LATIFOV F.S.

HIGH-FREQUENCY EIGEN-OSCILLATIONS OF A CYLINDRIC HULL IN AN INFINITE ELASTIC MEDIUM

Abstract

In this paper the high-frequency free oscillations of the cylindric hull with an infinite length in the infinite elastic isotropic homogeneous medium is considered. For the shell the P.S. Timoshenko type refined theory is used. The frequency equation is made up and numerically realized.

High-frequency free oscillations of an infinite length cylindric hull in an infinite elastic isotropic homogeneous medium are considered in the paper. To study a dynamic stress state of the hull, S.P. Timoshenko type refined theory is adopted, where along with ordinary membrane and bending factors we also considered the influence of intersecting forces and rotation inertia of a normal element of the hull. The motion of an elastic medium is described by Luame equations in permutations.

The frequency equation is made up and numerically realized.

A system of the motion equation of the hull in permutation components u, ϑ, w has the form [1]:

$$\frac{\partial^2 u}{\partial\xi^2}+\frac{1-\nu}{2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{1+\nu}{2}\frac{\partial^2\vartheta}{\partial\xi\partial\varphi}-\nu\frac{\partial w}{\partial\xi}-\gamma^2\frac{\partial^2 u}{\partial t^2}=0;$$

$$\frac{1+\nu}{2}\frac{\partial^2 u}{\partial\xi\partial\varphi}+\frac{\partial^2\vartheta}{\partial\varphi^2}+\frac{1-\nu}{2}\frac{\partial^2\vartheta}{\partial\xi^2}-\frac{\partial w}{\partial\varphi}-\gamma^2\frac{\partial^2\vartheta}{\partial t^2}=0;$$

$$-\nu\frac{\partial u}{\partial\xi}-\frac{\partial\vartheta}{\partial\varphi}+w+\frac{1-\nu}{2}k\left(\nabla^2 w+\frac{\partial\beta_x}{\partial\xi}+\frac{\partial\beta_s}{\partial\varphi}\right)+\gamma^2\frac{\partial^2 w}{\partial t^2}=\frac{1-\nu^2}{Eh}P; \qquad (1)$$

$$\frac{h^2}{12R^2}\left[\frac{\partial^2\beta_x}{\partial\xi^2}+\frac{1-\nu}{2}\frac{\partial^2\beta_x}{\partial\varphi^2}+\frac{1+\nu}{2}\frac{\partial^2\beta_s}{\partial\xi\partial\varphi}\right]-\frac{1-\nu}{2}k\left(\frac{\partial w}{\partial\xi}+\beta_x\right)-\gamma^2\frac{h^2}{12R^2}\frac{\partial^2\beta_x}{\partial t^2}=0;$$

$$\frac{h^2}{12R^2}\left[\frac{\partial^2\beta_s}{\partial\varphi^2}+\frac{1-\nu}{2}\frac{\partial^2\beta_s}{\partial\xi^2}+\frac{1+\nu}{2}\frac{\partial^2\beta_x}{\partial\xi\partial\varphi}\right]-\frac{1-\nu}{2}k\left(\frac{\partial w}{\partial\varphi}+\beta_s\right)-\gamma^2\frac{h^2}{12R^2}\frac{\partial^2\beta_s}{\partial t^2}=0.$$

Here $\gamma=\frac{1-\nu^2}{E}\rho$; $\nabla^2=\frac{\partial^2}{\partial\xi^2}+\frac{\partial}{\partial\varphi^2}$; k is a shear coefficient; β_x, β_s are inclination angles; $\xi=\frac{x}{R}$; $\varphi=\frac{y}{R}$; h is a thickness of the hull; R is a radius of the hull; ν is a Poisson's coefficient; E is a Young module of the hull material, ρ is the density of the hull material; t is a time; P is the external normal loading.

Motion of the elastic medium is described by the equation [2]

$$\nabla^2\vec{S}+\frac{1}{1-2\nu_s}graddiv\vec{S}+\rho_s\frac{\partial^2\vec{S}}{\partial t^2}=0, \qquad (2)$$

where S_ξ, S_φ, S_r are projections of the permutations vectors of the medium $\vec{S}$, ν_s is a Poisson coefficient of the medium material, t is time, ρ_s is the density of the medium material.

A motion equation of the hull and medium is complemented by contact conditions on the external surface of the cylinder. Here we consider such a contact, where it is assumed a free sliding of media on the contact surface, but isolation doesn't happen. Then kinematic contact conditions have the form

$$S_r = w, \quad r = R + h/2. \tag{3}$$

Dynamic contact conditions adopt the form $(r = R + h/2)$

$$\sigma_{rx} = 0, \quad \sigma_{r\varphi} = 0, \quad P = -\sigma_{rr}. \tag{4}$$

The components $\sigma_{rx}, \sigma_{r\varphi}, \sigma_{rr}$ of the stress vector $\vec{\sigma}_r$ in the medium are found from Hook's law [2].

We add to contact conditions (3) and (4) radiation conditions at infinity, i.e. for $r \to \infty$

$$S_\xi = S_\varphi = S_r = 0. \tag{5}$$

So, the problem on eigen-oscillations of a cylindric hull infinite elastic medium is led to the joint integration of hull theory equations (1) and three-dimensional elasticity theory (2) by fulfilling contact conditions (3), (4) on the contact surface.

We shall seek for the solutions of the motion equation in the form:

$$\begin{aligned} u &= A\sin m\xi \cos n\varphi \sin \omega t, \\ \vartheta &= B\cos m\xi \sin n\varphi \sin \omega t, \\ w &= C\cos m\xi \cos n\varphi \sin \omega t, \\ \beta_x &= D\sin m\xi \cos n\varphi \sin \omega t, \\ \beta_s &= F\cos m\xi \sin n\varphi \sin \omega t, \end{aligned} \tag{6}$$

where A, B, C, D, F are constants, m is a wave number in the longitudinal direction, n is the number of waves in the peripheral direction, ω is the desired eigen-frecuency of the considered system.

The solution of motion equations of the elastic medium (2) with regard to (5) an infinite elastic cylinder has the form [2]:

$$\begin{aligned} S_\xi &= \left(A_1 m K_n(\alpha r) - C_1 \beta^2 K_n(\beta r)\right)\sin n\xi \cos n\varphi \sin \omega t, \\ S_\varphi &= \left(-A_1 \frac{n}{r} K_n(\alpha r) - C_1 \frac{mn}{r} K_n(\beta r) - \frac{B_1}{n}\frac{\partial K_n(\beta r)}{\partial r}\right)\cos m\xi \sin n\varphi \sin \omega t, \\ S_r &= \left(A_1 \frac{\partial K_n(\alpha r)}{\partial r} - C_1 m \frac{\partial K_n(\beta r)}{\partial r} + \frac{B_1 n}{r} K_n(\beta r)\right)\cos m\xi \cos n\varphi \sin \omega t, \end{aligned} \tag{7}$$

where

$$\alpha = \left(m^2 - \tilde{A}\frac{\rho_s^* \lambda}{E_s^*}\right)^{1/2}, \quad \beta = \left(m^2 - \tilde{B}\frac{\rho_s^* \lambda}{E_s^*}\right)^{1/2}, \quad \tilde{A} = \frac{1 - \nu_s - 2\nu_s^2}{(1-\nu^2)(1-\nu_s)},$$

$$\tilde{B} = \frac{2(1+\nu_s)}{1-\nu^2}, \quad E_s^* = \frac{E_s R}{Eh}, \quad \rho_s^* = \frac{\rho_s R}{\rho h}, \quad \lambda = \frac{(1-\nu^2)R^2 \rho \omega^2}{E},$$

A_1, C_1, B_1 are constants.

By means of the formula (7) we can easily determine the components of stress vector $\vec{\sigma}_r$ in an elastic medium [2].

By using the solution of the motion equation of the hull (6) and elastic medium (7), and also contact conditions (3), (4) we arrive at the system of homogeneous linear algebraic equations with respect to the constants $A, B, C, D, F, A_1, C_1, B_1$. To exist the non-trivial solution we equate its principal determinant to zero, and as a result we arrive to the frequency equation for finding the frequency parameter x, which we shall write in the form of the determinant:

$$\det\left\| a_{ij} \right\| = 0 \quad (i, j = 1, 2, \ldots, 8). \tag{8}$$

The element a_{ij} has an awkward form, therefore we don't cite it here. We only note that they depend on mechanical and physical parameters characterizing the materials of the hull and medium. The equation (8) is transcendental and has infinite many solutions. This equation has been realized numerically. The parameters:
$\nu_s = 0{,}38$, $\nu = 0{,}3$, $E_s = 2 \cdot 10^7\ \mathrm{N/m^2}$, $\rho_s = 2 \cdot 10^2\ \mathrm{kg/m^3}$, $E = 2 \cdot 10^{11}\ \mathrm{N/m^2}$, $\rho = 7{,}8 \cdot 10^3\ \mathrm{kg/m^3}$, $m = 1$ were adopted.

The results of the count are represented at fig. 1 and fig.2.

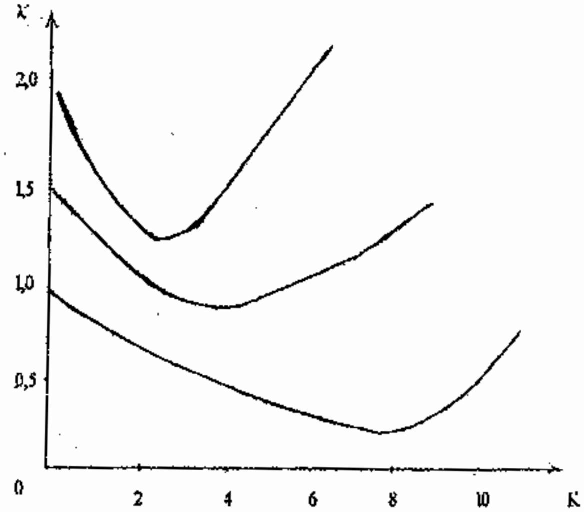

Fig. 1. Dependence of the eigen-frequency on the number of waves in the peripheral direction

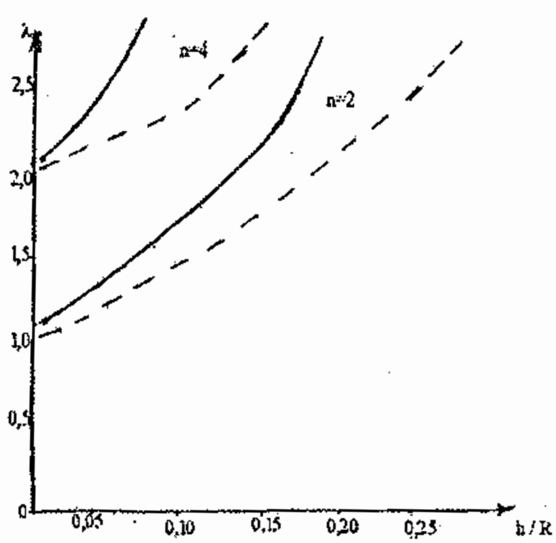

Fig. 2. Dependence of the eigen-frequency on the hull thickness.

The dependence of the frequency parameter λ on the wave number in the peripheral direction is shown in fig.1. We see from the figure that by increasing n , the frequency at first decreases, and then by achieving minimum begins to increase. Here, the number of waves n, to which minimal frequency corresponds, is less than for empty hull. The results of count show that the least increase of the eigen-frequency of the system in comparison with empty hull is achieved for $n=0$, for great numbers n this difference increases.

In fig. 2 the dependences of the frequency parameter λ on the thickness are presented. To dotted lines corresponds those λ, for the description of which of the hull deformation Kirghoff-Liav's hypotheses is used. We see from the figure that by increasing the hull thickness, the divergence between the frequency parameter of the considered system becomes essential.

References

[1]. Вольмир А.С. *Нелинейная динамика пластинки и оболочки.* «Наука», 1972, 432с.
[2]. Ильгамов М.А., Иванов В.А., Гулин Б.В. *Расчет оболочки с упругим заполнителем.* «Наука», 1987, 260с.

Latifov F.S.
Institute of Mathematics and Mechanics of AS Azerbaijan.
9, F. Agayeva str., 370141, Baku, Azerbaijan.
Tel.: 39-47-20.

Received July 19, 1999; Revised October 13, 1999.
Translated by Aliyeva E.T.