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ON INTER-INFLUENCE OF TWO CYLINDERS WITH RECTANGULAR CROSS SECTIONS UNDER THE STABILITY LOSS IN AN ELASTIC MATRIX.

Abstract

In the paper the stability of two cylinders arranged in the infinite elastic matrix parallel in close distance and whose transverse sections are rectangle is investigated.

Investigations are carried out in frame of piece-wise homogeneous model and using the equations of the theory of three-dimensional condition is considered. The materials of the matrix and the cylinders are taken isotropy and homogeneous.

Introduction.

In paper [3] in the frames of the piecewise homogeneous body model, by the attraction of three-dimensional linearized stability theory (4), for the first time it was suggested a method to study the stability in a structure of one-directed fibrous composite materials under the pressure along the reinforcing elements. At present, many results have been obtained, and their detailed presentation is in monograph [5]. However, in all these investigations, the cross sections of fibers considered to be circles. Hence, the results of these investigations are not applicable for tape composites. In these composites the tapes are shaped as infinite cylinders with non-circular cross-section of different form. So, we study a stability problem of non-circular cylinders is an elastic matrix for tape composites. The investigation method of these problems for isolated cylinders is suggested in [6]. In paper [7] this method is extended to the case when it is considered the inter-influence under the stability loss between the cylinders with non-circular cross sections, and the cross-sections of the cylinders are elliptic. In the given paper by method [7], stability of two cylinders with rectangular cross-sections in an elastic matrix under small sub-critical deformations with regard to inter-influence of cylinders under the stability loss is investigated. Note that, the similar problem for two cylinders with circular cross sections were studied in papers [1,2].

1. Problem statement and solving method.

Consider a problem on the stability of a balance state of an infinite elastic matrix reinforced by two rectangular parallel cylinders with the same rectangular cross sections under the pressure «in an infinity» along the axis of the cylinders with «dead» normal efforts of intensity $\widetilde{q}$.

We relate the body to Lagrangian coordinates that coincide with Cartesian ones in deformation. We shall denote the variables belonging to the matrix and filler correspondingly by super script (2) and (1) and use the denotations from [4]. On the plane of cross-section of cylinders we connect the local (x_{1q}, x_{2q}) and polar (r_q, θ_q) $(q = 1,2)$ coordinate system (fig.1) with the center of each cylinder. The connection between the coordinates are expressed as:

$$r_q \exp\theta_q = R_{qp} \exp i\varphi_{qp} + r_p \exp i\theta_p \quad (q = 1,2; \quad p = 1,2). \tag{1.1}$$

Here $R_{qp} = R_{pq}$ is a distance between the centers of cylinders, $\varphi_{12} = 0$, $\varphi_{21} = \pi$ is an angle between x_{1q} and R_{qp}.

It is assumed that a contour of a cross-section of cylinders is described by the equation

$$\left(\frac{x_{1q}}{a}\right)^8 + \left(\frac{x_{2q}}{b}\right)^8 = 1, \tag{1.2}$$

which is the equation of a rectangle with rounded off angles (fig.1)

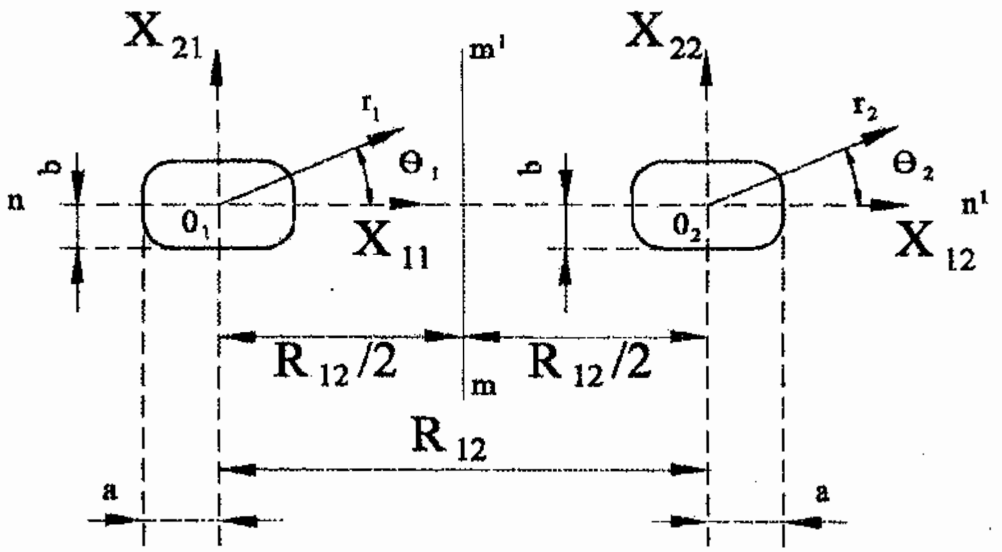

Fig. 1.

In (1.2), $2a$ is the length, and $2b$ is the width of the rectangle. Here, the constituent orthonormals to the contour of cross-section of the cylinder are of the form:

$$N_{1q} = \cos^7\theta_q \left[\cos^{14}\theta_q + \left(\frac{a}{b}\right)^{16}\sin^{14}\theta\right]^{1/2},$$

$$N_{2q} = \left(\frac{a}{b}\right)^8 \sin^7\theta \left[\cos^{14}\theta + \left(\frac{a}{b}\right)^{16}\sin^{14}\theta\right]^{1/2}. \tag{1.3}$$

We assume that materials of the matrix and filler are homogeneous and isotropic. And we adopt that the materials of cylinders are the same. We carry out our research in scopes of the second version of small subcritical deformations theory [4], when a subcritic (principal) stress state is determined on a geometrical linear elasticity theory. The characteristics of the material, i.e. a Young's module and Poisson's coefficient for the matrix correspondingly denote by $E^{(2)}$ and $\nu^{(2)}$,and for cylinders - by $E^{(1)}$ and $\nu^{(1)}$. Assume that $E^{(1)} > E^{(2)}$.

Only, by definition the components of principal (subcritical) stress-state we shall neglect the stresses arising on the square with normals perpendicular to the axis have ox_3, since these stresses have the order $\tilde{q}(\nu^{(1)} - \nu^{(2)})$. Note that under the pressure along sufficiently rigid components of the medium, this assumption may be considered valid. These assumptions are fulfilled exactly, when $\nu^{(1)} = \nu^{(2)}$. Here the variables characterizing the principal stress-state are determined as:

$$\varepsilon_{33}^{0(2)} = \varepsilon_{33}^{0(1)q} = \varepsilon; \quad \sigma_{11}^{0(2)} = \sigma_{11}^{0(1)q} = \sigma_{22}^{0(2)} = \sigma_{22}^{0(1)q} = 0;$$
$$\sigma_{33}^{0(2)} \neq 0, \quad \sigma_{33}^{0(1)q} \neq 0, \quad \sigma_{33}^{0(2)} = E^{(2)}\varepsilon; \quad \sigma_{33}^{0(1)} = E^{(1)}\varepsilon. \qquad (1.4)$$

Assuming that under the stability loss, a complete linking has been realized between the interfaces of materials of the cylinder and matrix, we can write the following equality:

$$p_N^{(2)}\Big|_{s_q} = p_N^{(1)q}\Big|_{s_q}; \quad p_\tau^{(2)}\Big|_{s_q} = p_\tau^{(1)q}\Big|_{s_q}; \quad p_3^{(2)}\Big|_{s_q} = p_3^{(1)q}\Big|_{s_q};$$
$$u_r^{(2)}\Big|_{s_q} = u_r^{(1)q}\Big|_{s_q}; \quad u_\theta^{(2)}\Big|_{s_q} = u_\theta^{(1)q}\Big|_{s_q}; \quad u_3^{(2)}\Big|_{s_q} = u_3^{(1)q}\Big|_{s_q}. \qquad (1.5)$$

Investigate the stability loss in a structure of the material when the length of the wave of the stability loss form is determined not by the length of the sample or the form of structural element, but by the relations between mechanical and geometrical characteristics of the cylinder and matrix. This phenomenon arises in the case, when a curve of dependence of contraction on the wave formation $\chi = \pi b / L$ (l is length of the wave along the axis ox_3 of stability loss form) has a minimum, excluding the case $\chi = 0$.

According to the general solution of equations for three-dimensional linearized stability theory for compressible bodies under small homogeneous subcritical deformations, the constituents of surface forces and displacements both for the matrix and cylinder are determined by the functions $\psi^{(q)}$ and $x^{(q)}$ [4,5], that are the solutions of the equations

$$\left(\Delta_q + \zeta_1^{(q)^2} \frac{\partial^2}{\partial x_3^2}\right)\psi^{(q)} = 0,$$
$$\left(\Delta_q + \zeta_2^{(q)^2} \frac{\partial^2}{\partial x_3^2}\right)\left(\Delta_q + \zeta_3^{(2)^2} \frac{\partial^2}{\partial x_3^2}\right)\chi^{(q)} = 0,$$
$$\Delta = \frac{\partial^2}{\partial r_q^2} + \frac{1}{r_q}\frac{\partial}{\partial r_q} + \frac{1}{r_q^2}\frac{\partial^2}{\partial \theta_q^2}. \qquad (1.6)$$

Here the constants $\zeta_i^{(q)}$ are defined by the constants of materials, and by the variables of subcritical stress-states.

2. Representation of solution and deduction of a characteristic equation.

According to [4] we shall investigate only the bending form of a stability loss of cylinders, assuming that a perturbation domain in a matrix is damping by moving off the cylinders. Under these conditions, the solution of equations (1.6) for the matrix is represented in the form:

$$\psi^{(2)} = \gamma \sin \gamma x_3 \sum_{n=1}^{\infty}\sum_{q=1}^{2}\left[A_{n1}^{(2)q}\cos n\theta_q + B_{n1}^{(2)q}\sin n\theta_q\right]K_n(\gamma\zeta_1^{(2)}r_q);$$
$$\chi^{(2)} = \cos \gamma x_3 \sum_{q=1}^{2}\sum_{s=2}^{3}\sum_{n=0}^{\infty}\left[A_{ns}^{(2)q}\cos n\theta_q + B_{ns}^{(2)q}\sin n\theta_q\right]K_n(\gamma\zeta_s^{(2)}r_q), \qquad (2.1)$$

where $K_n(x)$ and are McDonald's functions.

The solution to the equation (1.6) for cylinders we choose in the form:

$$\Psi_q^{(1)} = \gamma \sin \gamma x_3 \sum_{n-1}^{\infty} I_n\left(\gamma \zeta_{1,q}^{(1)} r_q\right)\left(A_{n1}^{(1)q} \sin n\theta_q + B_{n1}^{(1)q} \cos n\theta_q\right),$$

$$\chi_q^{(1)} = \cos \gamma x_3 \sum_{n-0}^{\infty}\sum_{s=2}^{\infty} I_n\left(\gamma \zeta_{s,q}^{(1)} r_q\right)\left[A_{ns}^{(1)q} \cos n\theta_q + B_{ns}^{(1)q} \sin n\theta_q\right], \qquad (2.2)$$

where $I_n(x)$ is a pure imaginary argument Bessel function.

Thus, by applying the addition theorem for the function (2.1), and later substituting the solutions (2.1) and (2.2) into the contact conditions (1.5) and expanding (1.5) in Fourier series of variable θ_q, we obtain an infinite, homogeneous system of linear algebraic equations with respect to unknown constants contained in (2.1) and (2.2). From the existence conditions of non-trivial solutions of this system we get a characteristic equation for the definition of critic contraction

$$D(\varepsilon, \chi) = 0. \qquad (2.3)$$

We don't cite the expression for the elements of characteristical determinant. As a result of solutions to the equation (2.3) we have the dependence $\varepsilon = \varepsilon(x)$. A critical contraction value is determined from the condition:

$$\varepsilon_{kp} = \min\{\varepsilon(\chi)\ . \qquad (2.4)$$

In addition, corresponding critical values of the wave formation parameter χ_{kq} is also determined.

Investigation of different variants of the stability loss of cylinders depending on what side the bending of the cylinder takes place after the stability loss is of great interest. Considering that a plane of the least bending rigidity of each cylinder coincides with the plane $x_{2q}\, x_{3q}$ we examine two most possible cases of the stability loss.

Case I. The cylinders are bending in the plane of least bending rigidity $x_{2q}\, x_{3q}$ in one direction (the stability loss in a phase, form I).

Case II. The cylinders are bending in the plane of the least bending rigidity $x_{2q}\, x_{3q}$ in the opposite direction (the stability loss in a contrary phase, form II).

The stated stability loss forms that have obvious physical sense settle all possible forms of the stability loss of two cylinders with rectangular cross-sections in an elastic matrix.

3. Numerical results.

Thus, consider the obtained numerical results for the stability loss of two neighboring for the stability loss of two neighboring cylinders with the same rectangular (rounded off angles) cross-sections in an elastic matrix under small subcritical deformations in the case, when $E^{(1)} / E^{(2)} = 100, \quad \nu^{(1)} = \nu^{(2)} = 0.3$

Note that these results were cited in table I for the form I of the stability loss (i.e. case I), and in table II for we form II of the stability loss (i.e. case II). Besides, note that in these tables, at different $\rho = R_{12} / \sqrt{ab}$, of a/b the values of ε_{kp} were cited, and in many cases under the value of ε_{kp} the corresponding values of χ_{kp} were shown in parenthesis.

We see from numerical results of tables I and II that the growth of a/b at both forms of the stability loss of cylinders leads to the decrease of ε_{kp} and χ_{kp}. However, the growth of distances between the cylinders, i.e. the growth of R_{12}/b at the form II of the stability loss leads to the increase of values of ε_{kp} and χ_{kp} but at the II form of the stability loss-to the decrease of ε_{kp} and χ_{kp} and the values ε_{kp} obtained for the I form of the stability loss is strictly less than corresponding values of ε_{kp} obtained for the II form of the stability loss of considered fibers and corresponding values of ε_{kp} obtained for one isolated fiber [6]. Besides, it follows from numerical results that in both cases with the growth of R_{12}/b the values ε_{kp} and χ_{kp} approximate to corresponding values of ε_{kp} and χ_{kp} obtained for one isolated fiber [6]. Consequently, in the considered case, a realizable form of the stability loss of two neighboring cylinders with rectangular cross sections in an elastic matrix under small subcritical deformations is the I form (i.e. case I) and here inter influence between the considered cylinders at the stability loss leads to the decrease of values of critical contractions.

Table 1.

	a/b				
$\rho = R_{12}/\sqrt{ab}$	1.0	1.2	1.5	1.7	2.0
2.5	0.0737 (0.30)	0.0690 (0.30)	0.0636 (0.30)	0.0610 (0.30)	0.0584 (0.30)
3	0.0763 (0.35)	0.0718 (0.30)	0.0663 (0.30)	0.0635 (0.30)	0.0606 (0.30)
4	0.0801 (0.35)	0.0761 (0.35)	0.0703 (0.30)	0.0676 (0.30)	0.0643 (0.30)
∞	0.0878	0.0828	0.0767	0.0740	0.0704

Table 2.

	a/b				
$\rho = R_{12}/\sqrt{ab}$	1.0	1.2	1.5	1.7	2.0
2.5	0.1101 (0.45)	0.1051 (0.45)	0.1000 (0.45)	0.0985 (0.40)	0.0987 (0.40)
3	0.1029 (0.45)	0.0979 (0.40)	0.0911 (0.40)	0.0883 (0.40)	0.0857 (0.40)
4	0.0962 (0.40)	0.0904 (0.40)	0.0842 (0.40)	0.0808 (0.40)	0.0771 (0.35)
∞	0.0878	0.0828	0.0767	0.0740	0.0704

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