KHANMAMEDOV A.Kh.

THE EXISTENCE OF THE ATTRACTOR FOR THE SEMI-LINEAR WAVE EQUATION WITH LOCALIZED DAMPING.

Abstract

In the paper the mixed problem with Diriclet homogeneous condition for the semi linear wave equation with local dissipation is considered. Using Massat theorem the existence of the minimal global attractor of the considered problem is proved.

The global behavior of solutions for a semi-linear wave equation in bounded domains was studied in paper [1], [2], [3] and etc. In these papers, the coefficient of the damping term of the equation had no degeneration. Exponential decrease of energy for wave equation with localized damping, where the coefficient of the damping term has no degeneration on a neighbourhood of the boundary of the domain, was studied in [4].

In the given paper, the behavior of solutions the semilinear wave equation with localized damping, where conditions on the degeneration domain are not imposed, are studied in one-dimensional case.

Consider the following mixed problem:

$$\left.\begin{array}{l} v_{tt}+\alpha(x)v_t-v_{xx}+f(v)=g(x), \quad (t,x)\in(0;+\infty)\times(0;1) \\ v(t,0)=v(t,1)=0, \quad t\in(0;+\infty) \\ v(0,x)=v_0(x), \quad v_t(0,x)=v_1(x), \quad x\in(0,1) \end{array}\right\} \tag{1}$$

where $\alpha(\cdot)\in C[0,1], \quad \alpha(x)\geq 0, \quad \alpha(\cdot)\not\equiv 0, \quad f(\cdot)\in C^2(R),$

$\varliminf_{|s|\to\infty}\frac{f(s)}{s}\geq 0, \quad g(\cdot)\in L_2(0,1), \quad v_0(x)$ and $v_1(x)$ - are given functions.

By means of the transformation $\theta=\begin{pmatrix} v \\ v_t \end{pmatrix}$ the problem (1) on the space $\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)$ is reduced to the following problem

$$\left.\begin{array}{l} \theta'(t)=A\theta(t)+F(\theta(t))+G, \quad t\in(0,+\infty) \\ \theta(0)=\theta_0 \end{array}\right\} \tag{2}$$

where

$$A=\begin{pmatrix} 0 & I \\ \frac{\partial^2}{\partial x^2} & -\alpha(\cdot)I \end{pmatrix}, D(A)=\left(W_2^2(0,1)\cap \overset{0}{W}{}_2^1(0,1)\right)\times \overset{0}{W}{}_2^1(0,1),$$

$$F(\theta(t))=\begin{pmatrix} 0 \\ f(v(t))-f(0) \end{pmatrix}, G=\begin{pmatrix} 0 \\ g+f(0) \end{pmatrix}.$$

According to the results of [5] the problem (2) under the given conditions has a unique solution $\theta(\cdot)\in C\left([0,+\infty); \overset{0}{W}{}_2^1(0,1)\times L_2(0,1)\right)$, that satisfies the following integral equation:

$$\theta(t)=\exp(tA)\theta_0+\int_0^t \exp((t-s)A)\big(F(\theta(s))+G\big)ds\,. \tag{3}$$

Therefore, the problem (1) generates a strongly continuous nonlinear semi-group $v(t)=U(t)+W(t)$, where

$$U(t)=\exp(tA),\quad W(t)\theta_0=\int_0^t \exp((t-s)A)\big(F(\theta(s))+G\big)ds.$$

Theorem 1. *There exit $M>0$ and $\varepsilon>0$, such that*

$$\|U(t)\|_{L\left(\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)\right)}\le M\cdot e^{-\varepsilon t},\quad t\ge 0\,. \tag{4}$$

Proof. By definition $\begin{pmatrix}u(t)\\ u_t(t)\end{pmatrix}=U(t)\theta$

where $u(t)$ in the solution of the following problem

$$\left.\begin{array}{r} u_{tt}+\alpha(x)u_t-u_{xx}=0,\quad (t,x)\in(0,+\infty)\times(0,1)\\ u(t,0)=u(t,1)=0,\quad t\in(0,+\infty)\\ u(0,x)=u_0(x),\quad u_t(0,x)=u_1(x),\quad x\in(0,1)\end{array}\right\} \tag{5}$$

By multiplying $(5)_1$ by u_t and integrating by parts over $(0,t)\times(0,1)$ we get:

$$\frac{1}{2}\left[\int_0^1 u_t^2dx+\int_0^1 u_x^2dx\right]+\int_0^t\int_0^1\alpha(x)u_t^2dxds\le\frac{1}{2}\left[\int_0^1 u_1^2dx+\int_0^1 u_{0x}^2dx\right],\quad t\ge 0\,. \tag{6}$$

Denote $\tilde u(t;)=\int_0^t u(s;)ds+\varphi(x)$, where $\varphi\in W_2^2(0,1)\cap \overset{0}{W}{}_2^1(0,1)$ and $\varphi_{xx}=u_1+\alpha u_0$. Then by integrating $(5)_1$ by t we get:

$$\left.\begin{array}{r} \tilde u_{tt}+\alpha\tilde u_t-\tilde u_{xx}=0,\quad (t,x)\in(0,+\infty)\times(9,1)\\ \tilde u(t,0)=\tilde u(t,1)=0,\quad t\in(0,+\infty)\\ \tilde u(0,x)=\varphi(x),\quad \tilde u_t(0,x)=u_0(x),\quad x\in(0,1)\end{array}\right\}$$

Hence it follows that the estimate similar to (6) is valid for $\tilde u$:

$$\int_0^t\int_0^1\alpha\tilde u_t^2dxds\le\frac{1}{2}\left[\int_0^1 u_0^2dx+\int_0^1\varphi_x^2dx\right],\quad t\ge 0\,. \tag{7}$$

Thus from (6) and (7) we get:

$$\int_0^t\int_0^1\alpha u^2dxds+\int_0^t\int_0^1\alpha u_t^2dxds\le c\cdot\left[\int_0^1 u_1^2dx+\int_0^1 u_{0x}^2dx\right],\quad t\ge 0\,. \tag{8}$$

Since the continuous function $\alpha(x)$ is non-negative on [0,1] and $\alpha(x)\not\equiv 0$, then exists $[a,b]\subset[0,1]$ such that $\alpha(x)\ge\delta>0$ for $\forall x\in[a,b]$. Then from (8) for $\forall t\ge 0$ we get:

$$\int_0^t\int_a^b\left(u^2+u_t^2\right)dxds\le\frac{c}{\delta}\int_0^1\left(u_1^2+u_{0x}^2\right)dx\,. \tag{9}$$

Let $\Psi(\cdot)\in C_0^2[a,b]$ and $\Psi(x)=1$ for $\forall x\in[a_1,b_1]\subset[a,b]$. By multiplying both sides of $(5)_1$ by $\Psi\cdot u$ and integrating by parts over $(0,t)\times(a,b)$, considering (9) we get:

$$\int_0^t\int_a^b \Psi\cdot u_x^2 dxds \le C_1\cdot\int_0^1\left(u_1^2+u_{0x}^2\right)dx,\quad t\ge 0$$

consequently

$$\int_0^t\int_{a_1}^{b_1} u_x^2 dxds \le C_1\cdot\int_0^1\left(u_1^2+u_{0x}^2\right)dx,\quad t\ge 0. \tag{10}$$

Let $p(\cdot)\in C_0^1[0,1]$, $p(x)=x$ for $\forall x\in[0,a_1]$ and $p(x)=x-1$ for $\forall x\in[b_1,1]$. By multiplying $(5)_1$ by $p(x)\cdot u_x$ and integrating by parts over $(0,t)\times(0,1)$ considering (6) and (10) we get:

$$\int_0^t\int_0^1\left(u_t^2+u_x^2\right)dxds \le C_2\int_0^1\left(u_{0x}^2+u_1^2\right)dx,\quad t\ge 0.$$

It follows from the latter that

$$\int_0^\infty \left\|U(t)\theta_0\right\|^2_{\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)} dt \le C_2\cdot\left\|\theta_0\right\|^2_{\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)}.$$

Hence according to [6] we get (4).

By virtue of reversibility of the operator A from theorem 1 we get the following

Corollary: *There exists a constant* $\mu>0$, *such that*

$$\left\|U(t)\right\|_{\mathcal{L}\left(W_2^2(0,1)\cap\overset{\circ}{W}{}_2^1(0,1)\times\overset{\circ}{W}{}_2^1(0,1)\right)} \le \widetilde{M}\cdot e^{-\mu t},\quad t\ge 0. \tag{11}$$

Theorem2. *For any bounded set* $B\subset \overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)$ *the subset* $\bigcup_{t\ge 0} W(t)B$ *is precompact in* $\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)$.

Proof. Let $\theta_0=B$. Then

$$\begin{aligned} W(t)\theta_0 &= \int_0^t \exp((t-s)A)\left(F(V(s)\theta_0)+G\right)ds = \\ &= \int_0^t \exp((t-s)A)F(V(s)\theta_0)ds + A^{-1}\left(\exp(tA)G-G\right). \end{aligned} \tag{12}$$

Since for $\forall t\ge 0$,

$$\left\|F(V)\theta_0\right\|_{W_2^2(0,1)\cap\overset{\circ}{W}{}_2^1(0,1)\times\overset{\circ}{W}{}_2^1(0,1)} \le C\left(\left\|\theta_0\right\|_{\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)}\right),$$

then from (4), (11) and (12) we get

$$\left\|W(t)\theta_0\right\|_{W_2^2(0,1)\cap\overset{\circ}{W}{}_2^1(0,1)\times\overset{\circ}{W}{}_2^1(0,1)} \le \widetilde{C}\left(\left\|\theta_0\right\|_{\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)}\right). \tag{13}$$

The last inequality proves theorem 2.

Now prove the pointwise damping of the semi-group $V(t)$.

Theorem 3. *On the space* $\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)$ *there exists a bounded set* $\mathcal{M}$ *such that for any* $\varphi\in\overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1)$

$$V(t)\varphi \xrightarrow[t\to\infty]{} \mathcal{M} \quad in \quad \overset{\circ}{W}{}_2^1(0,1)\times L_2(0,1).$$

Proof. By applying the results of paper [1], from theorems 1 and 2 we obtain that each point $\theta_0 \in \overset{0}{W}{}_2^1(0,1) \times L_2(0,1)$ has a compact ω - limit set

$$\omega(\theta_0) = \bigcap_{t\geq 0}\overline{\bigcup_{s>t} V(s)\theta_0} \subset \overset{0}{W}{}_2^1(0,1) \times L_2(0,1),$$

which is invariant with respect to the semi-group $V(t)$. It follows from the invariance of $\omega(\theta_0)$ that for $\forall \varphi = \begin{pmatrix}\varphi^1\\ \varphi^2\end{pmatrix} \in \omega(\theta_0)$ *there exist* $\{t_n\}_{n=1}^{\infty}$, $t_n \to +\infty$ and $\varphi_n = \begin{pmatrix}\varphi_n^1\\ \varphi_n^2\end{pmatrix} \in \omega(\theta_0)$ such that

$$V(t_n)\varphi_n = \varphi. \tag{14}$$

By passing to the limit on the space $\overset{0}{W}{}_2^1(0,1) \times L_2(0,1)$, considering (4) we get

$$\varphi = \lim_{n\to\infty} W(t_n)\varphi_n. \tag{15}$$

Denote $\|\omega(\theta_0)\|_H = \sup_{\varphi\in\omega(\theta_0)} \|\varphi\|_H$. By (15) according to (13) we have:

$$\|\varphi\|_{W_2^2(0,1)\cap \overset{0}{W}{}_2^1(0,1)\times \overset{0}{W}{}_2^1(0,1)} \leq \tilde{C}_1\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right).$$

Since φ is an arbitrary element from $\omega(\theta_0)$, if follows the last inequality that

$$\|\omega(\theta_0)\|_{W_2^2(0,1)\cap \overset{0}{W}{}_2^1(0,1)\overset{0}{W}{}_2^1(0,1)} \leq \tilde{C}_1\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right). \tag{16}$$

After substituting variables under the integral in the equation (3), for the vector function $\begin{pmatrix}v_n(t)\\ v_{nt}(t)\end{pmatrix} = V(t)\varphi_n$ we get:

$$\begin{pmatrix}v_n(t)\\ v_{nt}(t)\end{pmatrix} = \exp(tA)\varphi_n + \int_0^t \exp(sA)\begin{pmatrix}0\\ f(v_n(t-s)) + g\end{pmatrix} ds.$$

Since $\varphi_n \in \omega(\theta_0)$ and $f \in C^2(R)$ then by (16) the right hand side of the last equation has a derivative on t. By differentiating it we get:

$$\begin{pmatrix}v_{nt}(t)\\ v_{ntt}(t)\end{pmatrix} = \exp(tA)\left[A\varphi_n + \begin{pmatrix}0\\ f(v_n(0)) + g\end{pmatrix}\right] + \int_0^t \exp(sA)\begin{pmatrix}0\\ f(v_n(t-s))v_{nt}(t-s)\end{pmatrix} ds$$

By virtue of (14) $v_n(t_n) = \varphi^1$ *and* $v_{nt}(t_n) = \varphi^2$. Then taking into account that $v_n(t;)$ satisfies the equation $(1)_1$ we have:

$$v_{ntt}(t_n) \equiv \varphi^3 = \varphi_{xx}^1 - \alpha\varphi^2 - f(\varphi^1) + g$$

and we obtain:

$$\begin{pmatrix}\varphi^2\\ \varphi^3\end{pmatrix} = \exp(t_n A)\left[A\varphi_n + \begin{pmatrix}0\\ f(\varphi_n^1) + g\end{pmatrix}\right] + \int_0^{t_n} \exp(sA)\begin{pmatrix}0\\ f'(v_n(t_n - s))v_{nt}(t-s)\end{pmatrix} ds.$$

Here we pass to the limit in $W_2^1(0,1) \times L_2(0,1)$ for $n \to \infty$ by considering (4). Then

$$\lim_{n\to\infty}\int_0^{t_n}\exp(sA)\begin{pmatrix}0\\ f'(v_n(t_n-s))v_{nt}(t-s)\end{pmatrix}ds=\begin{pmatrix}\varphi^2\\ \varphi^3\end{pmatrix}. \tag{17}$$

Since by (16)

$$\left\|\begin{pmatrix}0\\ f'(v_n(t_n-s))v_{nt}(t_n-s)\end{pmatrix}\right\|_{W_2^2(0,1)\cap \overset{0}{W}{}_2^1(0,1)\times \overset{0}{W}{}_2^1(0,1)}\le \widetilde{C}_2\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right),$$

then from (17) it follows that

$$\|\varphi^2\|_{W_2^2(0,1)\cap \overset{0}{W}{}_2^1(0,1)}\le \widetilde{C}_3\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right).$$

Thus, for $\varphi=\begin{pmatrix}\varphi^1\\ \varphi^2\end{pmatrix}\in\omega(\theta_0)$

$$\|\varphi\|_{W_2^2(0,1)\times W_2^2(0,1)}\le \widetilde{C}_4\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right)$$

i.e.

$$\|\omega(\theta_0)\|_{W_2^2(0,1)\times W_2^2(0,1)}\le \widetilde{C}_4\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right). \tag{18}$$

Prove that there exist $c>0$ such that for $\forall\theta_0\in \overset{0}{W}{}_2^1(0,1)\times L_2(0,1)$

$$\|\omega(\theta)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\le c. \tag{19}$$

By definition of the set $\omega(\theta_0)$, for any $\varphi\in\omega(\theta_0)$ there exists $\{t_n\}_{n=1}^{\infty}$, $t_n\to+\infty$, such that

$$\begin{pmatrix}v(t_n)\\ v_t(t_n)\end{pmatrix}\equiv V(t_n)\theta_0\xrightarrow[n\to\infty]{}\varphi \text{ strongly in } \overset{0}{W}{}_2^1(0,1)\times L_2(0,1). \tag{20}$$

On the other hand, from the problem (1) we get that the Lyapunov function

$$L(V(t)\theta_0)=\frac{1}{2}\int_0^1 v_x^2dx+\frac{1}{2}\int_0^1 v_t^2dx+\int_0^1\Phi(v)dx+\int_0^1 gvdx$$

doesn't increase (here $\Phi(s)=\int_0^s f(t)dt$). And therefore there exists a finite limit

$$\lim_{t\to+\infty}L(V(t)\theta_0)=a.$$

Hence, by virtue of (20) it follows that for $\forall\varphi\in\omega(\theta_0)$

$$L(\varphi)\equiv\frac{1}{2}\int_0^1(\varphi_x^1)^2dx+\frac{1}{2}\int_0^1(\varphi^2)^2dx+\int_0^1\Phi(\varphi^1)dx+\int_0^1 g\varphi^1dx=a.$$

In other words, the function $L:\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)\to R$ on the set $\omega(\theta_0)$ is constant. Then it remains constant, and on the trajectory $\begin{pmatrix}\widetilde{v}(t)\\ \widetilde{v}(t)\end{pmatrix}\equiv V(t)\varphi \quad (\varphi\in\omega(\theta_0))$, i.e.

$$\frac{d}{dt}L(V(t)\varphi)=0,\quad \forall t\ge 0,\quad \forall\varphi\in\omega(\theta_0).$$

By virtue of the properties of the semigroup $V(t)$, it follows from the latter that $\widetilde{v}(t,\cdot)$ satisfies the following problem:

$$\left.\begin{array}{ll}\tilde{v}_{tt}-\tilde{v}_{xx}+f(\tilde{v})=g, & (t,x)\in(0,+\infty)\times(0,1)\\ \alpha\tilde{v}_t=0, & (t,x)\in(0,+\infty)\times(0,1)\\ \tilde{v}(t,0)=\tilde{v}(t,1)=0, & t\in(0,+\infty)\\ \tilde{v}(t,x)=\varphi^1,\ \tilde{v}_t(0,x)=\varphi^2 & x\in(0,1)\end{array}\right\} \qquad (21)$$

Therefore, by denoting $w=\tilde{v}_t$ and $\varphi^3=\varphi^1_{xx}-f(\varphi^1)+g$ we get

$$\left.\begin{array}{ll}w_{tt}-w_{xx}+f'(\tilde{v})w=0, & (t,x)\in(0,+\infty)\times(0,1)\\ w=0, & (t,x)\in(0,+\infty)\times(a_1,b_1)\\ w(t,0)=w(t,1)=0 & t\in(0,+\infty)\\ w(0,x)=\varphi^2,\ w_t(0,x)=\varphi^3 & x\in(0,1)\end{array}\right\} \qquad (22)$$

Since for $\forall t\geq 0$ $\begin{pmatrix}\tilde{v}(t)\\ \tilde{v}(t)\end{pmatrix}\in\omega(\theta_0)$, then from (18), (21), and (22) we get:

$$w(\cdot)\in C_b\left([0,+\infty);\ W_2^2(0,1)\cap\overset{0}{W}{}_2^1(0,1)\right),\quad w_t\in C_b([0,+\infty);\ L_2(0,1))$$

$$and\quad w_{tt}\in C_b([0,+\infty);\ L_2(0,1)).$$

Multiply $(22)_1$ by $x^n w_x (n\geq 1)$ and integrate by parts over $(0,t)\times(0,b)$. By using the Holder and Hardy inequality we get

$$\frac{n}{2}\int_0^t\int_0^{b_1}x^{n-1}w_x^2dxds+\frac{n}{2}\int_0^t\int_0^{b_1}x^{n-1}w_t^2dxds-\tilde{C}_5\left(\|\omega(\theta_0)\|_{\overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\right)\times\int_0^t\int_0^{b_1}x^{n-1}w_x^2dxds\leq$$

$$\leq\tilde{C}_6\left(\|\omega(\theta_0)\|_{W_2^2(0,1)\times W_2^2(0,1)}\right),\quad\forall t\geq 0$$

Choose n so that $n>4\tilde{C}_6\left(\|\omega(\theta_0)\|_{W_2^2(0,1)\times W_2^2(0,1)}\right)$.

Then we have

$$\frac{n}{4}\int_0^t\int_0^{b_1}x^{n-1}w_x^2dxds+\frac{n}{2}\int_0^t\int_0^{b_1}x^{n-1}w_t^2dxds\leq\tilde{C}_6\left(\|\omega(\theta_0)\|_{W_2^2(0,1)\times W_2^2(0,1)}\right). \qquad (23)$$

To complete the proof of theorem 3 we need the following lemma, whose validity is simply verified.

Lemma. *If* $y(\cdot)\in L_1(0,+\infty)\cap C^1[0,+\infty)$ *and* $y'(\cdot)\in L_\infty(0,+\infty)$ *then* $\lim_{t\to+\infty}y(t)=0$.

By applying the lemma to (23) we get

$$\lim_{t\to+\infty}\int_0^{b_1}x^{n-1}w^2dx=0,\quad\lim_{t\to+\infty}\int_0^{b_1}x^{n-1}w_t^2dx=0. \qquad (24)$$

Since

$w(\cdot)\in C_b([0,+\infty);\ W_2^2(0,1))$ and $w_t(\cdot)\in C_b([0,+\infty);L_2(0,1))$, then we get from (24) that $w(t)\xrightarrow[t\to+\infty]{}0$ weakly in $W_2^2(0,b_1)$ and $w_t(t)\xrightarrow[t\to\infty]{}0$ weakly in $L_2(0,b_1)$, therefore $\tilde{v}_t(t)\xrightarrow[t\to+\infty]{}0$ strongly in $L_2(0,b_1)$, and $\tilde{v}_{tt}(t)\xrightarrow[t\to+\infty]{}0$ weakly in $L_2(0,b_1)$.

Similarly we obtain that

$\tilde{v}_t(t)\xrightarrow[t\to+\infty]{}0$ strongly in $L_2(b_1,1)$ and $\tilde{v}_{tt}(t)\xrightarrow[t\to+\infty]{}0$ weakly in $L_2(b_1,1)$.

Thus

$$v_t(t)\xrightarrow[t\to+\infty]{}0 \text{ strongly in } L_2(0,1), \tag{25}$$

$$\tilde{v}_{tt}(t)\xrightarrow[t\to+\infty]{}0 \text{ weakly in } L_2(0,1). \tag{26}$$

By multiplying $(21)_1$ by $\tilde{v}(t,x)$ and integrating by parts over (0,1) get:

$$\int_0^1 \tilde{v}_x^2 dx \le c\left(1+\|g\|^2_{L_2(0,1)}\right)+\|\tilde{v}_{tt}\|_{W_2^{-1}(0,1)}\cdot\|\tilde{v}\|_{\overset{0}{W}{}_2^1(0,1)}.$$

By passing to the limit, by considering (26) we have

$$\overline{\lim_{t\to+\infty}}\int_0^1 \tilde{v}_x^2 dx \le c\left(1+\|g\|^2_{L_2(0,1)}\right). \tag{27}$$

On the other hand, as we have already stated

$$\frac{1}{2}\int_0^1 \left(\varphi_x^1\right)^2 dx+\frac{1}{2}\int_0^1\left(\varphi^2\right)^2 dx+\int_0^1 \Phi\left(\varphi^1\right)^2 dx+\int_0^1 g\varphi^1 dx=$$

$$=\frac{1}{2}\int_0^1 \tilde{v}_x^2 dx+\frac{1}{2}\int_0^1 \tilde{v}_t^2 dx+\int_0^1 \Phi(\tilde{v})dx+\int_0^1 g\tilde{v}dx .$$

By virtue of (25) and (27) from the last equality it follows (19) and the set $\mathcal{M}=\bigcup_{\theta_0\in \overset{0}{W}{}_2^1(0,1)\times L_2(0,1)}\omega(\theta_0)$ satisfies theorem 3.

As it is shown in [1] existence of the minimal global attractor follows from theorems 1-3.

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Khanmamedov A.Kh.

Institute of Mathematics and Mechanics of AS Azerbaijan.

9, F. Agayeva str., 370141, Baku, Azerbaijan.

Tel.: 39-47-20.

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