1999 VOL. X(XVIII)

KASUMOV N.Z.

ON THE SOLVABILITY OF THE HIGHER ORDER NONLINEAR PARABOLIC PROBLEM

Abstract

The investigation of different high order nonlinear problems have been the subject of a rather large number of papers (see [1], [2], [3,4]). In this article we consider one class of parabolic equations, which have strong nonlinearity. Here high order derivatives are contained in equation as some linear differential expression. It creates many difficults in their investigation. By proving the theorem of solvability we use successive application of elliptic regularization, substitution of prinsipal space and pseudomonotones method.

Let $\Omega \subset R^n (n>1)$ be a bounded domain with sufficiently smooth boundary $\partial\Omega$. In cylinder $Q=[0,T]\times\Omega$, $T>0-const$, we consider the following parabolic problem

$$\frac{\partial u}{\partial t}+F(u)=\frac{\partial u}{\partial t}+F\left(t,x,u,Du,...,D^{2m-1}u,Au\right)=h(t,x), \tag{1}$$

$$u(0,x)=0, \quad x\in\Omega, \tag{2}$$

$$D^{\alpha}u\big|_{\Gamma}=0, \quad |\alpha|\le m-1, \quad \Gamma=[0,T]\times\partial\Omega \tag{3}$$

(here $D^{\alpha}u$ is a vector-function, components of which are all partial derivatives $\partial^{|\alpha|}u/\partial^{\alpha_1}x_1...\partial^{\alpha_n}x_n$ of the function u, where $\alpha=(\alpha_1,...,\alpha_n)$ is a multiindex, $|\alpha|=\alpha_1+...+\alpha_n$).

Assume that the following conditions are satisfied:

1. $F(t,x,\xi_0,\xi_1,...,\xi_{2m-1},\eta)$ is the satisfied Caratheodory condition, that is measurable by t and x for every vector $\xi=(\xi_0,...,\xi_{2m-1},\eta)$ and continuously by ξ for a.e. $(t,x)\in Q$;
2. There exist constant $C>0, 1<p_i<p-1$ $\left(i=\overline{0,2m-1}\right)$, such that the inequality

$$\left|F(t,x,\xi_0,\xi_1,...,\xi_{2m-1},\eta)\right|\le C\left(\sum_{|\alpha|=i=0}^{2m-1}|\xi_\alpha|^{p_i}+|\eta|^{p-1}+1\right)$$

holds;

3. The function $F(t,x,\xi_0,\xi_1,...,\xi_{2m-1},\eta)$ is monotone by the last argument

$$\left(F(t,x,\xi_0,\xi_1,...,\xi_{2m-1},\eta)-F(t,x,\xi_0,\xi_1,...,\xi_{2m-1},\tilde\eta)\right)(\eta-\tilde\eta)\ge$$
$$\ge\Phi(t,x,\xi_0,\xi_1,...,\xi_{2m-1})\phi(\eta-\tilde\eta)$$

for any vector $(t,x,\xi_0,\xi_1,...,\xi_{2m-1})$;

the function $\Phi(t,x,\xi_0,\xi_1,...,\xi_{2m-1})$ is bounded

$$0<\delta<\Phi(t,x,\xi_0,\xi_1,...,\xi_{2m-1})<C_2;$$

and the function $\phi(r)$ is equivalent to $|r|^p, p>1$;

4. $2m$ - order linear differential expression A is a square of formal selfconjugate differential expression and with boundary conditions (3) generates evenly elliptic operator.

Theorem. *Let conditions 1-4 be satisfied. Then for any* $h(t,x)\in L_q(Q)$ *problem (1)-(3) has at any rate one solution* $u(t,x)\in L_p\left(0,T;W_p^{2m}(\Omega)\cap \overset{\circ}{W}{}_p^m(\Omega)\right)\cap$ $\cap \underset{\circ}{W}{}_q^1(0,T;L_q(\Omega))$, *which satisfies equation (1) almost everywhere in* Q.

By the proving the theorem of solvability we apply the method of elliptic regularization. The proof is carried out by means of the approximate method of Galerkin also. At first, we prove for $\varepsilon>0$ the solvability of the following auxiliary elliptic problem

$$-\frac{\partial^2 u_\varepsilon}{\partial t^2}+\frac{\partial u_\varepsilon}{\partial t}+F\left(t,x,u_\varepsilon,Du_\varepsilon,\ldots,D^{2m-1}u_\varepsilon,Au_\varepsilon\right)=h(t,x), \tag{4}$$

$$u_\varepsilon(0,x)=0,\quad u_\varepsilon'(T,x)=0, \tag{5}$$

$$D^\alpha u_\varepsilon|_\Gamma=0,\quad |\alpha|\le m-1. \tag{6}$$

The solution of the problem (4)-(6) it is necessary to understand in following sense

Definition. *A function* $u_\varepsilon(t,x)\in \underset{\circ}{W}{}_2^1(0,T;L_2(\Omega))\cap L_p(0,T;W_p^{2m}(\Omega)\cap \overset{\circ}{W}{}_p^m(\Omega))\cap$ $\cap\{u:u_\varepsilon'(T,x)=0\}$ *is called a solution of problem (4)-(6), if for each* $g(t,x)\in \underset{\circ}{W}{}_2^1(0,T;L_2(\Omega))\cap L_p(Q)$ *the following identity holds:*

$$\varepsilon\int_Q\frac{\partial u_\varepsilon}{\partial t}\frac{\partial g}{\partial t}dxdt+\int_Q\frac{\partial u_\varepsilon}{\partial t}g\,dxdt=$$
$$=\int_Q F\left(t,x,u,\ldots,D^{2m-1}u_\varepsilon,Au_\varepsilon\right)gdxdt=\int_Q h\,g\,dxdt. \tag{7}$$

The following reasonings explain the meaning of the condition $u_\varepsilon'(T,x)=0$ in definition.

If $h(t,x)\in L_q(Q)$, then from $u_\varepsilon(t,x)\in W_2^1(0,T;L_2(\Omega))\cap L_p\left(0,T;W_p^{2m}(\Omega)\cap\right.$ $\left.\cap \overset{\circ}{W}{}_p^m(\Omega)\right)$ and condition 2, by means of applying of equation (4), we obtain, that the solution $u_\varepsilon(t,x)$ is satisfied $\varepsilon\frac{\partial^2 u_\varepsilon}{\partial t^2}\in L(Q)$ also. All this allows to define the boundary meaning $u_\varepsilon'(T,x)$ for any fixed $\varepsilon>0$.

In order to prove the solvability of problem (4)-(6) at first we prove the solvability of the variational equation (7) for any $g(t,x)\in \overset{\circ}{W}{}_2^1(0,T;L_2(\Omega))\cap L_p(Q)$.

The isomorphism generated by the problem

$$Au=v(x),\quad x\in Q,$$
$$D^\alpha u|_{\partial\Omega}=0,\quad |\alpha|\le m-1 \tag{8}$$

gives the possibility to affirm the existence of full continuous in $L_p(\Omega)$ operator L, which is reverse for this operator and is full continuous from $L_p(\Omega)$ to $W_p^{2m-1}(\Omega)$. The fact, that the problem (8) generated isomorphism between these spaces, is known from the theory of linear differential equations (see [6]).

To this end, by means of substitution of principal space the problem (1)-(3) leads to the following equivalent problem

$$\frac{\partial L(v)}{\partial t}+F\left(t,x,L(v),DL(v),\ldots,D^{2m-1}L(v),v\right)=h(t,x), \tag{9}$$

$$v(0,x)=0, \tag{10}$$

where $u=L(v)$.

Accordingly, the problem (4)-(6) leads to the following regularized problem

$$-\varepsilon\frac{\partial^2 L(v_\varepsilon)}{\partial t^2}+\frac{\partial L(v_\varepsilon)}{\partial t}+F\left(t,x,L(v_\varepsilon),\ldots,D^{2m-1}L(v_\varepsilon),v_\varepsilon\right)=h(t,x) \tag{11}$$

$$v_\varepsilon(0,x)=0,\quad v_\varepsilon'(T,x)=0, \tag{12}$$

ε is some constant.

Then accordingly variational problem leads to the following variational problem

$$\varepsilon\int_Q\frac{\partial L(v_\varepsilon)}{\partial t}\frac{\partial g}{\partial t}dxdt+\int_Q\frac{\partial L(v_\varepsilon)}{\partial t}g\,dxdt+$$

$$+\int_Q F\left(t,x,L(v_\varepsilon),\ldots,D^{2m-1}L(v_\varepsilon),v_\varepsilon\right)g\,dxdt=\int_Q h\,g\,dxdt \tag{13}$$

$$\forall g(t,x)\in \overset{\circ}{W}{}_2^1(0,T;L_2(\Omega))\cap L_p(Q)$$

Now in order to solve equation (13) apply approximate Galerkin method (see [6]). Let $\{w_k(t,x)\}$ - be a complete system of functions in the $\overset{\circ}{W}{}_2^1(0,T;L_2(\Omega))\cap L_p(Q)$.

We look for approximate solutions of the problem (4)-(6) of the form

$$v_m(t,x)=\sum_{k=1}^m c_k^m w_k(t,x),$$

where the unknown coefficients c_k^m have to be determined from the system of algebraic equations

$$\varepsilon\int_Q\frac{\partial L(v_{\varepsilon m})}{\partial t}\frac{\partial w_k}{\partial t}dxdt+\int_Q\frac{\partial L(v_{\varepsilon m})}{\partial t}w_k\,dxdt+$$

$$+\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),v_{\varepsilon m}\right)w_k\,dxdt=\int_Q h w_k\,dxdt,\quad k=\overline{1,m} \tag{14}$$

The solvability of this system follows from the well-known lemma «on the acute angle» ([5]) fulfillment of conditions of which in view of continuous dependence of left part of (14) from $v_{\varepsilon m}$ follows from receiving of a priori estimates.

Pass to receiving a priori estimates. Let multiply the k-ht equation of system (14) to c_k^m and then take the sum by k. We get the following integral equality

$$\varepsilon\int_Q\frac{\partial L(v_{\varepsilon m})}{\partial t}\frac{\partial v_{\varepsilon m}}{\partial t}dxdt+\int_Q\frac{\partial L(v_{\varepsilon m})}{\partial t}v_{\varepsilon m}\,dxdt+$$

$$+\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),v_{\varepsilon m}\right)v_{\varepsilon m}\,dxdt=\int_Q h v_{\varepsilon m}\,dxdt$$

Taking into account, that operator $L^{1/2}$ is positive definite selfconjugate operator commutable with $\partial/\partial t$, we have

$$\varepsilon\int_Q \frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}\frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}dxdt+\int_Q \frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}L^{1/2}(v_{\varepsilon m})dxdt+$$

$$+\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),v_{\varepsilon m}\right)v_{\varepsilon m}\,dxdt=\int_Q hv_{\varepsilon m}\,dxdt$$

Hence, using properties of the function F, we will have following equivalent equality

$$\begin{aligned}&\varepsilon\int_Q\left(\frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}\right)^2 dxdt+\frac{1}{2}\int_\Omega L^{1/2}(v_{\varepsilon m}(x,T))^2 dx+\\&+\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),v_{\varepsilon m}\right)v_{\varepsilon m}\,dxdt-\\&-\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt=\\&=\int_Q hv_{\varepsilon m}\,dxdt-F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt\end{aligned}\tag{15}$$

Using condition 3, we shall estimate the left part of (15) from below.

$$\begin{aligned}&\varepsilon\int_Q\left(\frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}\right)^2 dxdt+\frac{1}{2}\int_\Omega\left(L^{1/2}(v_{\varepsilon m}(x,T))\right)^2 dx+\\&+\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),v_{\varepsilon m}\right)v_{\varepsilon m}\,dxdt-\\&-\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt\ge\\&\ge\varepsilon\int_Q\left(\frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}\right)^2 dxdt+\frac{1}{2}\int_\Omega\left(L^{1/2}(v_{\varepsilon m})\right)^2 dx+\\&+\int_Q \Phi\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m})\right)\phi(v_{\varepsilon m})dxdt\end{aligned}\tag{16}$$

For the right part in (15) we have

$$\begin{aligned}&\left|\int_Q hv_{\varepsilon m}dxdt-\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt\right|\le\\&\le\left|\int_Q hv_{\varepsilon m}dxdt\right|+\left|\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt\right|\le\\&\le C(\varepsilon)\int_Q |h(t,x)|^q\,dxdt+\varepsilon\int_Q |v_{\varepsilon m}|^p\,dxdt=\\&+\left|\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt\right|\end{aligned}\tag{17}$$

Using further condition 2 and Young's inequality, when we estimate the last addend of the right part of inequality (17), we obtain

$$\left|\int_Q hv_{\varepsilon m}dxdt\right|+\left|\int_Q F\left(t,x,L(v_{\varepsilon m}),\ldots,D^{2m-1}L(v_{\varepsilon m}),0\right)v_{\varepsilon m}\,dxdt\right|\le$$

$$\le c(\varepsilon)\iint_Q |h(t,x)|^q\,dxdt + \varepsilon \iint_Q |v_{\varepsilon m}|^p\,dxdt +$$

$$+c(\varepsilon_1)\iint_Q \left|F\left(t,x,L(v_{\varepsilon m}),\dots,D^{2m-1}L(v_{\varepsilon m}),0\right)\right|^q dxdt + \varepsilon_1 \iint_Q |v_{\varepsilon m}|^p\,dxdt \le$$

$$\le c(\varepsilon)\iint_Q |h(t,x)|^q\,dxdt + (\varepsilon+\varepsilon_1)\iint_Q |v_{\varepsilon m}|^p\,dxdt +$$

$$+c(\varepsilon_1)\int_Q c\left(\sum_{|\alpha|=i=0}^{2m-1}\left|D^\alpha L(v_{\varepsilon m})\right|^{p_i}\right)^q dxdt \le$$

$$\le c(\varepsilon)\iint_Q |h(t,x)|^q\,dxdt + (\varepsilon+\varepsilon_1)\iint_Q |v_{\varepsilon m}|^p\,dxdt +$$

$$+\tilde{c}\,c(\varepsilon)\sum_{|\alpha|=i=0}^{2m-1}\iint_Q \left|D^\alpha L(v_{\varepsilon m})\right|^{p_i q}\,dxdt \le$$

$$\le c(\varepsilon)\|h(x,t)\|^q_{L_q(Q)} + \varepsilon_2\|v_{\varepsilon m}\|^p_{L_p(Q)} + c(\varepsilon_2, mes\Omega).$$

Using (15), (16), and conditions, which were putting on the function $\Phi\left(t,x,L(v_{\varepsilon m}),\dots,D^{2m-1}L(v_{\varepsilon m})\right)$ and $\phi(v_{\varepsilon m})$ in 3, choosing enough small ε_2, we have

$$\varepsilon\iint_Q\left(\frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}\right)^2 dxdt + \frac{1}{2}\int_\Omega \left(L^{1/2}(v_{\varepsilon m}(x,T))\right)^2 dx + \|v_{\varepsilon m}\|_{L_{p}(Q)} \le c\left(\|h\|_{L_q(Q)}\right),$$

where $c\left(\|h\|_{L_q(Q)}\right)$ is a constant.

Hence follows a priori estimates

$$\varepsilon\left\|\frac{\partial L^{1/2}(v_{\varepsilon m})}{\partial t}\right\|_{L_2(Q)} \le C;\quad \|v_{\varepsilon m}\|_{L_p(Q)} \le C;\quad \left\|L^{1/2}(v_{\varepsilon m})\right\|_{L_2(\Omega)}(T) \le C \tag{18}$$

Thus it has been received a sequence of approximate solution $\{v_{\varepsilon m}\}$ of the equation (13), which uniform on m for every fixed ε satisfy to a priori estimates (17).

From these estimations indicating boundedness of a sequence $\{v_{\varepsilon m}(t,x)\}$ in space $L_p(Q)$ in view of weak compactness of a bounded set in $L_p(Q)$ the existence of the element $v_\varepsilon(t,x)\in L_p(Q)$ and subsequence of approximate solutions $\{v_{\varepsilon m_k}(t,x)\}$, designated in further again as $\{v_{\varepsilon m}(t,x)\}$, converging weakly to $v_\varepsilon(t,x)$ in $L_p(Q)$. Now we shall show, that limiting function $v_\varepsilon(t,x)$ is a solution of the equation (13). For this purpose it is necessary to investigate properties of a nonlinear part of the equation (11).

At first we shall show boundedness of an operator $Q(\cdot)=F\left(t,x,L(v),\dots,D^{2m-1}L(v),v\right): L_p(Q)\to L_q(Q)$, which immediately follows from the following reasonings:

$$\iint_Q \left|F\left(t,x,L(v),\dots,D^{2m-1}L(v),v\right)\right|^q dxdt \le$$

$$\le C\iint_Q\left(\sum_{|\alpha|=i=0}^{2m-1}\left|D^\alpha L(v)\right|^{p_i} + |v|^{p-1}+1\right)^q dxdt \le$$

$$\leq \widetilde{C}\left(\sum_{|\alpha|=i=0}^{2m-1} \int\limits_Q \left| D^\alpha L(v) \right|^{p,q} + \int\limits_Q |v|^p \, dxdt \right) + K .$$

Using further that operator L is a completely continuous operator from $L_p(\Omega)$ to $W_p^{2m-1}(\Omega)$ for any $\alpha(|\alpha| \leq 2m-1)$, we have

$$D^\alpha L : L_p(\Omega) \to L_p(\Omega) \text{ compactly.}$$

Therefore

$$D^\alpha L : L_p(\Omega) \to L_p(\Omega) \text{ restrictedly}$$

that is $\left\| D^\alpha L(v) \right\|_{L_p(Q)} \leq K \|v\|_{L_p(Q)}$.

From here it follows

$$\int\limits_Q \left| F\left(t, x, L(v), ..., D^{2m-1} L(v), v\right) \right|^q dxdt \leq \overline{C}\left(\|v\|_{L_p(q)}^p + 1 \right).$$

Thus, we have received boundedness of a sequence of images of an operator Q in space $L_p(Q)$. From here and from reflexivity of the space L_q it follows the existence of the function $\aleph_\varepsilon \in L_q(Q)$, such that

$$F\left(t, x, L(v_{\varepsilon m}), DL(v_{\varepsilon m}), ..., D^{2m-1} L(v_{\varepsilon m}), v_{\varepsilon m}\right) \to \aleph_\varepsilon \text{ weakly in } L_q(Q)$$

(possibly after choosing a subsequence). Consequently, we may to pass to the limit in (13) by m for every fixed k.

After passing to the limit we obtain

$$\int\limits_Q \frac{\partial L(v_\varepsilon)}{\partial t} \frac{\partial w_k}{\partial t} dxdt + \int\limits_Q \frac{\partial L(v_\varepsilon)}{\partial t} dxdt + \int\limits_Q \aleph_\varepsilon w_k dxdt = \int\limits_Q h w_k dxdt, \qquad \forall k \qquad (19)$$

We shall show now, that is holds the

$$F\left(t, x, L(v_\varepsilon), DL(v_\varepsilon), ..., D^{2m-1} L(v_\varepsilon), v_\varepsilon\right) = \aleph_\varepsilon .$$

At fist we notice, that in view of compactness of operators L and $D^\alpha L(|\alpha| \leq 2m-1)$ in $L_p(Q)$ we obtain, that it is possible to choose a subsequence from $\{v_{\varepsilon m}\}$, such that

$$L(v_{\varepsilon m}) \Rightarrow L(v_\varepsilon) \text{ strongly in } L_p(Q);$$

$$D^\alpha L(v_{\varepsilon m}) \Rightarrow D^\alpha L(v_\varepsilon) \text{ strongly in } L_p(Q).$$

Moreover, it was shown above, that

$$v_{\varepsilon m} \to v_\varepsilon \text{ weakly in } L_p(Q).$$

We shall take advantage of the following statement now.

Lemma. *Operator* $Q(\cdot) = F\left(t, x, L(v), ..., D^{2m-1} L(v), v\right)$ *is an operator of variational calculation.*

The of the lemma follows from direct verify of all properties of operator of variational calculation.

For operator of variational calculation is valid the following

Corollary. (see [5]) *«Q - is an operator of variational calculation»* $\Rightarrow$ *«Q - is a pseudomonotone operator».*

Further we shall use a fact, that the sum of monotone and pseudomonotone operators is a pseudomonotone operator ([5]).

Hence it follows, that the operator G, defined the expression

$$G(v_{\varepsilon m}) = -\varepsilon \frac{\partial^2 L(v_{\varepsilon m})}{\partial t^2} + \frac{\partial L(v_{\varepsilon m})}{\partial t} + F(t, x, L(v_{\varepsilon m}), \ldots, D^{2m-1} L(v_{\varepsilon m}), v_{\varepsilon m})$$

is a pseudomonotone operator

$$G : \overset{\circ}{W}{}^1_p(0,T; L_p(\Omega)) \to W_q^{-1}(0,T; L_q(\Omega)).$$

Then, using this property, from (19) we receive the following equality

$$F(t, x, L(v_\varepsilon), \ldots, D^{2m-1} L(v_\varepsilon), v_\varepsilon) = \aleph_\varepsilon .$$

It follows in force of further reasoning. Using the pseudomonotonicity of the operator, generated by problem (11)-(12), we obtain

$$\liminf \left(-\varepsilon \frac{\partial^2 L(v_{\varepsilon m})}{\partial t^2} + \frac{\partial L(v_{\varepsilon m})}{\partial t} + F(t, x, L(v_{\varepsilon m}), \ldots, D^{2m-1} L(v_{\varepsilon m}), v_{\varepsilon m}), v_{\varepsilon m} - \tilde{v} \right) \geq$$

$$\geq \left(-\varepsilon \frac{\partial^2 L(v_\varepsilon)}{\partial t^2} + \frac{\partial L(v_\varepsilon)}{\partial t} + F(t, x, L(v_\varepsilon), \ldots, D^{2m-1} L(v_\varepsilon), v_\varepsilon), v_\varepsilon - \tilde{v} \right),$$

$\forall \tilde{v} \in \overset{\circ}{W}{}^1_p(Q).$

Evidently, that

$$\lim_{m \to \infty} \left(-\varepsilon \frac{\partial^2 L(v_{\varepsilon m})}{\partial t^2} + \frac{\partial L(v_{\varepsilon m})}{\partial t} + F(t, x, L(v_{\varepsilon m}), \ldots, D^{2m-1} L(v_{\varepsilon m}), v_{\varepsilon m}), v_{\varepsilon m} - \tilde{v} \right) \geq$$

$$\geq \left(-\varepsilon \frac{\partial^2 L(v_\varepsilon)}{\partial t^2} + \frac{\partial L(v_\varepsilon)}{\partial t} + \aleph_\varepsilon, v_{\varepsilon m} - \tilde{v} \right), \quad \forall \tilde{v} \in W_p^1(Q).$$

Then we have

$$\left(F(t, x, L(v_\varepsilon), \ldots, D^{2m-1} L(v_\varepsilon), v_\varepsilon) - \varepsilon \frac{\partial^2 L(v_\varepsilon)}{\partial t^2} + \frac{\partial L(v_\varepsilon)}{\partial t}, v_\varepsilon - \tilde{v} \right) \geq$$

$$\geq \left(-\varepsilon \frac{\partial^2 L(v_\varepsilon)}{\partial t^2} + \frac{\partial L(v_\varepsilon)}{\partial t} + \aleph_\varepsilon, v_\varepsilon - \tilde{v} \right).$$

From here follows

$$(\aleph_\varepsilon - F(t, x, L(v_\varepsilon), \ldots, D^{2m-1} L(v_\varepsilon), v_\varepsilon), v_\varepsilon - \tilde{v}) \geq 0 .$$

Assume, that $\tilde{v} = v_\varepsilon - \lambda w, \quad \forall w \in \overset{\circ}{W}{}^1_p(Q), \lambda \to +0$. Then obtain

$$(\aleph_\varepsilon - F(t, x, L(v_\varepsilon), \ldots, D^{2m-1} L(v_\varepsilon), v_\varepsilon), w) \geq 0 .$$

Since the choice of $w(t,x)$ was free, the last inequality is fair as soon as

$$\aleph_\varepsilon = F(t, x, L(v_\varepsilon), \ldots, D^{2m-1} L(v_\varepsilon), v_\varepsilon).$$

Thus we receive the solvability of equation (13).

From a priori estimates and equation (13) for the first addend in (11) follows, that

$$\varepsilon \frac{\partial^2 L(v_\varepsilon)}{\partial t^2} \in L_q(0,T; L_q(\Omega))$$

for every $\varepsilon > 0$, because all other addends in (13) and the right part has a sense for every $g(t,x) \in L_p(Q)$.

Using this from the solvability of the equation (13) by means of standard steps (see [4]) we receive the solvability of the problem (11)-(12). Thus the solvability of the regularized problem is proved.

Then we'll reason a usually, i.e. analogous [1], [5], [4]. Namely, using the solvability of a regularized problem, at first we receive for $\frac{\partial L(v_\varepsilon)}{\partial t}$ uniform by ε estimates, and then pass to the limit by ε when $\varepsilon \to 0$. This concludes the proof of the considering problem.

References

[1]. Dubinski Y.A. *Nonlinear elliptic and parabolic equations.* Itogi nauki i techn.: sovremen. Probl. Matem, 9, 1976, 3-130.

[2]. Kasumov N.Z. *On the solvability of one hight order nonlinear problem.* Izv. Akad. Nauk Azerb., v.XVIII, No 4-5, 1997, 99-105.

[3]. Soltanov K.N. *Periodic solution of some nonlinear parabolic equations with an implicit degeneracy.* Dokl. Akad. Nauk SSSR, v.222, No 2, 1975, 637-641.

[4]. Soltanov K.N. *Nonlinear equations in non-reflexive Banach spaces and strongly nonlinear differential equations.* Preprint No 440, Berlin, 1998, 37 p.

[5]. Lions J.L. *Quelques methods de resolution des problems aux limites nonlineares.* Dunod, Paris, 1969.

[6]. Hormander L. *The analysis of linear partial differential operators,* 1983.

Kasumov N.Z.
Institute of Mathematics and Mechanics of AS Azerbaijan.
9, F. Agayev str., 370141, Baku, Azerbaijan.
Tel.: 39-47-20.

Received June 23, 1999; Revised September 3, 1999.
Translated by author.