MECHANICS

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STRENGTH OF THE SURROUND PIPE SUBJECTED TO THE ACTION OF THE INTERNAL PIECE-WISE DISTRIBUTED LOADING

Abstract

Calculation method of casing pipe subjected to the action of internal piecewise-distributed loading is worked out in the paper. The arrangement and distribution of stresses are found in a peripherical direction by Fourier transformation method on a pipe axis and expansion in series.

In the work it is considered the strength of the surround pipe subjected to the action of the internal loading. Action of the loading is modeled in the form of the piece-wise distributed loading. Because of that the mechanical model is represented in the form of the infinite long pipe in which inner surface the piece-wise distributed loading is applied (see fig. 1).

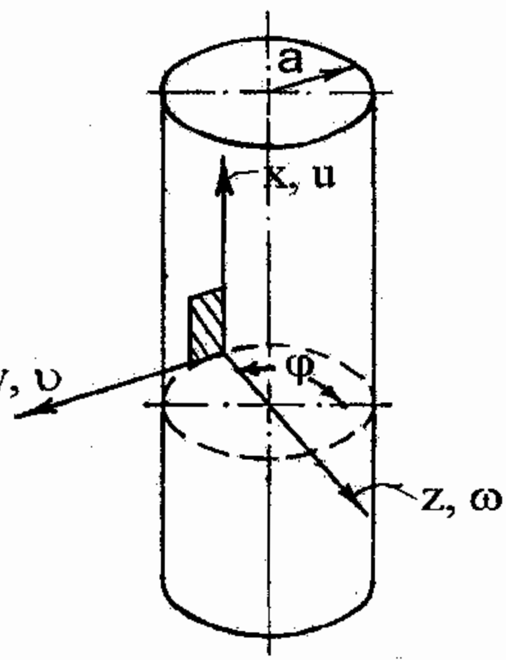

Fig.1.

The relation of the wall's thickness to the diameter of the surround pipe generally changes within the interval 0,03-0,06. So we can use as the equilibrium equation the equations for shells which have for such the pipe the form:

$$\left.\begin{aligned}
&\frac{\partial^2 u}{\partial x^2}+\frac{1-\nu}{2a^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{1+\nu}{2a}\frac{\partial^2 \upsilon}{\partial x\partial\varphi}-\frac{\nu}{a}\frac{\partial\omega}{\partial x}=0,\\
&\frac{1+\nu}{2}\frac{\partial^2 u}{\partial x\partial\varphi}+a\frac{1-\nu}{2}\frac{\partial^2 \upsilon}{\partial x^2}+\frac{1}{a}\frac{\partial^2 \upsilon}{\partial\varphi^2}-\frac{1}{a}\frac{\partial\omega}{\partial\varphi}+\frac{h^2}{12a}\left(\frac{\partial^3\omega}{\partial x^2\partial\varphi}+\frac{\partial^3\omega}{a^2\partial\varphi^3}\right)+\\
&+\frac{h^2}{12a}\left[(1-\nu)\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{a^2\partial\varphi^2}\right]=0,\\
&\nu\frac{du}{dx}+\frac{\partial\upsilon}{a\partial\varphi}-\frac{\omega}{a}-\frac{h}{12}\left(a\frac{\partial^4\omega}{\partial x^4}+\frac{2}{a}\frac{\partial^4\omega}{\partial x^2\partial\varphi^2}+\frac{\partial^4\omega}{a^3\partial\varphi^4}\right)-\\
&-\frac{h}{12}\left(\frac{2-\nu}{a}\frac{\partial^3\upsilon}{\partial x^2\partial\varphi}+\frac{\partial^3\upsilon}{a^3\partial\varphi^3}\right)=-\frac{aq(1-\nu^2)}{Eh}.
\end{aligned}\right\} \quad (1)$$

Where u,υ,ω are the components of the displacement vector correspondingly in the directions of the axes x, y, z; a is the radius of the mean surface of pipe's cylinder, ν is Poisson's coefficient, h is the thickness of pipe's wall, q is the intensivity of the piece-wise continuous distributed loading. φ is the polar angle. Function of distribution of the pressure force of the dies by φ is shown in fig. 2, and distribution by x is shown in fig. 3.

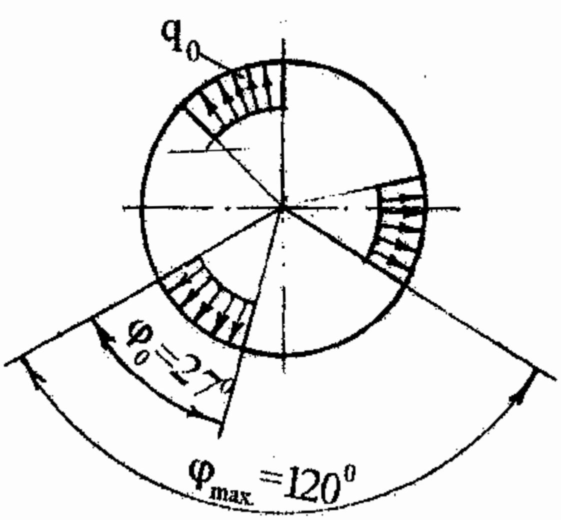

Fig.2.

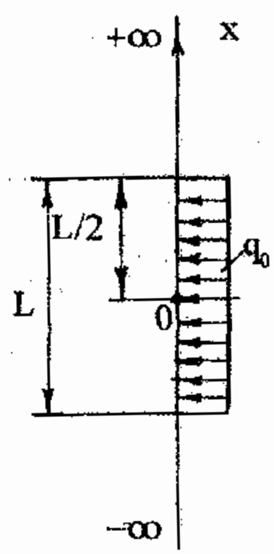

Fig.3.

After using the Fourier cosine transform the system (1) obtains the form:

$$\left.\begin{aligned}
&-s^2\overline{u}+\frac{1-\nu}{2a^2}\cdot\frac{\partial^2\overline{u}}{\partial\varphi^2}+i\frac{1+\nu}{2a}\cdot s\frac{\partial\overline{\upsilon}}{\partial\varphi}-\frac{\nu}{a}is\overline{\omega}=0,\\
&i\frac{1+\nu}{2}\cdot s\frac{\partial\overline{u}}{\partial\varphi}-a\frac{1-\nu}{2}s^2\overline{\upsilon}+\frac{1}{a}\frac{\partial^2\upsilon}{\partial\varphi^2}-\frac{1}{a}\frac{\partial\overline{\omega}}{\partial\varphi}+\frac{h^2}{12a}\left(-s^2\frac{\partial\overline{\omega}}{\partial\varphi}+\frac{\partial^3\overline{\omega}}{a^2\partial\varphi^3}\right)+\\
&+\frac{h^2}{12a}\left[-(1-\nu)s^2\overline{\upsilon}+\frac{\partial^2\overline{\upsilon}}{a^2\partial\varphi^2}\right]=0,\\
&i\nu s\overline{u}+\frac{\partial\overline{\upsilon}}{a\partial\varphi}-\frac{\overline{\omega}}{a}-\frac{h^2}{12}\left(a^2s^4\overline{\omega}-\frac{2}{a}s^2\frac{\partial^2\overline{\omega}}{\partial\varphi^2}+\frac{\partial^4\overline{\omega}}{a^3\partial\varphi^4}\right)-\\
&-\frac{h^2}{12}\left(-\frac{2-\nu}{a}s^2\frac{\partial\overline{\upsilon}}{\partial\varphi}+\frac{\partial^3\overline{\upsilon}}{a^3\partial\varphi^3}\right)=-\frac{a(1-\nu^2)}{Eh}\overline{q}.
\end{aligned}\right\}\qquad(2)$$

The following expansion by φ has been taken.

$$\left.\begin{aligned}
\overline{\omega}&=\sum_{n=0}^{\infty}\overline{\textit{в}}_n\cos 3n\varphi\\
\overline{u}&=\sum_{n=0}^{\infty}\overline{c}_n\cos 3n\varphi\\
\overline{\upsilon}&=\sum_{n=0}^{\infty}\overline{\ell}_n\sin 3n\varphi
\end{aligned}\right\}\qquad(3)$$

Taking into account (3) from (2) we obtain

$$\left.\begin{aligned}
&-\frac{\nu}{a}is\overline{b}_n+\left(-s^2-\frac{1-\nu}{2a^2}9n^2\right)\overline{c}_n+i\frac{1+\nu}{2a}3ns\overline{\ell}_n=0,\\
&\left(\frac{3n}{a}+\frac{nh^2}{4a}s^2+\frac{9n^3h^2}{4a^3}\right)\overline{b}_n-i\frac{1+\nu}{2}3ns\overline{C}_n+\left(-\frac{h^2(1-\nu)}{12a}s^2\right.\\
&\left.-\frac{3h^2n^2}{4a^3}-a\frac{1-\nu}{2}s^2-\frac{9n^2}{a}\right)\overline{\ell}_n=0,\\
&-\left(\frac{1}{a}+\frac{h^2a^2}{12}s^4+\frac{3n^2h^2}{2a}s^2+\frac{27h^2n^4}{4a^3}\right)\overline{b}_n+\nu is\overline{c}_n-\\
&-\left(-\frac{3n}{a}-\frac{h^2(2-\nu)n}{2a}s^2-\frac{9n^3h^2}{4a^3}\right)\overline{\ell}_n=-\frac{a(1-\nu^2)}{Eh}\cdot q_0\left(\frac{2}{\pi}\right)^{\frac{3}{2}}\cdot\frac{\sin s\frac{\ell}{2}}{sn}\cdot\sin\left(n\frac{9\pi}{40}\right).
\end{aligned}\right\}\qquad(4)$$

It is considered that the displacements in directions x, y, that is u, υ are infinite small, it is sufficient to find the radial displacement ω. From the algebraic system of equations (4) the coefficients $\overline{\textit{в}}_n$ are determined in the form

$$\overline{\textit{в}}_n=\frac{\Delta\textit{в}}{\Delta},\qquad(5)$$

where

$$\Delta_{в} = -\frac{1}{s}\sin(65\cdot s)\sin(0{,}70685832\cdot n)\cdot\Big(1{,}2499705\frac{1}{n}s^4 +$$
$$+ 1{,}9400838\cdot 10^{-5} ns^2 + 7{,}899976\cdot 10^{-9} n^3\Big);$$
$$\Delta = -a_4 s^8 - a_3 s^6 n^2 + a_2 s^4 + a_1 s^2 + a_0 ;$$

a_0, a_1, a_2, a_3, a_4 are constant numbers depending on n.

In order to expand the fraction in the right-hand side of (5) we find the roots of the equation

$$\Delta = -a_4 s^8 - a_3 s^6 n^2 + a_2 s^4 + a_1 s^2 + a_0 = 0 . \tag{6}$$

Equation (6) has been solved numerically for $n = 0, 1, 2, \ldots, 10$.

Let us rewrite equation (5) in the following form

$$\bar{в}_n = \frac{\Delta_в}{\Delta} = \frac{a_4' \sin(65\cdot s)}{a_4\cdot s}\cdot\frac{s^4 + a_2' s^2 + a_0'}{\left(s^2 - s_1^2\right)\cdot\left(s^2 - s_2^2\right)\cdot\left(s^2 - s_3^2\right)\cdot\left(s^2 - s_4^2\right)} = \frac{B}{s}\left[\frac{1}{\left(s^2 - s_3^2\right)\cdot\left(s^2 - s_4^2\right)} +\right.$$
$$\left. + \frac{a_2''}{\left(s^2 - s_2^2\right)\cdot\left(s^2 - s_3^2\right)\cdot\left(s^2 - s_4^2\right)} + \frac{a_2'' s_1 + a_0''}{\left(s^2 - s_1^2\right)\cdot\left(s^2 - s_2^2\right)\cdot\left(s^2 - s_3^2\right)\cdot\left(s^2 - s_4^2\right)}\right], \tag{7}$$

where $s_1^2, s_2^2, s_3^2, s_4^2$ are the roots of equation (6); $B = \dfrac{a_4' \sin(65\cdot S)}{a_4}$; $a_4', a_4, , a_2'', a_0'', a_2'$, a_0' are the constants depending on n.

The components of equation (7) were expanded into the rational fractions whose denominators have the second order. After that the inverse Fourie transform was used and the coefficients $в_n$ were found in the form:

$$в_n = \frac{a_4'}{a_4}\left[\frac{A_3}{s_3} + \frac{A_4}{s_4} + a_2''\left(\frac{B_2}{s_2} + \frac{B_3}{s_3} + \frac{B_4}{s_4}\right) + \left(a_2'' s_1 + a_0''\right)\cdot\left(\frac{C_1}{s_1} + \frac{C_2}{s_2} + \frac{C_3}{s_3} + \frac{C_4}{s_4}\right)\right]\times$$
$$\times(1-e)\cdot e^{-\frac{x}{65}} .$$

For ω the following expression finally was obtained:

$$\omega = \sum_{n=0}^{\infty} b_n(0)\cdot(1-e)e^{-\frac{2x}{\ell}}\cdot\cos 3n\varphi , \tag{8}$$

where ℓ is the length of the dies.

With help of the geometric correlations the components of deformation were found and by Hook's law were found the stress components. For determination of the coefficients $в_n$ and carrying out the calculations for strength by the numerical way the following values of the parameters were:

$\ell = 130\,\text{mm}$,

$a = 105{,}1\,\text{mm}$,

$q_0 = 21{,}7\,\text{MPa}$.

As the strength condition the Mizes's condition were taken in the form

$$\left(\sigma_{xx} - \sigma_{rr}\right)^2 + \left(\sigma_{rr} - \sigma_{\varphi\varphi}\right)^2 + \left(\sigma_{\varphi\varphi} - \sigma_{xx}\right)^2 + 6\left(\sigma_{r\varphi}^2 + \sigma_{rx}^2 + \sigma_{\varphi x}^2\right) \le 2\sigma_T^2 , \tag{9}$$

where σ_{xx} and so on are the components of the stress tenzor; σ_T is the limit of the fluidity.

After substitution of the expression for stresses in (9) it was obtained

$$\left(\sum_{n=0}^{4} e_n(0)\right)^2 \cdot \left(\frac{4}{r^2} + \frac{3}{4225}\right) \le \frac{\sigma_T^2}{2(1-e)^2 \cdot \mu^2}, \qquad (10)$$

where $\mu = \dfrac{E}{2(1+\nu)}$; ν is the Poisson coefficient, $e = 2{,}718282$.

The given data correspond to the extremal variant for the acting loadings, geometric sizes and mechanical characteristics of the pipe's material. It was established that for the given values of the parameters the strength of the surround pipe was provided big reserve.

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