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ON REPRESENTATION OF SUMMABLE FUNCTIONS BY SINGULAR INTEGRALS

Abstract

The conditions imposed on summation method for the representation of functions from $L_{1,\mu}$ class in generalized Lebesgue points by singular integrals are found in the paper.

This paper is the continuation of paper [1], where singularity conditions of integrals generated by summation of Fourier-Hegenbaner series have been found.

Convergence conditions of singular integrals at Lebesgue's R - points are clarified.

Denote by $L_\mu[-1,1]$ a space of summable with weight $\mu(x)=\left(1-x^2\right)^{\lambda-\frac{1}{2}}$ function $f(x)$ with the norm

$$\|f\|_\mu=\frac{\Gamma(\lambda+1)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\lambda+\frac{1}{2}\right)}\int_{-1}^{1}\left(1-x^2\right)^{\lambda-\frac{1}{2}}|f(x)|dx\,.$$

Compare the function $f\in L_\mu[-1,1]$ with Fourier- Hegenbaner series

$$f(x)\sim\sum_{n=0}^{\infty}a_n^\lambda(f)P_n^\lambda(x)\,. \qquad (1)$$

The summation of series (1) by $\{\varphi_n(\tau)\}$ $(\varphi_0(\tau)=1, n=1,2,\ldots)$ method leads to the integral of the form (see [1])

$$\mathcal{L}_\tau^\lambda(f;x)=\frac{\Gamma(\lambda)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\lambda+\frac{1}{2}\right)}\int_0^\pi \mathcal{K}_\tau^\lambda(t)f_t(x)\sin^{2\lambda}t\,dt\,, \qquad (2)$$

where

$$\mathcal{K}_\tau^\lambda(t)=\sum_{n=0}^{\infty}\varphi_n(\tau)(n+\lambda)P_n^\lambda(\cos t)\,, \qquad (3)$$

and

$$f_t(x)=\frac{\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma(\lambda)}\int_0^\pi(\sin\varphi)^{2\lambda-1}f\left(x\cos t+\sqrt{1-x^2}\sin t\cos\varphi\right)d\varphi$$

is a generalized shear function.

The point $x\in[-1,1]$ is called Lebesgue's R - point of the function $f\in L_\mu[-1,1]$ if

$$\int_0^\gamma\left|f_t(x)-f(x)\right|\sin^{2\lambda}t\,dt=0\left(\gamma^{2\lambda+1}\right),\quad \gamma\to0\,.$$

Lemma 1. *Let* $f(x)\sin^{2\lambda} t \in L[0,\pi]$ *and*

$$M = \sup_{0<h\le\pi}\left\{\frac{1}{h^{2\lambda+1}}\left|\int_0^h f(u)(\sin u)^{2\lambda}\,du\right|\right\} < \infty. \tag{4}$$

Then for any non-negative non-increasing on $[0,\pi]$ *function* $g(t)$ *is so that* $t^{2\lambda} g(t) \in L[0,\pi]$

$$\int_0^\pi f(t)g(t)\sin^{2\lambda} t\,dt \tag{5}$$

exists and the inequality

$$\int_0^\pi f(t)g(t)\sin^{2\lambda} t\,dt \le M(2\lambda+1)\int_0^\pi t^{2\lambda} g(t)\,dt \tag{6}$$

is valid.

Proof. Clarifying the conditions of lemma, note that if $g(0)=\infty$, then integral (5) exists as improper and if $g(0)<\infty$, the function $g(t)$ is bounded and integral (5) exists as ordinary Lebesgue integral. Unlimiting the generality we can adopt $g(\pi)=0$. Let $0<\alpha<\pi$. On the segment $[\alpha,\pi]$ the function $g(t)$ is limited and the integral

$$\int_\alpha^\pi f(t)g(t)\sin^{2\lambda} t\,dt \tag{7}$$

trivially exists. If we put

$$F(t) = \int_0^t f(u)\sin^{2\lambda} u\,du,$$

then integral (7) will be inscribed in the form of

$$\int_\alpha^\pi f(t)g(t)\sin^{2\lambda} t\,dt = \int_\alpha^\pi g(t)\,dF(t) = -F(\alpha)g(\alpha) - \int_\alpha^\pi F(t)\,dg(t).$$

But by virtue of (4) we have

$$|F(t)| \le M t^{2\lambda+1}, \tag{8}$$

and since $g(t)$ doesn't increase, then

$$\int_0^\alpha t^{2\lambda} g(t)\,dt \ge g(\alpha)\int_0^\alpha t^{2\lambda}\,dt = \frac{\alpha^{2\lambda+1}}{2\lambda+1} g(\alpha). \tag{9}$$

Now, from (8) and (9) we have

$$|F(\alpha)g(\alpha)| \le M(2\lambda+1)\int_0^\alpha t^{2\lambda} g(t)\,dt. \tag{10}$$

Since, $-g(t)$ is not decreasing, then from (8) and (9)

$$\left|\int_\alpha^\pi F(t)\,d[-g(t)]\right| \le M\int_\alpha^\pi t^{2\lambda+1}\,d[-g(t)] = M\alpha^{2\lambda+1}g(\alpha) + M(2\lambda+1)\int_\alpha^\pi t^{2\lambda} g(t)\,dt \le$$

$$\le M(2\lambda+1)\int_0^\pi t^{2\lambda} g(t)\,dt \tag{11}$$

follows.

Taking into account (10) and (11) in (7), we get

$$\left|\int_{\alpha}^{\pi} f(t)g(t)\sin^{2\lambda} t dt\right| \le M(2\lambda+1)\left\{\int_{0}^{\alpha} t^{2\lambda} g(t)dt + \int_{0}^{\pi} t^{2\lambda} g(t)dt\right\}.$$

Here, going over to the limit for $\alpha \to 0$, we obtain the assertion of the lemma.

Denote by X one of spaces $L_{p,\mu}[-1,1]$ or $C[-1,1]$. In paper [2] it is proved

Lemma 2. *A generalized shear function $f_t(x)$ is a linear operator acting from X to X with the norm*

$$\|f_t\|_{[x,x]} = 1.$$

Put $\psi_t(x) = f_t(x) - f(x)$. Below by c we shall denote positive constants, and by C_λ - positive constants depending on extracted indices, generally speaking, different at various cases.

Theorem 1. *Let $f \in L_\mu[-1,1]$. If*

$$\left|\varkappa_\tau^\lambda(x)\right| \le c, \quad x \in [-1,1], \tag{a}$$

$$\lim_{\tau \to \tau_0} \int_a^b \left(1-x^2\right)^{\lambda-\frac{1}{2}} \varkappa_\tau^\lambda(x)dx = 0, \quad -1 \le a < b \le 1, \tag{b}$$

then

$$\lim_{\tau \to \tau_0} \int_{-1}^{1} \left(1-x^2\right)^{\lambda-\frac{1}{2}} \psi_t(x)\varkappa_\tau^\lambda(x)dx = 0.$$

Proof. Let first $f \in C[-1,1]$, then by lemma 2 and $\psi_t \in C[-1,1]$ $\forall t \in [0,\pi]$. By Cantour theorem decompose the segment $[-1,1]$, $-1 = x_0 < x_1 < \cdots < x_m = 1$, so that $\forall \varepsilon > 0, \exists \delta(\varepsilon) > 0$, that $\forall x \in [x_k, x_{k+1}]\ \max_k |x_k - x_{k+1}| < \delta$,

$$\left|\psi_t(x) - \psi_t(x_k)\right| < \varepsilon. \tag{12}$$

And therefore

$$|J_\tau| \le \int_{-1}^{1} \mu(x)\left|\psi_t(x) - \psi_t(x_k)\right|\left|\varkappa_\tau^\lambda(x)\right|dx + \left|\int_{-1}^{1} \mu(x)\psi_t(x_k)\varkappa_\tau^\lambda(x)dx\right| =$$

$$= \sum_{k=1}^{m} \int_{x_{k-1}}^{x_k} \mu(x)\left|\psi_t(x) - \psi_t(x_k)\right|\left|\varkappa_\tau^\lambda(x)\right|dx + \left|\sum_{k=1}^{m} \psi_t(x_k) \int_{x_{k-1}}^{x_k} \mu(x)\varkappa_\tau^\lambda(x)dx\right| = J_\tau^{(1)} + J_\tau^{(2)}. \tag{13}$$

By virtue of (12) and the condition (a) of the theorem

$$J_\tau^{(1)} < \varepsilon \cdot c \int_{-1}^{1} \left(1-x^2\right)^{\lambda-\frac{1}{2}} dx = c_\lambda \cdot \varepsilon. \tag{14}$$

And by virtue of the condition (b) of the theorem

$$\lim_{\tau \to \tau_0} J_\tau^{(2)} = 0. \tag{15}$$

Taking into account (14) and (15) in (13), we obtain, that

$$\lim_{\tau \to \tau_0} J_\tau = 0. \tag{16}$$

Let now $f(x)$ be a measurable bounded function

$$|f(x)| \le c, \quad x \in [-1,1].$$

Since, in addition $\forall t \in [0,\pi]$

$$|f_t(x)| \le c,$$

then $f_t(x)$ is a measurable bounded function, but then $\psi_t(x)$ is also a measurable bounded function, as

$$|\psi_t(x)| = |f_t(x) - f(x)| \le |f(x)| + |f_t(x)| \le 2c. \tag{17}$$

By virtue of absolute continuity of the integral $\forall e \in [-1,1]$ with measure $me < \delta$

$$\int_e \left(1 - x^2\right)^{\lambda - \frac{1}{2}} dx < \varepsilon. \tag{18}$$

Further, by N.N. Luzin theorem ([3], p. 118) there exists such a function $v(x) \in C[-1,1]$, that $me = mE\ (\psi_t \ne v) < \delta$

$$|v(x)| \le c. \tag{19}$$

According to (16)-(19) we have

$$|J_\tau| = \left|\int_{-1}^{1} \mu(x)\psi_t(x)\mathcal{K}_\tau^\lambda(x)dx\right| \le \left|\int_{-1}^{1} \mu(x)(\psi_t(x) - v(x))\mathcal{K}_\tau^\lambda(x)dx\right| +$$

$$+\left|\int_{-1}^{1} \mu(x)v(x)\mathcal{K}_\tau^\lambda(x)dx\right| < \varepsilon + c\int_e \mu(x)dx < \varepsilon(1 + c). \tag{20}$$

Now, let $f \in L_\mu[-1,1]$, then by virtue of lemma 2 and $f_t \in L_\mu[-1,1]$, moreover $\|f_t\|_\mu \le \|f\|_\mu$, but then $\|\psi_t\|_\mu = \|f_t - f\|_\mu \le 2\|f\|_\mu$.

Take $\varepsilon > 0$ and using the absolute continuity of an integral find such $\delta > 0$, that for any measurable set $e \in [-1,1]$ with measure $me < \delta$ there was

$$\int_e |\psi_t(x)|\mu(x)dx < \varepsilon. \tag{21}$$

Now find such a measurable bounded function ([3], p. 113) that

$$mE(\psi_t \ne v) < \delta, \quad |v(x)| \le c. \tag{22}$$

Then from (20), (21) and (22) and condition (a) of the theorem we have

$$|J_\tau| \le \left|\int_{-1}^{1} (\psi_t(x) - v(x))\mu(x)\mathcal{K}_\tau^\lambda(x)dx\right| + \left|\int_{-1}^{1} \mu(x)v(x)\mathcal{K}_\tau^\lambda(x)dx\right| <$$

$$< \int_{E(\psi_t \ne v)} \mu(x)|\psi_t(x)||\mathcal{K}_\tau^\lambda(x)|dx + \int_{E(\psi_t \ne v)} \mu(x)|v(x)||\mathcal{K}_\tau^\lambda(x)|dx + \varepsilon <$$

$$< c \int_{E(\psi_t \ne v)} \mu(x)|\psi_t(x)|dx + c \int_{E(\psi_t \ne v)} \mu(x)dx + \varepsilon < c \cdot \varepsilon.$$

The theorem proved.

The function $\phi(t)$ is called a majorant of the function $Q(t)$ on $[a,b]$ if $\forall t \in [a,b]$, $|Q(t)| \le \phi(t)$ and $\phi(t)$ doesn't increase on $[a,b]$.

Theorem 2. *Let the integral (2) be singular, and the kernel* $\mathcal{K}_\tau^\lambda(t)$, *has on* $[0,\delta]$ *the majorant* $\phi_\tau(t)$, *such that*

$$\int_0^\delta \phi_\tau(t)t^{2\lambda}dt \le c, \quad \delta \in (0,\pi) \tag{23}$$

and

$$\left|\mathcal{K}_\tau^\lambda(t)\right| \le c\,,\quad t \in [\delta,\pi] \tag{24}$$

then $\forall f \in L_\mu[-1,1]$ *at each Lebesgue's* R*-point* $x \in [-1,1]$ *the equality*

$$\lim_{\tau\to\tau_0} \mathcal{L}_\tau^\lambda(f;x) = f(x)$$

holds.

Proof. $\forall \varepsilon > 0$, $\exists \delta(\varepsilon) > 0$ is such that for $0 < h \le \delta$

$$\int_0^h \left|f(x) - f_t(x)\right| \sin^{2\lambda} t dt < \varepsilon \cdot h^{2\lambda+1}.$$

Hence with regard to lemma 1 we shall have:

$$\left|f(x) - \mathcal{L}_\tau^\lambda(f;x)\right| \le \int_0^\delta \left|f(x) - f_t(x)\right|\left|\mathcal{K}_\tau^\lambda(t)\right| \sin^{2\lambda} t dt +$$

$$+\left|\int_\delta^\pi \left[f(x) - f_t(x)\right]\mathcal{K}_\tau^\lambda(t) \sin^{2\lambda} t dt\right| <$$

$$< c_\lambda \cdot \varepsilon \int_0^\delta t^{2\lambda}\phi_\tau(t)dt + J_\delta(\tau) < c_\lambda \cdot \varepsilon + J_\delta(\tau). \tag{25}$$

From (24), the singularity of integral, and theorem 1

$$\lim_{\tau\to\tau_0} J_\delta(\tau) = 0 \tag{26}$$

follows.

Taking into account (26) in (25), we obtain the assertion of the theorem.

Theorem 3. *Let* $0 < \lambda < 1$. *If the sequence satisfies the conditions*

$$\lim_{\tau\to\tau_0} \varphi_n(\tau) = 1, \tag{27}$$

$$\sum_{n=0}^{\infty} (n+1)^\gamma \left|\Delta^2 \varphi_n(\tau)\right| \le c,\quad 0 < \gamma < 2 \tag{28}$$

then $\forall f \in L_\mu[-1,1]$ *at each Lebesgue's* R*-point* $x \in [-1,1]$ *the equality*

$$\lim_{\tau\to\tau_0} \mathcal{L}_\tau^\lambda(f;x) = f(x).$$

Proof. The kernel (3) may be represented in the form of (see [1])

$$\mathcal{K}_\tau^\lambda(t) = \sum_{n=0}^{\infty} (n+1)\Delta^2\varphi_n(\tau)F_n^\lambda(t).$$

Appling the Christophell- Darboux formula we can write

$$F_n^\lambda(t) = \frac{1}{2(n+1)(1-\cos t)} \sum_{\nu=0}^{n} (\nu + 2\lambda)P_\nu^\lambda(\cos t) - (\nu+1)P_{\nu+1}^\lambda(\cos t) =$$

$$= \frac{\sin^{-2}\frac{t}{2}}{n+1}\left\{\sum_{\nu=0}^{n} 2\lambda P_\nu^\lambda(\cos t) + \sum_{\nu=0}^{n}\left[\nu P_\nu^\lambda(\cos t) - (\nu+1)P_{\nu+1}^\lambda(\cos t)\right]\right\} =$$

$$= \frac{\sin^{-2}\frac{t}{2}}{n+1}\left\{2\lambda \sum_{\nu=0}^{n} 2\lambda P_\nu^\lambda(\cos t) - (n+1)P_{\nu+1}^\lambda(\cos t)\right\}.$$

Here, taking into account the inequality ([4], p. 179)

$$\sin^{\lambda} t \left|P_n^{\lambda}(\cos t)\right| < \frac{2^{1-\lambda}}{\Gamma(\lambda)} n^{\lambda-1}, \quad t \in [0,\pi]$$

we obtain

$$F_n^{\lambda}(t) = O(1)\frac{1}{\sin^2 \frac{t}{2}}\left(\frac{1}{n+1}\sum_{\nu=0}^{n}\left|P_\nu^{\lambda}(\cos t)\right| + \left|P_{n+1}^{\lambda}(\cos t)\right|\right) =$$

$$= O(1)\frac{n^{\lambda-1}}{\sin^2 \frac{t}{2}\sin^{\lambda} t} = O(1)\frac{n^{\lambda-1}}{\left(\sin\frac{t}{2}\right)^{2+\lambda}}, \quad t \in (0,\pi). \tag{29}$$

On the other hand ([4], p.178)

$$P_n^{\lambda}(\cos t) = O\left(n^{2\lambda-1}\right), \quad t \in [0,\pi]. \tag{30}$$

But then

$$F_n^{\lambda}(\cos t) = O(1)\left(\sin\frac{t}{2}\right)^{-2}\left(\frac{1}{n+1}\sum_{\nu=0}^{n}(\nu+1)^{2\lambda-1} + (n+1)^{2\lambda-1}\right) =$$

$$= O(1)\left(\sin\frac{t}{2}\right)^{-2} n^{2\lambda-1} = O\left(n^{2\lambda-1}\right), \quad t \in [\delta,\pi], \tag{31}$$

$$F_n^{\lambda}(t) = O\left(n^{2\lambda+1}\right), \quad t \in [0,\pi]. \tag{32}$$

From (29) and (32) note, that the functions

$$\phi_n^{\lambda}(t) = \begin{cases} cn^{2\lambda+1}, & 0 \le t \le \pi; \\ cn^{\lambda-1}\left(\sin\frac{t}{2}\right)^{2+\lambda}, & 0 < t < \pi; \end{cases}$$

do not increase on $(0,\pi)$, moreover

$$\int_0^{\pi} t^{2\lambda}\phi_n^{\lambda}(t)dt = O(1)\left(\int_0^{1/n} n^{2\lambda+1}\cdot t^{2\lambda}dt + \int_{1/n}^{\pi} n^{\lambda-1}t^{\lambda-2}dt\right) = O(1),$$

i.e. the functions $\phi_n^{\lambda}(t)$ are integrable majorants of the functions $F_n^{\lambda}(t)$.

But then with regard to the conditions of the theorem we have:

$$\int_0^{\delta}\left|\mathcal{K}_\tau^{\lambda}(t)\right|\sin^{2\lambda} t dt = O(1)\sum_{n=0}^{\infty}(n+1)\left|\Delta^2\varphi_n(\tau)\right|\int_0^{\delta} t^{2\lambda}\phi_n^{\lambda}(t)dt =$$

$$= O(1)\sum_{n=0}^{\infty}(n+1)\left|\Delta^2\varphi_n(\tau)\right| = O(1),$$

i.e. the condition (23) of theorem 2 is fulfilled. And from (31) and the condition (28) of the theorem it follows, that

$$\mathcal{K}_\tau^{\lambda}(t) \le \sum_{n=0}^{\infty}(n+1)\left|\Delta^2\varphi_n(\tau)\right|\left|F_n^{\lambda}(t)\right| \le c\sum_{n=0}^{\infty}(n+1)^{2\lambda}\left|\Delta^2\varphi_n(\tau)\right| \le C, \quad \delta \le t \le \pi,$$

i.e. the condition (24) of Theorem 2 is fulfilled. Thus, all conditions of theorem 2 are fulfilled and so, the assertion of theorem 3 follows.

Theorem 4. *Let* $\mathcal{L}_\tau^{\lambda}(f;x)$ *be a singular integral. If* $\mathcal{K}_\tau^{\lambda}(t) \ge 0$ *at each Lebesque's* R*-point* $x \in [-1,1]$ *the equality*

$$\lim_{\tau\to\tau_0} \mathcal{L}_\tau^\lambda(f;x)=f(x)$$

holds.

Proof. Let x be the Lebesgue's R-point, the $\forall\varepsilon>0$, $\exists\delta>0$ is such that for $0<h\le\delta$ the inequality

$$\int_0^h |f(x)-f_t(x)|\sin^{2\lambda} t dt<\varepsilon\cdot h^{2\lambda+1}$$

holds.

But then by lemma 1

$$\left|\int_0^\delta [f(x)-f_t(x)]\mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt\right|\le\int_0^\delta |f(x)-f_t(x)|\mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt\le$$

$$\le\varepsilon\cdot c_\lambda\int_0^\delta \mathcal{K}_\tau^\lambda(t)t^{2\lambda}dt\le$$

(since $t\le\frac{\pi}{2}\sin t$ for $0\le t\le\frac{\pi}{2}$)

$$\le\varepsilon\cdot c_\lambda\left(\frac{\pi}{2}\right)^{2\lambda}\int_0^\delta \mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt<\varepsilon\cdot c_\lambda\left(\frac{\pi}{2}\right)^{2\lambda}\int_0^\pi \mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt=\varepsilon\cdot c_\lambda. \tag{33}$$

On the other hand, by virtue of non-increasing of $\mathcal{K}_\tau^\lambda(t)$ we can write

$$\mathcal{K}_t^\lambda(t)\le\mathcal{K}_\tau^\lambda(\delta)\le(2\lambda+1)\delta^{-2\lambda-1}\int_0^\delta \mathcal{K}_\tau^\lambda(t)t^{2\lambda}dt\le$$

$$\le\left(\frac{\pi}{2}\right)^{2\lambda}\frac{2\lambda+1}{\delta^{2\lambda+1}}\int_0^\pi \mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt=\left(\frac{\pi}{2}\right)^{2\lambda}\frac{2\lambda+1}{\delta^{2\lambda+1}},$$

that means, that the function $\mathcal{K}_\tau^\lambda(t)$ is uniformly bounded on $[\delta,\pi]$. But then from the singularity of integral, and theorem 1 we obtain, that

$$\lim_{\tau\to\tau_0}\int_0^\pi [f(x)-f_t(x)]\mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt=0. \tag{34}$$

Furthermore,

$$|f(x)-\mathcal{L}_\tau^\lambda(f;x)|\le\int_0^\delta |f(x)-f_t(x)|\mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt+$$

$$+\left|\int_\delta^\pi [f(x)-f_t(x)]\mathcal{K}_\tau^\lambda(t)\sin^{2\lambda} t dt\right|=J_\delta^{(1)}(\tau)+J_\delta^{(2)}(\tau). \tag{35}$$

Taking into account (33) and (34) at (35) we obtain the assertion of the theorem.

Corollary 4.1. *Let* $0<\lambda<1$. *Then* $\forall f\in L_\mu[-1,1]$ *at each Lebesgue's* R*-point* $x\in[-1,1]$

$$\lim_{\tau\to 1-0}\mathcal{P}_\tau^\lambda(f;x)=f(x).$$

Here $\mathcal{P}_\tau^\lambda(f;x)$ *is Poisson's singular integral (see [1]).*

Corollary 3.1. *Let* $0<\lambda<1$. *Then* $\forall f\in L_\mu[-1,1]$ *at each Lebesgue's* R*-point* $x\in[-1,1]$ *for* $\alpha\ge 2\lambda$

$$\lim_{n\to\infty}\sigma_n^{\alpha,\lambda}(f;x)=f(x).$$

Here $\sigma_n^{\alpha,\lambda}(f;x)$ *are Chezarov's means of the series (1) (see [1]).*

Corollary 3.2. *Let* $0<\lambda\le\frac{1}{2}$. *Then* $\forall f\in L_\mu[-1,1]$ *at each Lebesgue's* R*-point* $x\in[-1,1]$

$$\lim_{n\to\infty}B_n^{\lambda}(f;x;h_n)=f(x).$$

Here $B_n^{\lambda}(f;x;h_n)$ *are «averaged means» of Bernstein- Rogozin series (1) (see [1]).*

Corollary 3.3. *Let* $0<\lambda<1$. *Then* $\forall f\in L_\mu[-1,1]$ *at each Lebesgue's* R*-point* $x\in[-1,1]$

$$\lim_{n\to\infty}V_n^{\lambda}(f;x)=f(x).$$

Here $V_n^{\lambda}(f;x)$ *is Valle-Poisson's generalized operator (see [1]).*

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