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INVESTIGATION OF GROWTH OF THE FATIGUE CRACK IN THE TWOLAYER HALFPLANE

Abstract

The problem when the elastic half-plane consists of two rigidly enchained along the plane $x = H$ homogeneous isotropic elastic mediums and contains a boundary crack is considered. Stress intensity factor had been determined. For some materials number of cycles before fracture had been calculated.

In work [1] the plane problem of the theory of elasticity is considered when the halfplane consists of two rigidly enchained along the plane $x = H$ of the homogeneous isotrop elastic mediums and contains the surface through crack with length ℓ, $\ell < H$ (the boundary crack is perpendicular to the bound of separation of two mediums and to the bound of the halfplane, fig.1). It is supposed, that on the bound of the halfplane $x = 0$, $|y| < \infty$ the tangential stress is equal to zero and the normal stress is different from zero. The crack's bounds are free of external loadings. At infinity stresses are equal to zero and the displacement disappears.

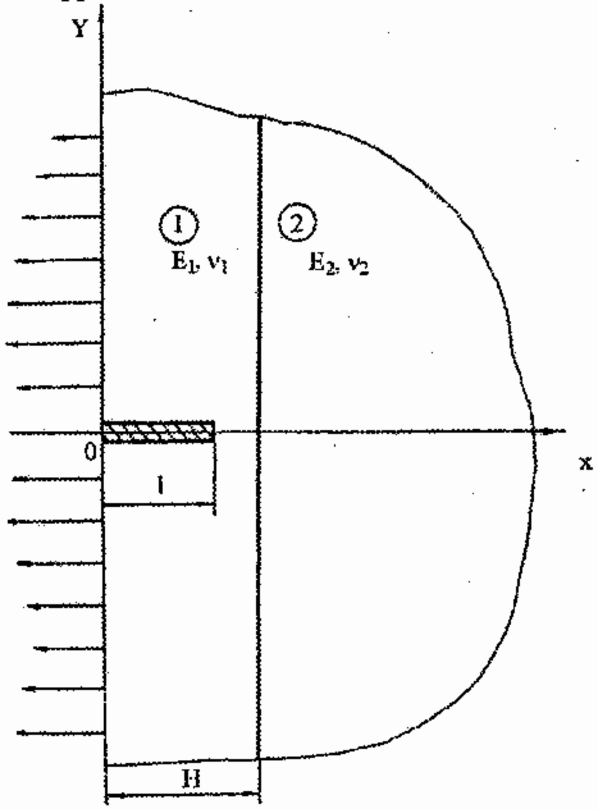

Fig. 1.

The formulated problem is considered as superposition of two problems: problem A and problem B.

Formulation of problem A.

$$\begin{aligned} & x=0, \quad |y|<\infty \quad \sigma_x = P_0(x,y), \quad \tau_{xy}=0 \\ & x=H, \quad |y|<\infty \quad [\sigma_x]=[\tau_{xy}]=0, \quad [u]=[\vartheta]=0 \\ & y=0, \quad 0<x<H \quad \tau_{xy}=0, \quad \vartheta=0 \\ & y=0, \quad x>H \quad \tau_{xy}=0, \quad \vartheta=0 \end{aligned} \tag{1}$$

The conditions at infinity for $r=\sqrt{x^2+y^2}$

$$\sigma_x, \tau_{xy}, \sigma_y \to 0, \quad u, \vartheta \sim O(r^\alpha) \quad (\alpha<0)$$

here $\sigma_x, \tau_{xy}, \sigma_y$ are the components of the stress tensor, u, ϑ are the components of the displacements vector; [F] is the jump of the quantity F; $P_0(y)$ is some given integrated function in the interval $|y|<\infty$.

Formulation of problem B.

$$\begin{aligned} & x=0, \quad |y|<\infty \quad \sigma_x=0, \quad \tau_{xy}=0 \\ & x=H, \quad |y|<\infty \quad [\sigma_x]=[\tau_{xy}]=0, \quad [u]=[\vartheta]=0 \\ & y=0, \quad 0<x<\ell \quad \sigma_y=-p(x), \quad \tau_{xy}=0 \\ & y=0, \quad \ell<x<H, \quad \tau_{xy}=0, \quad v=0 \\ & y=0, \quad x>H \quad \tau_{xy}=0, \quad v=0 \end{aligned} \tag{2}$$

The conditions at infinity for $r\to\infty$, σ_x, τ_{xy}, $\sigma_y \to 0$, $u, \vartheta \sim 0\left(r^\alpha\right)$ $(\alpha<0)$

The condition on the crack's bounds

$$K_I = \lim_{x\to \ell+0} [\sqrt{2\pi(x-\ell)} \quad \sigma_y(x,0)]$$

Here $p(x)$ is the continuous function. It is determined particularly from the solution of the boundary-valued problem A (in this case we come to the above formulated problem), and in the general case, it is determined from the solution of the symmetric first boundary-valued problem of the theory of elasticity for the two layer halfplane without any crack under action of some given external loading at infinity and on the bound of the two layer halfplane; K_I is stress intensivity (factor), which is subjected to determination.

The solution of Problem B was constructed in [2] and the value of Factor of stress intensivity K_I was found in the top of the crack ($y=0$, $0<x<\ell$), when the length of the crack ℓ is less than the width of the strip H

$$K_I=\sqrt{\pi\ell}\,\psi(\ell) \quad (\ell<H). \tag{3}$$

Function $\psi(\ell)$ is determined from Fredholm equation of the second type

$$\psi(t)=\int_0^\ell \psi(u)K(t,u)du+\frac{2}{\pi}\int_0^t \frac{p(x)}{\sqrt{t^2-u^2}}du \quad (0\le t\le \ell) \tag{4}$$

Analysis of the solution of problem B shows that for $\rho(x)=\sigma=const$, then from (4) we obtain

$$\psi_1(t)=\int_0^\ell \psi(u)K(t,u)du+1, \quad (0\le t\le \ell), \tag{5}$$

$$\psi_1(t)=\psi(u)/\sigma.$$

Introducing the denotations

$$K_0(q,\tau)=\ell K(\ell q,\ell\tau),\quad \psi_1(\ell q)=\psi(q),\qquad (6)$$
$$(0\le q,\tau\le 1)$$

and taking into account (6) in (5), we find

$$\psi(q)=\int_0^1\psi(\tau)K_0(q,\tau)d\tau+1$$

On that K_I is determined so:

$$K_I=\sigma\sqrt{\pi\ell}\,\psi(1)$$

It should note that function $\psi(1)$ depends on $\ell/H,\quad K,\nu_1,\quad \nu_2$, i.e.

$$\psi(1)=\psi(1,\ell/H,k,\nu_1,\nu_2)=\psi(\cdot)$$

Here $K=\dfrac{G_1}{G_2}$.

Let's investigate the fatigue growth of the crack when on the crack's bounds (the boundary-valued problem B) the cyclic stress changing during the time was applied.

$$\sigma_y=-\rho(x,t),\ \rho(x,t)=\sigma\sin\omega t\ ,$$
$$\sigma=const,\quad (0\le x\le\ell).$$

It is kept in mind that linking of the crack's bounds doesn't happen. Let $K>1$. Then assuming that $\psi(\cdot)=\psi_1(\cdot)\sin\omega t$, we come to the former equation (6) only for function $\psi(\cdot)$. On that

$$K_i=\sigma\sin(\omega t+\varphi)\sqrt{\pi\ell}\,\psi_1(\cdot),\qquad (7)$$

where φ is the initial phase. Here and further we take the physical possible assumptions:

a) the external loading and the properties of the material are such that $(\ell_{cr}/H)<1$;

b) the first material $0\le x\le H,\quad |y|<\infty$ is homogeneous and isotropy as by mechanical properties as by strength properties;

c) if the first material was failed sailed because of the crack then it is supposed that the bimaterial was failed because of the crack.

In order to investigate the durability (the number of the cycles N_f before fracture) of the bimaterials the function $\Psi_1(\cdot)$ had been approximated so:

$$\Psi_1(1,\ell/H,k,\nu_1,\nu_2)=\sum_{m=0}^{4}b_m(k,\nu_1,\nu_2)(\ell/H)^m.\qquad (8)$$

The values of $b_m=(m=\overline{0,4})$ are reduced in the Table:

	b_m				
E_1/E_2	b_0	b_1	b_2	b_3	b_4
1,0	1,12	0	0	0	0
0,1	1,08	0,53	-3,67	5,93	-3,31
10,0	1,45	-5,33	26,65	-42,50	22,75

From (7) and (8) we find

$$K_{\mathrm{Im}\,ax} = \sigma_{\max}\sqrt{\pi\ell}\sum_{m=0}^{4} b_m(k,\nu_1,\nu_2)\ \left(\frac{\ell}{H}\right)^m . \qquad (9)$$

Equaling $K_{\mathrm{Im}\,ax} = K_{I*}$, where K_{I*} is the cyclic crack resistance of the first material, we find the critic length of the crack.

From (9) we find

$$B_0 = \frac{\sigma_{\max}\sqrt{\pi H}}{K_{I*}} = \frac{1}{\sqrt{\frac{\ell_{cr}}{H}}\cdot\sum_{m=0}^{4} b_m(k,\nu_1,\nu_2)\left(\ell_{kp}/H\right)^m} . \qquad (10)$$

In fig. 2. the graphics of the dependence ℓ_{cr}/H on B_0 is reduced. Hence it follows that with increase of the external loading the critic length of the crack decreases.

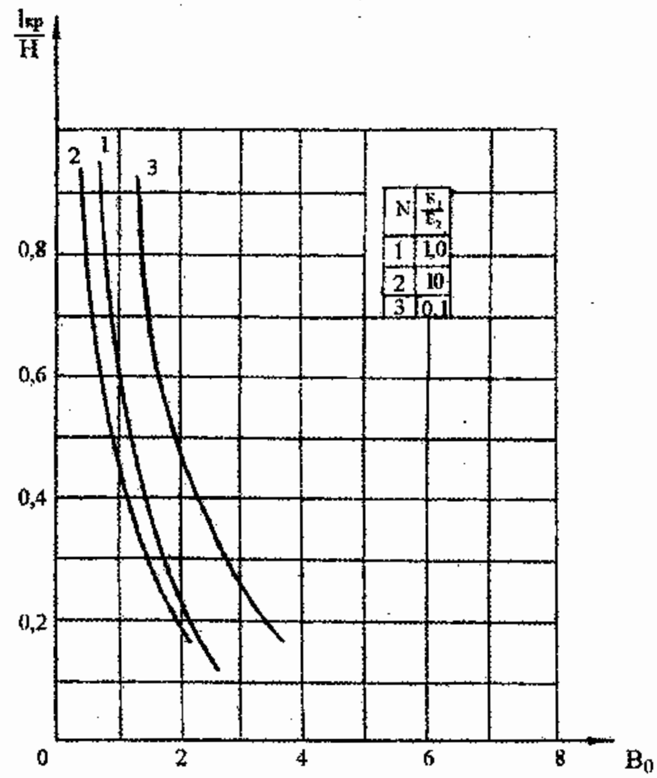

Fig. 2.

As the formula of growth which decribes the cracks we can take the formula by Cherepanov-Kuliyev (for $K_{\mathrm{Im}\,in} \geq 0$)

$$\frac{d\ell}{dn} = -\beta\left[\frac{K_{\mathrm{Im}\,ax}}{K_{I*}^2} + \ln\left(1 - \frac{K_{\mathrm{Im}\,ax}}{K_{I*}^2}\right)\right] + \frac{\vartheta}{\omega}\exp[\lambda K_{\mathrm{Im}\,ax}]\cdot I_0(\lambda, K_{\mathrm{Im}\,ax}). \qquad (11)$$

Here $\beta, \vartheta_0, \lambda, K_{I*}$ are determined from the experiment [3].

Neglecting the kinematics effects from (11) we find

$$\frac{\beta N_f}{H} = \int_{\ell_{cr}/H}^{\ell_0/H} \frac{dx}{B_0^2 x\left[\sum_{m=0}^{4} b_m x^m\right]^2 + \ln\left[1 - xB_0^2(\sum_{m=0}^{4} b_m x^m)^2\right]} .$$

Here ℓ_0 is the initial length of the crack. The dependence of the function on $\beta N_f / H$ and ℓ_0 / H is reduced in fig.3.

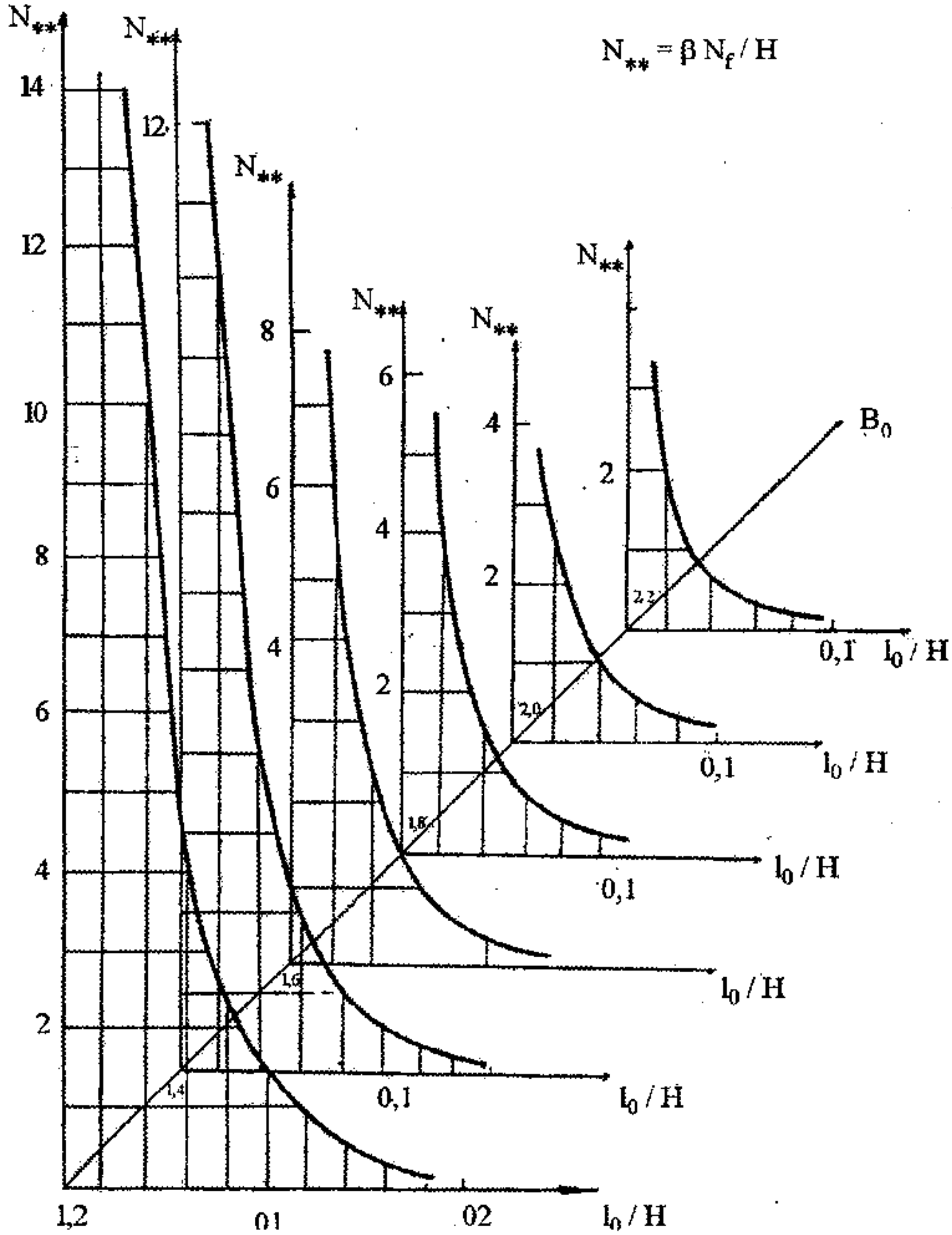

Fig.3.

These curves represent the analogies of Veler's curves. From fig.3. it follows, that with increase of ℓ_0 / H for non-changeable values of other parameters of the problem the number of cycles before fracture N_f decreases. The above suggested approach gives the possibility to estimate the number of the cycles before fracture of the bimaterials.

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