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STRONG GENERALIZED SOLVABILITY OF FIRST BOUNDARY VALUE PROBLEM FOR GILBARG-SERRIN EQUATION

Abstract

In present paper the class of elliptic equations of the second order of Gilbarg-Serrin's type was considered. The strong generalized solvability of first boundary value problem for such equations is proved.

Let D be bounded domain in n-dimensional Euclidian space E_n of points $x=(x_1,...,x_n)$, $n\ge 3$, $0\in D$, and ∂D be boundary of D. Consider in D Gilbarg-Serrin's operator

$$L=\Delta+\mu(r)\sum_{i,j=1}^{n}\frac{x_i x_j}{r^2}\frac{\partial^2}{\partial x_i \partial x_j},$$

where $r=|x|$, $b_1\le\mu(r)\le b_2$, $b_1>-1$, $b_2<\infty$.

It is easy to see, that operator L is uniformly elliptic in domain D. However, if $\inf\limits_{x\in D}\mu(|x|)>n-2$, then solutions of equation $Lu=0$ find the series of qualitative new properties extrinsic, for example, for solutions of Laplace equation (see [1-2]). The aim of present paper is to prove the uniqueness of strongly generalized Dirichlet problem for class of equations of Gilbarg-Serrin's type. It must be noted, that in case if $\mu\equiv const$ the analogous results was obtained in [3-5]. The paper [6] could be used as reference for the question on weak solvability of first boundary value problem for considered equations.

Now we will describe some denotations and definitions. By $L_{2,\gamma}(D)$, $W^1_{2,\gamma}(D)$ and $W^2_{2,\gamma}(D)$ we will denote Banach spaces of functions, giving on D, for which final norms are

$$\|u\|_{L_{2,\gamma}(D)}=\left(\int_D r^{\gamma-2}u^2dx\right)^{1/2},$$

$$\|u\|_{W^1_{2,\gamma}(D)}=\left(\int_D\left(r^{\gamma-2}u^2+r^{\gamma}|\nabla u|^2\right)dx\right)^{1/2}$$

and

$$\|u\|_{W^2_{2,\gamma}(D)}=\left(\int_D\left(r^{\gamma-2}u^2+r^{\gamma}|\nabla u|^2+r^{\gamma+2}\sum_{i,j=1}^{n}\left(\frac{\partial^2 u}{\partial x_i\partial x_j}\right)^2\right)dx\right)^{1/2}.$$

Here ∇u is vector of gradient of function $u(x)$. By $\dot{W}^2_{2,\gamma}(D)$ we will denote subspace $W^2_{2,\gamma}(D)$ the dense set in which are all infinitively differential in $\overline{D}$ functions, which vanishes on ∂D, the corresponding norms of which are finite. For $i,j=\overline{1,n}$ $u_i=\frac{\partial u}{\partial x_i}$, $u_{ij}=\frac{\partial^2 u}{\partial x_i\partial x_j}$. Everywhere further under the record $c(...)$ we will understand that constant c, which depends only on content of brackets. Suppose for function $\mu(r)$ following conditions

$$b_0 \le \mu(r) \le b_2;\ b_0 > 2n-3,\ b_2 < \infty, \tag{1}$$

$$|\mu'(r)| \le \frac{c_1}{r};\ \ -\frac{c_2}{r^2} \le \mu''(r) \le 0;\ r \in (0, diam\, D). \tag{2}$$

We will use following lemma, that what was proved in [6].

Lemma 1. *Let D be bounded domain in E_n, in which the coefficients of operator L satisfying to conditions (1)-(2) are determined. Then if $\gamma = 1-n$, then for any function $u(x) \in W_{2,\gamma}^2(D) \cap C_0^\infty(D)$ the following estimation is valid*

$$-\int_D r^\gamma u L u dx \ge C_3(\mu) \int_D r^\gamma |\nabla u|^2 dx. \tag{3}$$

Lemma 2. *If conditions (1)-(2) holds with respect to coefficients of operator L, $\gamma = 1-n$, then for any function $u(x) \in W_{2,\gamma}^2(D) \cap C_0^\infty(D)$ inequality*

$$\|u\|_{W_{2,\gamma}^2(D)} \le C_4(n,\mu) \|Lu\|_{L_{2,\gamma+2}(D)} \tag{4}$$

holds.

Proof. We fix k, $1 \le k \le n$ and suppose in estimation (3) ru_k instead of function u. We have

$$\sum_{k=1}^n \int_D r^\gamma |\nabla(ru_k)|^2 dx = \sum_{k=1}^n \int_D r^\gamma u_k^2 dx + \sum_{i,k=1}^n \int_D r^\gamma x_i (u_k^2)_i dx +$$

$$+ \sum_{i,k=1}^n \int_D r^{\gamma+2} u_{ik}^2 dx = \sum_{k=1}^n \int_D r^\gamma u_k^2 dx - (\gamma+n) \sum_{k=1}^n \int_D r^\gamma u_k^2 dx +$$

$$+ \sum_{i,k=1}^n \int_D r^{\gamma+2} u_{ik}^2 dx = \sum_{i,k=1}^n \int_D r^{\gamma+2} u_{ik}^2 dx. \tag{5}$$

So as $\gamma = 1-n$. From the other side,

$$L(ru_k) = rLu_k + u_k Lr + 2\sum_{i,j=1}^n a_{ij} r_i u_{kj} \tag{6}$$

where $a_{ij} = \delta_{ij} + \frac{\mu(r) x_i x_j}{r^2}$. Here δ_{ij} is Kroneker's symbol. Moreover, $Lr = \frac{n-1}{r}$,

$2\sum_{i,j=1}^n a_{ij} r_i u_{kj} = 2\sum_{i=1}^n \frac{x_i}{r} u_{ki} + 2\mu(r) \sum_{j=1}^n \frac{x_j u_{kj}}{r}$. Thus, from (6) we get

$$I = -\int_D r^{\gamma+1} u_k L(ru_k) dx = -\int_D r^{\gamma+2} u_k L u_k dx -$$

$$-\int_D r^{\gamma+1} u_k \frac{n-1}{r} u_k dx - 2\sum_{i=1}^n \int_D r^{\gamma+1} u_k \frac{x_i}{r} u_{ki} dx -$$

$$-2\sum_{i=1}^n \int_D r^{\gamma+1} u_k \frac{x_i}{r} u_{ki} \mu(r) dx = i_1 + i_2 + i_3 + i_n. \tag{7}$$

It is easy to see, that

$$i_2 = -(n-1) \sum_{i=1}^n \int_D r^\gamma u_k^2 dx,$$

$$i_3 = \sum_{i=1}^{n} \int_D \left(r^\gamma x_i\right)_i u_k^2 dx = \gamma \sum_{i=1}^{n} \int_D r^{\gamma-2} x_i^2 u_k^2 dx +$$
$$+ n \int_D r^\gamma u_k^2 dx = \int_D r^\gamma u_k^2 dx ,$$

i.e.

$$i_2 + i_3 = (\gamma + 1) \int_D r^\gamma u_k^2 dx = -n \int_D r^\gamma u_k^2 dx \leq 0 \tag{8}$$

We have further

$$i_4 = \sum_{i=1}^{n} \int_D \left(r^\gamma x_i \mu(r)\right)_i u_k^2 dx = \int_D r^\gamma \mu(r) u_k^2 dx +$$
$$+ \int_D r^{\gamma+1} \mu'(r) u_k^2 dx . \tag{9}$$

Finally,

$$i_1 = -\int_D r^{\gamma+2} u_k (Lu)_k dx + \int_D r^{\gamma+2} u_k \mu'(r) \frac{x_k}{r} \sum_{i,j=1}^{n} \frac{x_i x_j}{r^2} u_{ij} dx +$$
$$+ 2 \int_D r^{\gamma+2} u_k \mu(r) \sum_{i=1}^{n} \frac{x_i u_{ik}}{r^2} dx - 2 \int_D r^{\gamma+2} u_k \mu(r) \sum_{i,j=1}^{n} \frac{x_i x_j}{r^4} u_{ij} dx =$$
$$= j_1 + j_2 + j_3 + j_4 . \tag{10}$$

But from the other side, according to conditions (1)-(2)

$$j_1 = \int_D r^{\gamma+2} u_{kk} Lu dx + (\gamma + 2) \int_D r^\gamma x_k u_k Lu dx \leq$$
$$\leq \int_D r^{\gamma+2} u_{kk} Lu dx + |\gamma + 2| \int_D r^{\gamma+1} |u_k| \cdot |Lu| dx , \tag{11}$$

$$j_2 \leq C_1 \sum_{i,j=1}^{n} \int_D r^{\gamma+1} |u_k| \cdot |u_{ij}| dx , \tag{12}$$

$$j_3 \leq 2b_2 \sum_{i=1}^{n} \int_D r^{\gamma+1} |u_k| \cdot |u_{ik}| dx , \tag{13}$$

$$j_4 \leq 2b_2 \sum_{i,j=1}^{n} \int_D r^{\gamma+1} |u_k| \cdot |u_{ij}| dx . \tag{14}$$

Taking account of (5)-(9) and (11)-(14) in (10), we obtain that for any $\varepsilon > 0$

$$C_3 \sum_{i,j=1}^{n} \int_D r^{\gamma+2} u_{ij}^2 dx \leq \sum_{i=1}^{n} \int_D r^{\gamma+2} u_{ii} Lu dx +$$
$$+ (c_1 + b_2) \sum_{i=1}^{n} \int_D r^\gamma u_i^2 dx + |\gamma + 2| \sum_{i=1}^{n} \int_D r^{\gamma+1} |u_i| |Lu| dx +$$
$$+ c_1 \sum_{i,j,k=1}^{n} \int_D r^{\gamma+1} |u_k| |u_{ij}| dx + 2b_2 \sum_{i,k=1}^{n} \int_D r^{\gamma+1} |u_k| |u_{ik}| dx +$$
$$+ 2b_2 \sum_{i,j,k=1}^{n} \int_D r^{\gamma+1} |u_k| |u_{ij}| dx \leq \frac{\varepsilon}{2} \sum_{i=1}^{n} \int_D r^{\gamma+2} u_{ii}^2 dx +$$
$$+ \frac{n}{2\varepsilon} \int_D r^{\gamma+2} (Lu)^2 dx + (c_1 + b_2) c_6 \int_D r^{\gamma+2} (Lu)^2 dx +$$
$$+ \frac{n-3}{2} \int_D r^\gamma |\nabla u|^2 dx + \frac{(n-3)n}{2} + \int_D r^{\gamma+2} (Lu)^2 dx +$$

$$+\frac{c_1\varepsilon n}{2}\sum_{i,j=1}^{n}\int_D r^{\gamma+2}u_{ij}^2dx+\frac{c_1n^2}{2\varepsilon}\int_D r^{\gamma}|\nabla u|^2dx+$$

$$+\frac{2b_2\varepsilon}{2}\sum_{i,j=1}^{n}\int_D r^{\gamma+2}u_{ij}^2dx+\frac{2b_2n}{2\varepsilon}\int_D r^{\gamma}|\nabla u|^2dx+$$

$$+\frac{2b_2\varepsilon n}{2}\sum_{i,j=1}^{n}\int_D r^{\gamma+2}u_{ij}^2dx+\frac{2b_2n^2}{2\varepsilon}\int_D r^{\gamma}|\nabla u|^2dx\le$$

$$\le\varepsilon c_7(\mu,n)\sum_{i,j=1}^{n}\int_D r^{\gamma+2}u_{ij}^2dx+c_8(\varepsilon,\mu,n)\int_D r^{\gamma+2}(Lu)^2dx\,.$$

Now we choose and fix $\varepsilon=\frac{c_3}{2c_7}$. Then

$$\sum_{i,j=1}^{n}\int_D r^{\gamma+2}u_{ij}^2dx\le c_9(\mu,n)\int_D r^{\gamma+2}(Lu)^2dx \tag{15}$$

From the other side from (3) it follows that for any $\varepsilon_1>0$ it is valid inequality

$$\frac{\varepsilon_1}{2}\int_D r^{\gamma-2}u^2dx+\frac{1}{2\varepsilon_1}\int_D r^{\gamma+2}(Lu)^2dx\ge c_3\int_D r^{\gamma}|\nabla u|^2dx\,. \tag{16}$$

Moreover, according to [6]

$$\int_D r^{\gamma-2}u^2dx\le c_{10}(\mu,n)\int_D r^{\gamma}|\nabla u|^2dx\,. \tag{17}$$

Fix now $\varepsilon_1=\frac{c_3}{c_{10}}$. Then from (16) and (17) we have

$$\int_D r^{\gamma}|\nabla u|^2dx\le c_{11}(\mu,n)\int_D r^{\gamma+2}(Lu)^2dx \tag{18}$$

Thus, from (15), (17) and (18) follows the requested estimation (4). Lemma is proved.

For $x^0\in E_n$ and $R>0$ we denote by $Q_R^{x^0}$ the ball $\{x:|x-x^0|<R\}$.

Lemma 3. *Let $\overline{Q}_R^{x^0}\subset D,\gamma=1-n$ and with respect to coefficients of operator L conditions (1)-(2) hold. Then for any function $u(x)\in W_{2,\gamma}^2\left(Q_R^{x^0}\right)\cap C^{\infty}\left(\overline{Q}_R^{x^0}\right)$ for any $r\in(0,R)$ inequality*

$$\|u\|_{W_{2,\gamma}^2\left(Q_r^{x^0}\right)}\le C_{12}(R,\mu,n)\left(1-\frac{r}{R}\right)^{-2}\left(\|Lu\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)}+\|u\|_{W_{2,\gamma}^2\left(W_{2,\gamma}^1\left(Q_R^{x^0}\right)\right)}\right). \tag{19}$$

is valid.

Proof. Let $u(x)\in W_{2,\gamma}^2\left(Q_R^{x^0}\right)\cap C^{\infty}\left(\overline{Q}_R^{x^0}\right)$. Consider auxiliary function $\eta(x)$ such that $\eta(x)=1$ for $x\in Q_r^{x^0}$, $\eta(x)=0$ for $x\notin Q_{\frac{R+r}{2}}^{x^0}$, $0\le\eta(x)\le1, \eta(x)\in C_0^{\infty}\left(Q_R^{x^0}\right)$. For this we could assume, that for $i,j=\overline{1,n}$

$$|\eta_i|\le\frac{C_{13}(n)}{R-r},\ |\eta_{ij}|\le\frac{C_{13}}{(R-r)^2}\,. \tag{20}$$

Suppose $v(x)=u(x)\cdot\eta(x)$. It is clear, that $v(x)\in C_0^{\infty}\left(Q_R^{x^0}\right)\cap W_{2,\gamma}^2\left(Q_R^{x^0}\right)$. Therefore for this function we can apply estimation (4). We have, taking account of (20)

$$\|u\|^2_{W^2_{2,\gamma}\left(Q_r^{x^0}\right)} \le c_{14}(n,\mu)\left(\int\limits_{Q_R^{x^0}} (Lu)^2|x|^{\gamma+2}dx + \frac{c_{15}(n)}{(R-r)^4}\int\limits_{Q_R^{x^0}} u^2|x|^{\gamma-2}dx + \frac{c_{16}(n,\mu)}{(R-r)^2}\int\limits_{Q_R^{x^0}} |x|^{\gamma}|\nabla u|^2 dx\right).$$

From here immediately follows the requested estimate (19). Lemma is proved.

Lemma 4. *If conditions of previous lemma are valid, then for any function* $u(x)\in W^2_{2,\gamma}\left(Q_R^{x^0}\right)\cap C^\infty\left(\overline{Q}_R^{x^0}\right)$ *for any* $r\in(0,R)$ *following inequality is true*

$$\|u\|_{W^2_{2,\gamma}\left(Q_r^{x^0}\right)} \le c_{17}(R,r,n,\mu)\left(\|Lu\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)} + \|u\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)}\right). \tag{21}$$

Proof. Suppose $A=\sup\limits_{r\in(0,R)}\left\{\left(1-\frac{r}{R}\right)^2\|u\|_{W^2_{2,\gamma}\left(Q_r^{x^0}\right)}\right\}$. Then there exists R_1, $0<R_1<R$ such that

$$A\le 2\left(1-\frac{R_1}{R}\right)^2\|u\|_{W^2_{2,\gamma}\left(Q_{R_1}^{x^0}\right)}.$$

According to previous lemma for any $R_2, R_1<R_2<R$ we have

$$A\le 2\left(1-\frac{R_1}{R}\right)^2 C_{18}(\mu,n)R_2^{-2}\left(1-\frac{R_1}{R_2}\right)^{-2}\times$$

$$\times\left(\|Lu\|_{L_{2,\gamma+2}\left(Q_{R_2}^{x^0}\right)} + \|u\|_{W^1_{2,\gamma}\left(Q_{R_2}^{x^0}\right)}\right)\le 2C_{18}R_2^{-2}\times$$

$$\times\left(1-\frac{R_1}{R}\right)^2\left(1-\frac{R_1}{R_2}\right)^{-2}\left[\|Lu\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)} + \|u\|_{W^1_{2,\gamma}\left(Q_{R_2}^{x^0}\right)}\right].$$

Applying now interpolational inequality, we have for any $\varepsilon>0$

$$A\le 2C_{18}R_2^{-2}\left(1-\frac{R_1}{R}\right)^2\left(1-\frac{R_1}{R_2}\right)^{-2}\left[\|Lu\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)} + \varepsilon\|u\|_{W^2_{2,\gamma}\left(Q_{R_2}^{x^0}\right)} + C_{19}(\varepsilon,n)\|u\|_{L_{2,\gamma+2}\left(Q_{R_2}^{x^0}\right)}\right]\le$$

$$\le 2C_{18}R_2^{-2}\left(1-\frac{R_1}{R}\right)^2\left(1-\frac{R_1}{R_2}\right)^{-2}\left(1-\frac{R_2}{R}\right)^{-2}\varepsilon A +$$

$$+2C_{18}R_2^{-2}\left(1-\frac{R_1}{R}\right)^2\left(1-\frac{R_1}{R_2}\right)^{-2}\|Lu\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)} +$$

$$+2C_{18}C_{19}R_2^{-2}\left(1-\frac{R_1}{R}\right)^2\left(1-\frac{R_1}{R_2}\right)^{-2}\|u\|_{L_{2,\gamma+2}\left(Q_R^{x^0}\right)} \tag{22}$$

Suppose that $\delta=1-\frac{R_1}{R}$ and choose $R_2\in(R_1,R)$ such, that $1-\frac{R_2}{R}=\frac{\delta}{2}$. Now we fix this R_2. It is easy to see, that $\frac{\delta}{2}<1-\frac{R_1}{R_2}<\delta$. Choose now ε such that

$$32C_{18}R_2^{-2}\delta^{-2}\varepsilon=\frac{1}{2}.$$

Taking account of fact that $C_{19}=\frac{C_{20}(n)}{\varepsilon}$, from (22) we obtain estimation (21). Lemma is proved.

Corollary. *Let for* $\rho>0$ $D_\rho=\{x: x\in D, dist(x,\partial D)>\rho\}$. *Then if conditions of lemma are satisfied, then for any function* $u(x)\in W_{2,\gamma}^2(D)$ *for any sufficiently small* ρ *the following estimation is valid*

$$\|u\|_{W_{2,\gamma}^2(D_\rho)}\le C_{21}(D,\rho,\mu,n)\left(\|Lu\|_{L_{2,\gamma+2}(D)}+\|u\|_{L_{2,\gamma+2}(D)}\right).$$

Lemma 5. *Let conditions (1)-(2) holds with respect to coefficients of operator* L *and* $\gamma=1-n$. *Then for any function* $u(x)\in \dot{W}_{2,\gamma}^2(D)$ *for any sufficiently small* ρ *inequality holds*

$$\|u\|_{W_{2,\gamma}^2(D\setminus D_\rho)}\le C_{22}(D,\rho,\mu,n)\left(\|Lu\|_{L_{2,\gamma+2}(D)}+\|u\|_{L_{2,\gamma+2}(D)}\right). \tag{23}$$

Proof. It is enough to prove estimation (23) for functions $u(x)\in W_{2,\gamma}^2(D)\cap C^\infty(\overline{D})$, $u|_{\partial D}=0$. We will use method of local straightening of boundary (see [7]). Now let $x^0\in\partial D$ and ρ is fix enough small number. So as boundary ∂D belongs to class C^2, then there exists coordinates transformation $x\leftrightarrow y$ such, that if y^0 is image of point x^0, then in some neighbourhood of y^0 the image of boundary is determined by equation $y_n=0$. Denote by $\hat{Q}_{2\rho}^{x^0}$ the subset D, which for such mapping will maps to the half ball $Q_{2\rho,+}^{y^0}=\{y: y\in Q_{2\rho}^{y^0}, y_n>0\}$. For this the operator L will transforms to the some elliptic operator $\tilde{L}$ of second order with continuous coefficients, so as $x^0\ne 0$. Let $\tilde{u}(y)$ be image of $u(x)$ for such mapping. We continue function $\tilde{u}(y)$ by odd order through hyperplane $y_n=0$ into half ball $Q_{2\rho}^{y^0}\setminus Q_{2\gamma,+}^{y^0}$ and denote obtained continuation again by $\tilde{u}(y)$. According to previous lemma

$$\|\tilde{u}\|_{W_{2,\gamma}^2\left(Q_\rho^{y^0}\right)}\le C_{17}\left(\|Lu\|_{L_{2,\gamma+2}\left(Q_{2\rho}^{y^0}\right)}+\|u\|_{L_{2,\gamma+2}\left(Q_{2\rho}^{y^0}\right)}\right).$$

Taking into account fact, that function $\tilde{u}(y)$ continues by odd order, returning to variables x and covering $\overline{D\setminus D_\rho}$ by sets $\hat{Q}_{2\rho}^{x^0}$, we obtain estimation (23). Lemma is proved.

Corollary. *If conditions of lemma holds, then for any function* $u(x)\in \dot{W}_{2,\gamma}^2(D)$ *inequality*

$$\|u\|_{W_{2,\gamma}^2(D)}\le C_{23}(D,\mu,n)\left(\|Lu\|_{L_{2,\gamma+2}(D)}+\|u\|_{L_{2,\gamma+2}(D)}\right)$$

is true.

Theorem 1. *Let with respect to coefficients of operator* L *conditions (1)-(2) satisfies. Then for any function* $u(x)\in \dot{W}_{2,\gamma}^2(D)$ *the following estimation holds*

$$\|u\|_{W_{2,\gamma}^2(D)}\le C_{24}(D,\mu,n)\|Lu\|_{L_{2,\gamma+2}(D)}. \tag{24}$$

Proof. As it follows from [6], the inequality (3) is valid for any function $u(x)\in W_{2,\gamma}^2(D)\cap C^\infty(\overline{D})$, $u|_{\partial D}=0$. From (3) for any $\sigma>0$ we have

$$\frac{\sigma}{2}\int_D r^{\gamma-2}u^2dx+\frac{1}{2\sigma}\int_D r^{\gamma+2}(Lu)^2dx\ge C_3\int_D r^\gamma|\nabla u|^2dx. \tag{25}$$

Taking into account (17), from (25) we obtain

$$\frac{1}{2\sigma}\int_D r^{\gamma+2}(Lu)^2dx \geq \left(\frac{c_3}{c_{10}}-\frac{\sigma}{2}\right)\int_D r^{\gamma-2}u^2dx. \qquad (26)$$

Choosing now $\sigma=\frac{c_3}{c_{10}}$ and taking into account that $r^{\gamma-2}\geq(diamD)^{-4}r^{\gamma+2}$, from corollary of lemma 5 and (26) we obtain requested estimation (24). Theorem is proved.

Theorem 2. *Let in domain D are determined coefficients of operator L, satisfying to conditions (1)-(2) and $\gamma=1-n$. Then first boundary value problem*

$$Lu=f,\ x\in D;\ u\in \dot W^2_{2,\gamma}(D),$$

is uniquely solvable in space $\dot W^2_{2,\gamma}(D)$ for any $f\in L_{2,\gamma+2}(D)$. For this for the solution of $u(x)$ the following estimation is valid

$$\|u\|_{W^2_{2,\gamma}(D)}\leq C_{24}\|f\|_{L_{2,\gamma+2}(D)} \qquad (27)$$

Proof. For proof of this theorem it is enough to apply standard procedure (see, for example, [3]). Using theorem 1 and estimation (27) becomes a corollary of inequality (24).

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