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PROLONGED STRENGTH OF THE TWISTED HOLLOW SHAFT

Abstract

On the base of the known model of deformation and failure by Suvorova and Akhundov the process of the scattered failure of the twisted hollow shaft was investigated. The incubation was determined, the equation of the motion of the failure front was constructed. By the numerical method for the constant and singular kernel of the damaging the curvatures showing the principle of the motion of the failure front.

Hollow shafts are one of the most extended elements of machines and constructions. Using of the working resource of hollow shafts at torsion requires the studying of the fracture process taking into account the time factor. Using of the Theory of damaging, which considers the factor of accumulation of the different defects in the material, violations in the structure, gives such possibility. In [1] the theory of damaging was suggested in which the processes of deformation and damaging are interconnected and the fracture is represented as the final critic stage of deformation. Other important singularity of this theory is that for the cases when loading-off zones absent the obtained problems are analogous the problems of the hereditary elasticity to which the corresponding methods of solving are applicable. Particularly when only the forces were given on the surface of the body then distribution of stresses is such as for the elastic material. The mentioned predefined the using of namely the theory [1] for investigation of this problem on prolonged strength of the twisted shaft.

As it has already been pointed out, as also in the elastic problem the only tangential stress different from zero is determined by the classic formula

$$\tau = \frac{2M_k}{\pi\left(b^4 - a^4\right)} r, \tag{1}$$

where a and b are correspondingly the internal and external radiuses of the circle surfaces of the hollow shaft, r is the moving radius, M_k is the torsion moment.

By [1] the fracture criterion will be

$$(1 + K^*)\tau = \tau_0, \tag{2}$$

where K^* is the damaging operator of the hereditary type; τ_0 is the limit of strength for shear of the defectless material. For the case when the torsion moment is constant or increases monotonely and there is not loading-off, the damaging operator has the structure of the ordinary operator of visco-elasticity.

$$K^*\tau = \int_0^t K(t,s)\tau(s)ds, \tag{3}$$

where $K(t,s)$ is the kernel of the damaging operator.

Fracture will happen for the first time where the tangential stress τ has the maxsimal value. That is the external surface of the shaft $r = b$, where

$$\tau_{max} = \frac{2M_k b}{\pi\left(b^4 - a^4\right)}. \tag{4}$$

Substituting (4) into the fracture criterion (2) we will obtain the algebraic equation for determination of time t_0 of fracture of the external surface layer - incubation.

$$\int_0^t K(t_0,s)ds = \frac{\pi(b^4 - a^4)}{2b}\frac{\tau_0}{M_k} - 1. \qquad (5)$$

As it was determined [1] differ can be used as damaging kernels. Then we can give the following view to (5):

$$\int_0^t K(s)ds = \frac{b_0^4 - 1}{2\beta_0} g - 1, \qquad (6)$$

where

$$\beta_0 = \frac{b}{a}; \quad g = \frac{\pi a^3 \tau_0}{2M_k}. \qquad (7)$$

In order to write-out the explicit expression for t_0 it is necessary to concretize the type of the kernel. If $K(t) = kt^{-\alpha}$, then

$$t_0 = \left[\frac{1-a}{k}\left(\frac{\beta_0^4 - 1}{\beta_0} g - 1\right)\right]^{\frac{1}{1-a}}. \qquad (8)$$

If $K(t) = \lambda k t^{-\delta t}$, then

$$t_0 = -\frac{1}{\delta}\ln\left\{1 + \frac{\delta}{\lambda}\left(1 - \frac{\beta_0^4 - 1}{\beta_0} g\right)\right\}. \qquad (9)$$

If $K(t) = k$, then

$$t_0 = \frac{1}{k}\left[\frac{\beta_0^4 - 1}{\beta_0} g - 1\right]. \qquad (10)$$

Analysis of these expressions shows that in dependence on sizes of the hollow shaft-pipe, that is on $\beta_0 = a / b$ the quantity t_0 can take different values. For example, if $(\beta_0^4 - 1)g / \beta_0 = 1$, then according to (8)-(10) the fracture will begin immediately since the application of the torsion moment and further the fracture front will propagate to the inner surface of the hollow shaft. Moreover, for the singular or constant kernel of damaging even the least value of the torsion moment will reduce to failure, though the time t_0 increases with decrease M_k. But in the case of the regular kernel there is always such a lower bound that for $M_k < M_k^*$ failure doesn't happen. It should note also that for $M_k > M_{k0}$ (where M_{k0} - determines the quantity of the torsion moment corresponding to the zero value of t_0) failure happens instantly.

For $t > t_0$ in the hollow shaft the exterior ring zone of the failure is formed which extends inside.

In order to follow the motion of the failure front - the surface of separation of the failed and the unfailed parts of the hollow shaft - it is necessary to consider the external radius depending on time. It means that $\beta / \alpha = \beta(s)$, $r / a = \beta(t)$ where t - is the given moment of time and $t_0 < s < t$. Then the expression for the tangential stress will be:

$$\tau(t,s) = \frac{2M_k}{\pi a^3} \frac{\beta(t)}{\beta^4(s)-1}. \tag{11}$$

Taking into account this expression in the failure criterion (2), we have:

$$\frac{\beta(t)}{\beta^4(t)-1} + \int_0^t K(t-s)\frac{\beta(t)}{\beta^4(s)-1}ds = g. \tag{12}$$

For finding out the picture of the process let's take $K(t-s) = k = const$ as a damaging kernel for definiteness. Then (12) will be:

$$\frac{1}{\beta^4(t)-1} + k\int_0^t \frac{ds}{\beta^4(s)-1} = \frac{g}{\beta(t)}. \tag{13}$$

Differentiating (13) by time we will obtain the following differential equation of the first order with respect to the dimensionless radius $\beta(t)$ of the failure front:

$$\frac{d\beta}{dt} = \frac{k\beta^2(\beta^4-1)}{4\beta^5 - g(\beta^4-1)^2}. \tag{14}$$

The initial condition to (14) is the correlation (10), that is $\beta = \beta_0$ for $t = t_0$, where β_0 are the given initial geometrical sizes of the hollow shaft.

Equation (14) shows that the motion of the failure front that is $d\beta / dt < 0$ fulfilled and it is possible only when

$$\frac{4\beta^5}{(\beta^4-1)^2} < g. \tag{15}$$

This condition connecting the geometrical parameters of hollow shaft and the torsional moment and pointing out its limit value for which the failure beginning from the exterior radius will spread during the time to the inner surface. For all values of the torsional moment M_k exceeding the value determined by (15) the failure will happen instantly though in the interval

$$\frac{4\beta}{\beta^4-1} < g < \frac{4\beta^5}{(\beta^4-1)^2} \tag{16}$$

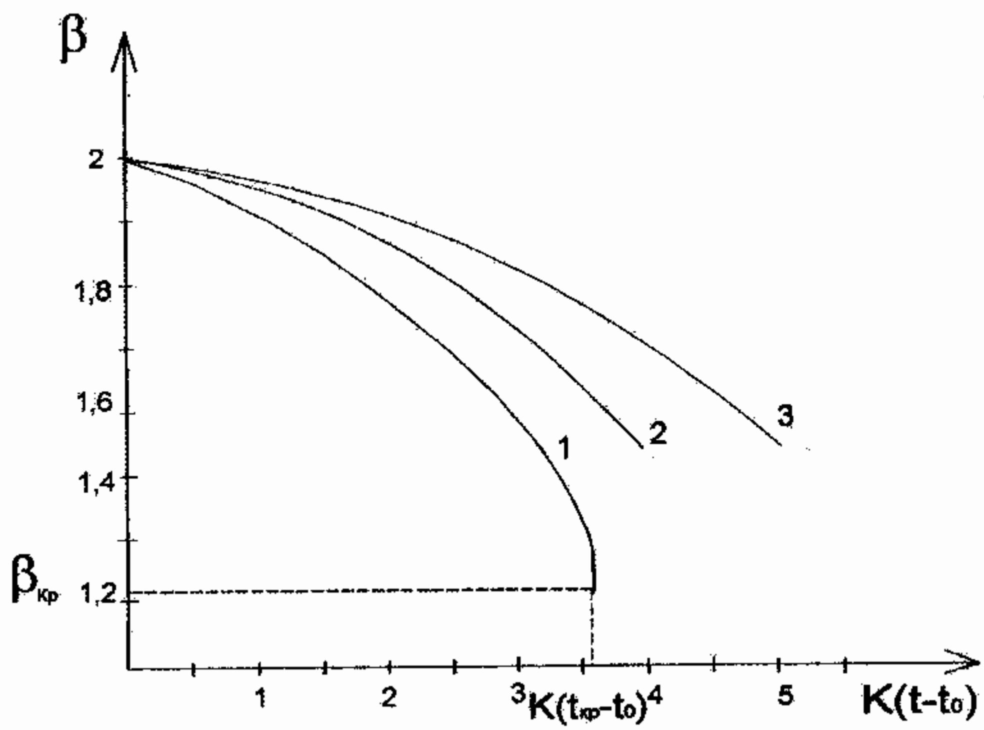
there is some lag time of the start of the process which can be calculated from (8)-(10).

Therefore, we obtain the following picture of the failure process. For some value of the torsional moment satisfying (15) at time moment t_0 on the exterior surface failure begins. The failure front further propagates with increasing velocity. When the front reaches the situation determined by the condition

$$\frac{4\beta_{kp}^5}{(\beta_{kp}^4-1)^2} = \frac{\pi a_0^3 \tau_0}{2M_k} \tag{17}$$

the velocity becomes infinite great and the instant failure of the whole model happens.

From (14) we find the following principle of the motion of the failure front:



Moving curve of front destruction
$\beta_0 = 2; g = 3;$
$1 - \alpha = 0; 2 - \alpha = 0{,}1; 3 - \alpha = 0{,}15$

Fig.1.

$$k(t - t_0) = \ln\frac{\beta - 1}{\beta_0 - 1} + \frac{g}{3}\left(\beta_0^3 - \beta^3\right) + g\left(\frac{1}{\beta_0} - \frac{1}{\beta}\right), \tag{18}$$

where t_0 is determined by (10).

In fig. the curvatures showing the character of the motion of the failure front were given for the constant by (18) and for the singular by (12) kernels of the damaging operator for the following values of the process parameters: $\beta_0 = 2$; $g = 3$; $\alpha = 0{,}5$. For the chosen values of the parameters for the constant kernel of the damaging operator for the incubation according to (10) we will obtain $Kt_0 = 21{,}5$. From fig.1. it follows that the period of the explicit failure process makes about 17% from latent incubation for $\alpha = 0$, 18,5% for $\alpha = 0{,}1$ and 19,6% for $\alpha = 0{,}15$. Moreover the failure front moves during the most time of the explicit period of the failure process with almost constant velocity, only at the final stage growing sharply its velocity up to the finite great value after that the instant failure of the rest of the part of the hollow shaft happens. Contribution of the singularity of the kernel of the damaging operator is particularly in decreasing of the velocity of the motion of the failure front.

References

[1]. Суворова Ю.В., Ахундов М.Б. *Длительное разрушение изотропной среды в условиях сложного напряжённого состояния.* Машиноведение, АН СССР, 1986 №4, с.40-46

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Received October 12, 1999; Revised January 05, 2000.
Translated by Soltanova S.M.