha\beta};$$
$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right); \quad i,j,\alpha,\beta = 1,2,3. \tag{4}$$

In (4) generally accepted denotations were used.

Suppose that the plate for $x_1 = 0;\ l_1$ is rigidly fixed and for $x_3 = 0;\ l_3$ is hinge beared. Taking into account above mentioned we formulate the boundary conditions:

$$u_1 = u_2 = u_3 = 0;\ \text{for}\ x_1 = 0; l_1, \tag{5}$$

$$u_2 = 0;\ \sigma_{33} = 0;\ \text{for}\ x_3 = 0; l_3, \tag{6}$$

$$\sigma_{i2}\big|_{x_2=0;\,h} = 0 \quad (i = 1,2,3) \tag{7}$$

So, investigation of natural oscillations of the considered plate is reduced to solving of natural problem (4)-(7). For solving that problem in the frame of variation principle by Hamilton and Ostrogradsky the half analytic version of the Finite Elements Method (FEM) is used [9, 10]. Displacements of the plate are represented in a view

$$u_i = \vartheta_i(x_1, x_2)\left[\left(\delta_i^1 + \delta_i^2\right)\sin\frac{\pi x_3}{l_3} + \delta_i^3\cos\left(\frac{\pi x_3}{l_3}\right)\right]e^{i\omega t}. \tag{8}$$

Substituting correlations (8) into (4)-(7) we find all the equations (4) and conditions (5)-(7) with respect to x_3 are fulfilled automatically. For determination of function $\vartheta_i(x_1, x_2)$ $(i = 1,2,3)$ we use FEM. Doing the well- known procedures we divide $\Omega = \{0 \le x_1 \le l_1; 0 \le x_1 \le h\}$ into the finite number of elements Ω_j. Suppose, that Ω_j are rectangle quadratic elements of Lagrange family [9, 10].

On finding natural frequencies we do on this way. We substitute instead of conditions (7) for σ_{22} under $x_2 = h$ by the condition $\sigma_{22}\big|_{x_2=h} = pe^{i\omega t}\sin\left(\pi\frac{x_3}{l_3}\right)$ and deriving the equation

$$\left(K - \omega^2 m\right)\vec{a} = \vec{r}, \tag{9}$$

where K is a rigidity matrix, M is a mass matrix, $\vec{a}$ is a vector which components displaced in nodal points, $\vec{r}$ is a vector of external forces. We take value of ω under which solutions of equation (9) with respect to vector $\vec{a}$ approach to infinity as a natural value of frequencies.

Above expressed let study the problems on natural and forced oscillations at the same time using FEM. So, begin to analysis of the numerical results of natural frequencies of the plate.

If should note, that concrete numerical results were obtained in the case when plate's material consists of alternating layers of two isotrop homogeneous materials with elastic properties E_1, E_2 (Young's modulus) and ν_1, ν_2 (Poisson coefficients). We took $\nu_1 = \nu_2 = 0.3$ and the mechanical properties A_{ij}^0 in (2) were determined by means of E_1, E_2, ν_1, ν_2 by well-known formulas given in many monographs on composite materials. Take, that $\eta_1 = \eta_2 = 0.5$, where η_1 and η_2 are concentrations of the matrix and the filler correspondingly.

At all the numerical investigations it was supposed $\delta = \frac{\pi}{2}$; $l_1/\Delta_1 = 8$. Moreover, on using of FEM the symmetries of the considered problem with respect to $x_1 = l_1/2$ had been considered and all procedures of numerical solving were carried out only for the part $\{0 \le x_1 \le l_1/2; 0 \le x_2 \le h\}$ of area Ω. The pointed out area is divided into 80 rectangle elements with 369 nodal points and 1089 degrees of freedom. By [7] we introduce the indimensional reduced frequency $\overline{\omega}^2 = \rho\omega^2 l_1^2 / A_{22}^0$ and investigate influence of geometrical and mechanical parameters $\gamma = l_1/l_3$; h/l_1 ; ε; E_1/E_2 to value of natural frequencies of the plate. Moreover, note that here the denotations had been taken from [7].

Therefore, consider the numerical results, reduced for different values of parameter ε, which were obtained for $E_2/E_1 = 50$, $h/l = 0.1$, $\gamma = 1$ (Table 1). As it follows from this table in all the considered modes with increase of value ε the values of frequencies decrease monotonely and it is explained so that existence of curvatures in structure of plate's material reduces to decrease of its rigidity. The pointed out decreases grow with increase of the amplitude of the form of local curvature.

In Table 2 influence of changing of parameter h/l to the value of natural frequencies was showen. $E_2/E_1 = 50$; $\gamma = 1$ were taken and for parameter ε two values were chosen. As it follows from the results, increase of h/l value of reduce to increase of frequencies of natural oscillations of the plate and existence of curvature reduces to decrease of values of frequencies. The results obtained for respectively great h/l show that influence of existence of the curvature in plate's material to the value of the investigated natural frequencies is insignificant. Influence of change of γ to value of natural frequencies is showen in Table 3. Here it was also supposed $E_2/E_1 = 50$ and ε takes values 0.00, 0.10 and $h/l = 0.1$. Having analysis of the results by this table we conclude that increase of γ reduces to increase of values of natural frequencies. The values obtained for $\gamma = 0.1$ with great precision coincide with the corresponding results obtained for the plate, which is infinite in direction of axis Ox_3 [5]. In table 4 for $h/l = 0.1$ and $\gamma = 1$ the values of natural frequencies were reduced for different E_2/E_1. Note, that case $E_2/E_1 = 1$ corresponds to the isotrop homogeneous plate and case $E_2/E_1 > 1$ to the anisotrop one; for $\varepsilon = 0$ it corresponds to the homogeneous plate, for $\varepsilon = 0.5$ to the non-homogeneous one (that is to the plate which structure has a local curvature). As it follows from tables, increase of E_2/E_1 reduces to monotone increase of values of natural frequencies. Note that the results given in the last three tables are explained so that increase of parameters h/l, γ and E_2/E_1 reduces to increase to plate's rigidity. And almost at all the considered cases the existence of the curvature in structure of the plate's material reduces to decrease of value of rigidity and it reduced to decrease of value of the considered of modes of natural frequencies.

Table 1

$E_2/E_1 = 50$; $h/l = 0.1$; $\gamma = 1$

ε	MODES		
	$\overline{\omega}_{I}^{2}$	$\overline{\omega}_{II}^{2}$	$\overline{\omega}_{III}^{2}$
0.0	2.62	8.79	20.16
0.1	2.62	8.79	20.15
0.3	2.60	8.78	20.08
0.5	2.58	8.76	19.96
0.7	2.54	8.74	19.82

Table 2

$E_2/E_1 = 50$; $\gamma = 1$

h/l	ε	MODES		
		$\overline{\omega}_1^2$	$\overline{\omega}_{11}^2$	$\overline{\omega}_{111}^2$
0.10	0.00	2.62	8.79	20.16
	0.5	2.58	8.76	19.96
0.15	0.0	3.45	10.44	22.80
	0.5	3.43	1046	23.26
0.20	0.0	3.96	11.35	24.26
	0.5	3.95	11.42	24.32

Table 3

$E_2/E_1 = 50$; $h/l = 0.1$

γ	ε	MODES		
		$\overline{\omega}_1^2$	$\overline{\omega}_{11}^2$	$\overline{\omega}_{111}^2$
1.5	0.0	4.60	11.27	22.98
	0.5	4.58	11.27	22.98
1.3	0.0	3.65	10.14	21.71
	0.5	3.62	1012	21.56
1	0.0	2.62	8.79	20.16
	0.5	2.58	8.76	19.96
0.5	0.0	1.77	7.41	18.52
	0.5	1.72	7.39	18.26
0.3	0.0	1.64	7.13	18.18
	0.5	1.59	7.11	17.91
0.1	0.0	1.58	6.99	18.00
	0.5	1.54	6.97	17.73

Table 4

$h/l = 0.1$; $\gamma = 1$.

E_2/E_1	ε	MODES		
		$\overline{\omega}_1^2$	$\overline{\omega}_{11}^2$	$\overline{\omega}_{111}^2$
1	0.0	0.49	2.43	7.16
10	0.0	1.17	4.99	13.07
	0.5	1.15	4.98	12.97
20	0.0	1.71	6.61	16.28
	0.5	1.69	6.59	16.12
50	0.0	2.62	8.79	20.16
	0.5	2.58	8.76	19.96
100	0.0	3.32	10.20	22.48
	0.50	3.28	10.19	22.37

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