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GEOMETRICAL NON-LINEAR TORSION OF THE THICK-WALLED PIPE OF THE VISCO-ELASTIC MATERIAL

Abstract

A problem on geometrically non-linear torsion of a thick- walled pipe made of visco-elastic material is solved by a sequential approximation method.

Let us consider torsion of the thick-walled pipe of the visco-elastic material, which is under action of torsional moment (fig. 1). Let's use the initial state coordinates. The origin of coordinates is chosen in one of the end-walls of the bar. The external radius of across-section is denoted by R, and the internal radius by r; S_R and S_r are lateral surfaces, S is square of the end-wall; l is length of the bar considered sufficient big.

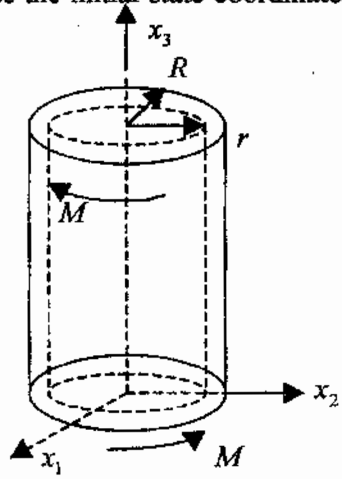

Fig. 1.

We represent displacements in the form [1]:

$$u_1(x_1,x_2,x_3,t) = -\theta(t)x_2x_3 + \theta^2(t)\left[-\frac{1}{2}x_1x_2^2 + V_1(x_1,x_2,t)\right],$$

$$u_2(x_1,x_2,x_3,t) = -\theta(t)x_1x_3 + \theta^2(t)\left[-\frac{1}{2}x_2x_1^2 + V_2(x_1,x_2,t)\right],$$

$$u_3(x_1,x_2,x_3,t) = \theta(t)\varphi(x_1,x_2) + \theta^2(t)cx_3, \tag{1}$$

where $\theta(t)$ is torsion angle, $V_1(x_1,x_2,t)$, $V_2(x_1,x_2,t)$ are required functions, $c(t)$ is constant subjected to determination, $\varphi(x_1,x_2)$ is function of torsion which is equal to zero in the considered case.

The components of deformation tenzor are expressed by the components of displacement vector in the form:

$$2e_{ij} = u_{i,j} + u_{j,i} + u_{s,i}\cdot u_{s,j}. \tag{2}$$

It is supposed that, the mechanical properties of the pipe's material are described by the correlations of the physical-linear visco-elasticity [2]:

$$\frac{S_{ij}}{2G_0} = \text{э}_{ij} - \int_0^t R(t-\tau)\,\text{э}_{ij}(\tau)d\tau, \tag{3}$$

$$\frac{\sigma}{K_0} = \theta. \tag{4}$$

Here $\text{э}_{ij} = e_{ij} - e\delta_{ij}$ is deviator of deformation tenzor, $S_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ is deviator of stress tenzor, $\theta = 3e = e_{kk}$ is the relative change of the volume, $\sigma = \sigma_{kk}/3$ is the mean stress, G_0 is the instant shear modulus, K_0 is the instant modulus of the volume deformation, function $R(t-\tau)$ is the relaxation nuclear. The volume deformation is considered as ideal elastic.

The equilibrium equation [3]

$$\left[\sigma_{ij}\left(\delta_{ki} + U_{k,j}\right)\right]_{,j} = F_k$$

taking into account (1)-(4) passes into the following correlations:

$$G\nabla^2 V_1 + a_1\Delta_{,1} = -(F_{11} + F_{12})x_1\,,$$
$$G\nabla^2 V_2 + a_1\Delta_{,2} = -(F_{21} + F_{22})x_2\,. \qquad (5)$$

The boundary conditions [3] after analogous computations are represented in the form:

$$\left[(a_2V_{1,1} + a_3V_{2,2})x_1 + G(V_{1,2} + V_{2,1})x_2\right]_{S_R} + \left[(R_{\nu 11} + R_{\nu 12})x_1 + (R_{\nu 21} + R_{\nu 22})x_2\right]_{S_R} =$$
$$= \left[(a_2V_{1,1} + a_3V_{2,2})x_1 + G(V_{1,2} + V_{2,1})x_2\right]_{S_r} + \left[(R_{\nu 11} + R_{\nu 12})x_1 + (R_{\nu 21} + R_{\nu 22})x_2\right]_{S_r},$$
$$\left[(a_2V_{2,2} + a_3V_{1,1})x_2 + G(V_{1,2} + V_{2,1})x_1\right]_{S_R} + \left[(R_{\nu 21} + R_{\nu 22})x_1 + (R_{\nu 31} + R_{\nu 32})x_2\right]_{S_R} =$$
$$= \left[(a_2V_{2,2} + a_3V_{1,1})x_2 + G(V_{1,2} + V_{2,1})x_1\right]_{S_r} + \left[(R_{\nu 21} + R_{\nu 22})x_1 + (R_{\nu 31} + R_{\nu 32})x_2\right]_{S_r}, \qquad (6)$$

where $\Delta = V_{1,1} + V_{2,2}$, $\nabla^2 V_i = V_{i,11} + V_{i,22}$ is Laplace operator, $F_{11} = a_3x_1$, $F_{21} = a_3x_2$, $R_{\nu 11} = R_{\nu 31} = a_3\left(c + \frac{1}{2}(x_1^2 + x_2^2)\right)$, $R_{\nu 21} = 0$, $F_{12} = -2G_0\int_0^t (R(t-\tau)\ni_{11}(\tau))_{,1}\,d\tau - 2G_0 \times$
$\times \int_0^t (R(t-\tau)\ni_{12}(\tau))_{,2}\,d\tau$, $F_{22} = -2G_0\int_0^t (R(t-\tau)\ni_{22}(\tau))_{,2}\,d\tau - 2G_0\int_0^t (R(t-\tau)\ni_{12}(\tau))_{,1}\,d\tau$,

$$R_{\nu 12} = -2G_0\int_0^t R(t-\tau)\ni_{11}(\tau)d\tau\,,$$

$$R_{\nu 22} = -2G_0\int_0^t R(t-\tau)\ni_{12}(\tau)d\tau\,,\ R_{\nu 32} = -2G_0\int_0^t R(t-\tau)\ni_{22}(\tau)d\tau\,. \qquad (7)$$

Equation (5) and boundary condition (6) are linear with respect to unknown functions $V_1(x_1,x_2)$, $V_2(x_1,x_2)$. Let's solve the problem by the Successive approximation method. Take as the zero approximation the solution of the considered problem in the case of the elastic material of the pipe that is for $R(t-\tau)=0$. The pointed out solution was given in [4]. Following that work we write out:

$$V_1^{(0)}(x_1,x_2) = b_1^{(0)}b^2x_1 + 3b_2^{(0)}x_1x_2^2 + b_3^{(0)}x_1^3\,,$$
$$V_2^{(0)}(x_1,x_2) = b_1^{(0)}b^2x_2 + 3b_2^{(0)}x_2x_1^2 + b_3^{(0)}x_2^3\,, \qquad (8)$$

where $b_i^{(0)} = (R_0, G_0, R, r)$, $b^2 = R^2 + rR + r^2$.

Let's represent $V_i(x_1,x_2)$ in (1) and taking them into account in (5), (6), (7), we obtain the equation and the boundary conditions for determination of the first approximation $V_i^{(1)}$. We obtain the torsion problem in geometrical- nonlinear formulation with the imaginary first approximation of the force, which had already been determined. And in this case the problem is solved analogously the zero approximation. For any n-th approximation there exist:

The equilibrium equation and the boundary conditions

$$G\nabla^2 V_1^{(n)} + a_1\Delta_{,1}^{(n)} = -\left(F_{11} + F_{12}^{(n-1)}\right)x_1\,,$$
$$G\nabla^2 V_2^{(n)} + a_1\Delta_{,2}^{(n)} = -\left(F_{21} + F_{22}^{(n-1)}\right)x_2\,, \qquad (9)$$

$$\left[\left(a_2V_{1,1}^{(n)} + a_3V_{2,2}^{(n)}\right)x_1 + G\left(V_{1,2}^{(n)} + V_{2,1}^{(n)}\right)x_2\right]_{S_R} + \left[\left(R_{\nu 11}^{(n-1)} + R_{\nu 12}^{(n-1)}\right)x_1 + \left(R_{\nu 21}^{(n-1)} + R_{\nu 22}^{(n-1)}\right)x_2\right]_{S_R} =$$
$$\left[\left(a_2V_{1,1}^{(n)} + a_3V_{2,2}^{(n)}\right)x_1 + G\left(V_{1,2}^{(n)} + V_{2,1}^{(n)}\right)x_2\right]_{S_r} + \left[\left(R_{\nu 11}^{(n-1)} + R_{\nu 12}^{(n-1)}\right)x_1 + \left(R_{\nu 21}^{(n-1)} + R_{\nu 22}^{(n-1)}\right)x_2\right]_{S_r},$$
$$\left[\left(a_2V_{2,2}^{(n)} + a_3V_{1,1}^{(n)}\right)x_2 + G\left(V_{1,2}^{(n)} + V_{2,1}^{(n)}\right)x_1\right]_{S_R} + \left[\left(R_{\nu 31}^{(n-1)} + R_{\nu 32}^{(n-1)}\right)x_2 + \left(R_{\nu 21}^{(n-1)} + R_{\nu 22}^{(n-1)}\right)x_2\right]_{S_R} =$$

$$\left[\left(a_2 V_{2,2}^{(n)} + a_3 V_{1,1}^{(n)}\right)x_2 + G\left(V_{1,2}^{(n)} + V_{2,1}^{(n)}\right)x_1\right]_{S_r} + \left[\left(R_{\nu 31}^{(n-1)} + R_{\nu 32}^{(n-1)}\right)x_2 + \left(R_{\nu 21}^{(n-1)} + R_{\nu 22}^{(n-1)}\right)x_1\right]_{S_r}. \quad (10)$$

Moreover, $F_{ij}^{(n-1)}$, $R_{\nu ij}^{(n-1)}$ are determined by the foregoing $(n-1)$-th approximation. The equation (9) and boundary condition (10) distinct from the corresponding equations of the geometric non-linear problem of elastic torsion by that the known forces $F_{11}^{(n-1)}$, $F_{22}^{(n-1)}$, $R_{\nu 11}^{(n-1)}$, $R_{\nu 12}^{(n-1)}$, $R_{\nu 21}^{(n-1)}$, $R_{\nu 22}^{(n-1)}$, $R_{\nu 31}^{(n-1)}$, $R_{\nu 32}^{(n-1)}$ take part in them. The solution of this problem considered in [4]. Starting from this we obtain

$$V_1^{(n)}(x_1,x_2,t) = b_1^{(n)} b^2 x_1 + 3b_2^{(n)} x_1 x_2^2 + b_3^{(n)} x_1^3,$$
$$V_2^{(n)}(x_1,x_2,t) = b_1^{(n)} b^2 x_2 + 3b_2^{(n)} x_2 x_1^2 + b_3^{(n)} x_2^3,$$

where $b_2^{(n)} = b_2^{(n)}\left(b_2^{(n-1)}, K_0, G_0, R(t-\tau), R, r, t\right)$.

Substituting $V_i^{(n)}$ into (1) we determine the components of displacement vector for n-th approximation:

$$u_1^{(n)}(x_1,x_2,x_3,t) = -\theta(t)x_2x_3 + \theta^2(t)\left[-\frac{1}{2}x_1x_2^2 + V_1^{(n)}(x_1,x_2,t)\right],$$
$$u_2^{(n)}(x_1,x_2,x_3,t) = \theta(t)x_1x_3 + \theta^2(t)\left[-\frac{1}{2}x_2x_1^2 + V_2^{(n)}(x_1,x_2,(t))\right],$$
$$u_3^{(n)}(x_1,x_2,x_3,t) = \theta^2(t)C^{(n)}(t)x_3.$$

The quantity C is determined from the condition [3]:

$$\iint_S \sigma_{33}\,dx_1dx_2 = 0.$$

For example, for the quantity C for the first approximation we have:

$$C^{(1)} = \frac{3-4\nu}{3-5\nu}\left\{\frac{\nu}{1-\nu}\left[b_1^{(1)}b^2 + \frac{1}{2}\left(3b_2^{(1)} + 3b_3^{(1)} - \frac{1-\nu}{\nu}\right)\left(R^2-r^2\right)\right] + \right.$$
$$\left. + \frac{1-2\nu}{1-\nu}\left[\frac{1}{2}\left(C_1^{(0)} - b_{11}^{(0)}\right)b^2 - \frac{R^2-r^2}{2}\left(3b_2^{(1)} + 3b_3^{(1)} - \frac{1-\nu}{1-2\nu}\right)\right]\int_0^t R(t-\tau)d\tau\right\},$$

where $C_1^{(0)} = C^{(0)}b^{-2}$.

Moreover, from the integral condition [3]

$$\iint_S \left(x_1\sigma_{23} - x_2\sigma_{13}\right)dx_1dx_2 = M$$

we have

$$M = \frac{1}{2}\pi G\left(R^4 - r^4\right)\left(\theta^{(n)}(t) - \int_0^t R(t-\tau)\theta^{(n-1)}(t)d\tau\right) +$$
$$+ G\iint_{S_1}\left\{\theta^{(n)}(t)\left[\left(\frac{1}{2}x_1x_3^2 - x_2V_{1,2}^{(n)} + x_1V_{2,2}^{(n)}\right)x_1 - \left(-\frac{1}{2}x_2x_3^2 - x_2V_{1,2}^{(n)} + x_1V_{2,2}^{(n)}\right)x_2\right] - \right.$$
$$- \int_0^t R(t-\tau)\theta^{(n-1)}(t)\left[x_1\left(\frac{1}{2}x_1x_3^2 - x_2V_{1,2}^{(n-1)} + x_1V_{2,2}^{(n-1)}\right) - \right.$$
$$\left.\left. - x_2\left(-\frac{1}{2}x_2x_3 - x_2V_{1,1}^{(n-1)} + x_1V_{2,1}^{(n-1)}\right)\right]d\tau\right\}dx_1dx_2.$$

Numerical calculation had been carried out. In fig.2 the graphics of dependence of torsion $\theta(t)$ on time t are represented for various values of the torsional moment M;

fig. 3 the graphics of dependence $\theta \sim M$ are represented for various values of t, from which it follows that the influence of the properties of the visco-elastic medium is sugnificiant for torsion determination.

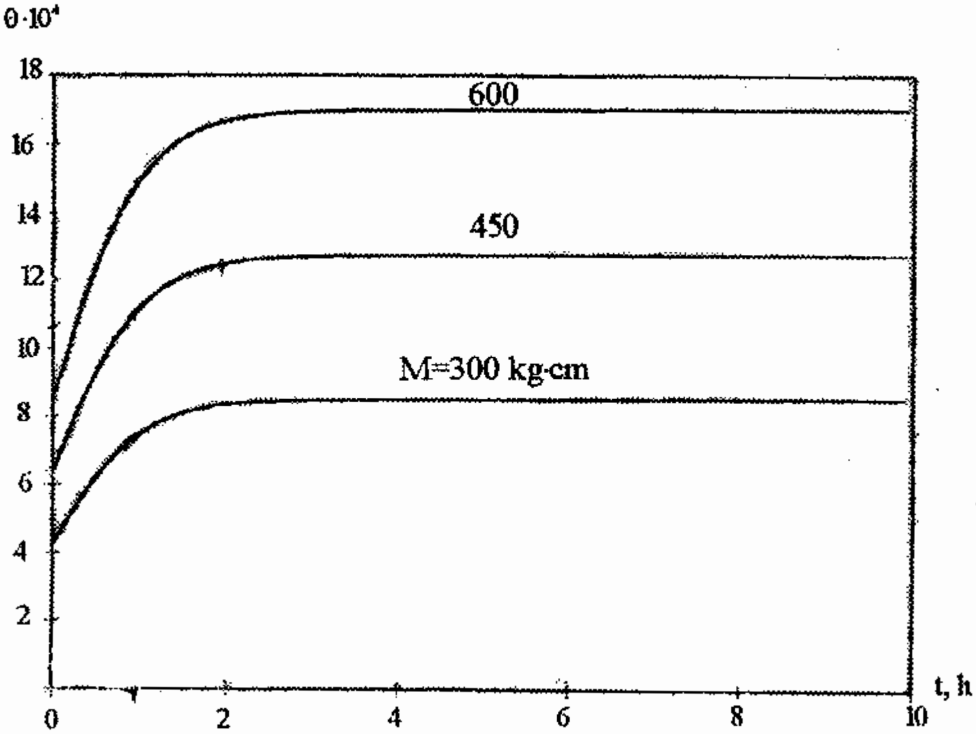

Fig. 2.

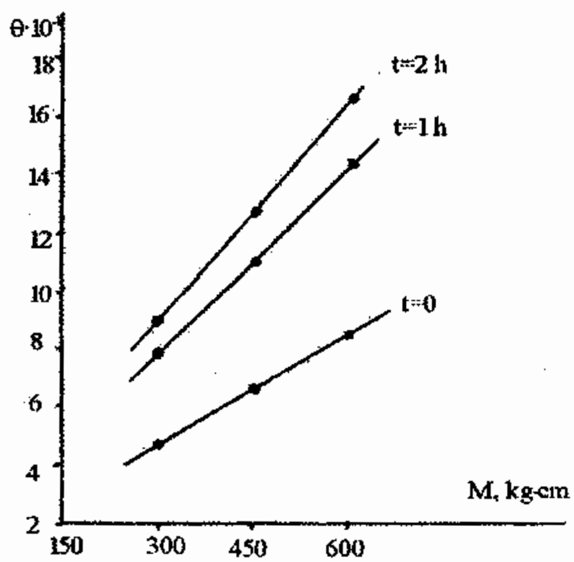

Fig. 3.

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