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SOLUTION OF THE PLANE PROBLEM OF HYDROELASTICITY AT FREE IMMERSING OF THE CYLINDRICAL SHELL IN THE LIQUID

Abstract

Problem of determination of a shell to strain-stress condition of the circular cylindrical shell, come down into the liquid by the lateral surface on a surface of the compressible liquid is considered. The forming of the cylinder is parallel to the surface of the liquid, on sufficient distance from territories of the shell of the pressure and the deformations do not depend on the axial coordinate.

The received results testify to significant qualitative and quantitative distinction of reactions of the pipeline shipped in the liquid, calculated taking into account and not taking into account the hydroelastic interaction.

The problem of determination of the strain-stress state of the circle cylindrical shells coming down by the lateral surface on the surface of the compressible liquid. The element of the cylinder is parallel to the surface of the liquid on the sufficient distance from the bounds of the shell the stresses and deformations don't depend on the axial coordinate, so the problem of penetrating of the long cylindrical shell (the long section of the pipe-line lash at the free immersing) can be considered as the plane.

The hydrodynamic pressure on the surface of the cylindrical shell was determined by the author [1] in the form

$$F(\theta,t)=\frac{2}{\pi}\exp\left(-\frac{t}{2}\right)\left(t^2-\theta^2\right)^{-0,5}ch\left(\sqrt{t^2-\theta^2}\,/\,2\right)H(t-\theta), \qquad (1)$$

where H $(t-\theta)$ is the Havyside unique function. This pressure is chosen so that at each given moment of the area on which the loading was applied to the bar, was equal to the area of the damped surface of the shell.

In fig. 1. the cross-section of the thin elastic shell with thickness h and the radius R.

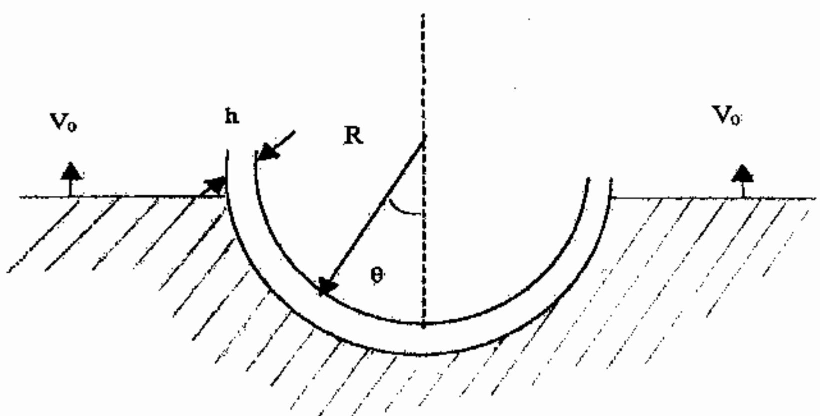

Fig. 1.

When the shell penetrates into the water at free loading with the velocity V_0. Here θ the polar angle. Keeping in mind the relativity of motion of the shell and the liquid, we can consider either penetrating of the shell into the quiescent liquid or climbing of the liquid onto the liquid as it was shown in fig. 1.

During the short time following the moment of the primary contact of the shell with the liquid the penetrating shell displaces the liquid at the expense of its compressity. Behaviour of the compressible liquid in the acoustic approximation is described by the wave potential φ satisfying the equation

$$\Delta\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} \tag{2}$$

and the formula

$$p = -\rho\frac{\partial\varphi}{\partial t} . \tag{3}$$

Here c is the velocity of the sound in the liquid; ρ is the density of the liquid; Δ is the Laplace operator.

For the free loading under the action of the natural weight the velocity of the shell V_0 is much less than the velocity of the sound c.

Penetrating into the liquid the shell rouses in it the non-stationary wave with the curved front.

At the initial moment of the immersion this wave is approximated [2] by the system of the plane acoustic waves and that let write the expression for the pressure on the bound of separation of the wall and the liquid in the from

$$p = \rho c V(t), \quad V(0) = V_0 . \tag{4}$$

On the damped surface of the shell

$$p(\theta,t) = \rho c V_0 \cos\theta . \tag{5}$$

On the dry surface of the shell the pressure is equal zero.

Both there cases can be described by the formula

$$p(\theta,t) = \rho c V_0 \cos\theta \cdot H\left(t - \frac{R(1-\cos\theta)}{V_0}\right),$$

where H(Z) is Hevyside unique formula [3].

For small deepnesses of immersion when the angle θ is small we can write

$$p(\theta,t) = \rho c V_0 H\left[t - \frac{Z^2}{2RV_0}\right], \tag{6}$$

where the quantity $Z = R\theta$ is the coordinate along the surface of the shell. Equation (6) is valid for $0 < \theta < 1$ or for the interval of time

$$0 < t < \frac{R}{2V_0} .$$

Considering the changing of the pressure of the interaction caused by deformation of the shell [1] the equation (6) we rewrite in the form:

$$p(\theta,t) = \rho c V_0 H\left[t - \frac{Z^2}{2RV_0}\right] - \rho c\frac{\partial W}{\partial t}, \tag{7}$$

where W is the radial displacement of the shell.

The second component in equation (7) has a small value out of the contact area with the liquid.

The equation for the deflection of the shell taking into account the dynamic loading (7) applied to it will have the form

$$\frac{\partial^4 y}{\partial x^4} + \lambda^4 \frac{\partial^2 y}{\partial t^2} = kH\left(t - \frac{x^2}{2RV_0}\right) - \beta \frac{\partial y}{\partial t}, \tag{8}$$

where y is the deflection (positive down); λ, k and β are constants:

$$\lambda^4 = 12\rho_1 / (Eh^2); k = 12\rho_1 c V_0 / \left(Eh^3\right); \beta = 12\rho_1 c / \left(Eh^3\right).$$

Here E, S_1 are correspondingly the modulus of elasticity and the density of the shell's material.

We find stress in an arbitrary point b_0 in the exterior layers of the shell by the formula

$$\sigma = \pm 0{,}5Eh \frac{\partial^2 y}{\partial x^2},, \tag{9}$$

where $x \in (-\infty; +\infty), t > 0$.

We obtain the solution of equation (8) by use of Fourier integral transform by x and Laplace integral transform by t [4]. Taking into account homogeneous initial conditions the eq. (8) is transformed to the from

$$\left(\xi^4 + \lambda^4 s^2 + \beta s\right)\overline{\overline{y}}(\xi, s) = k\sqrt{2\pi s V_0 s^{-3/2}}\, \ell^{-\frac{b\xi^2}{s}},, \tag{10}$$

where ξ is the parameter of Fourier transform; s is the parameter of Laplace transform; $b = RV_0 / 2$; $\overline{\overline{y}}$ is the representation of Fourier-Laplace function y.

Determining hence $\overline{\overline{y}}(s, \xi)$ and using the inversion of Fourier transform we obtain

$$\overline{y}(x, s) = \sqrt{\frac{b}{\pi}} \int_{-\infty}^{\infty} \frac{e^{i\xi x - \frac{b\xi^2}{s}}}{s^{3/2}\left(\xi^4 + \lambda^4 s^2 + \beta s\right)} d\xi. \tag{11}$$

Turn of the Laplace transform into the expression (11) can be fulfilled with help of the standard program by the methodic suggested in [4], with use of Lejandre orthogonal multinomials.

If we determine the deformation of the shell not taking into account the elastic interaction with the liquid, then in (1) it should put $\beta = 0$. Then

$$\overline{\sigma}(x, s) = \frac{Ehk}{2} \sqrt{\frac{b}{\pi}} \int_{-\infty}^{\infty} \frac{\xi^2 e^{-\frac{b\xi^2}{s}}}{s^{3/2}\left(\xi^4 + \lambda^4 s^2\right)} ds. \tag{12}$$

Using the known correlation's of the theory of Fourier transform and denoting

$$\frac{\xi^2 b}{s} = \eta^2; K = \sqrt{3} \frac{RV_0}{h\sqrt{E/\rho}}; \sigma_0 = \frac{Eh}{2R}; t_0 = \frac{\rho_1 \cdot h}{\rho R}$$

we obtain

$$\sigma = 2\sigma_0 \cdot K\Phi(K) \cdot \frac{t}{t_0}, t > 0, \tag{13}$$

where $\Phi(K) = \frac{K}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\eta^2 e^{\eta^2}}{\eta^4 + K} d\eta$ or according to [5]

$$\Phi(K) = \sqrt{2\pi K}\left[-C(K)\cos K - S(K)\sin K + \frac{\cos K}{2} + \frac{\sin K}{2}\right],$$

where C(K) and S(K) are Frenel integrals.

If $K >> 1$, that takes place in many practice cases, formula (13) is simplified, that is for $K \to \infty; 2K \cdot \Phi(K) \to 1$, whence

$$\frac{\sigma}{\sigma} \to \frac{t}{t_0}. \tag{14}$$

Hence it follows that for big K the stress doesn't depend on the velocity of penetration.

For $\beta \neq 0$, that is, taking into account the interaction of the shell with the liquid, the stress is determined by the formula

$$\sigma = \sigma_0 \left(1 - \frac{4}{\sqrt{\pi}} K^{3/2} \int_0^\infty \eta^2 e^{-K\eta^2 - \frac{t}{t_0(\eta^4+1)}} d\eta \right) \sigma_0 = \frac{Eh}{2R}. \tag{15}$$

The results of the numerical calculations by (15) are represented in fig. 2.

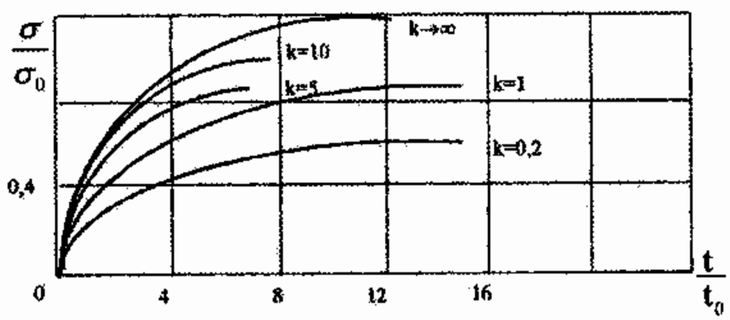

Fig. 2.

The obtained results testify to the great quantitative and qualitative difference of the reactions of the pipe-line immersed into the liquid, calculated taking into account and not taking into account the hydroelastic interaction.

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