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ON THE SOLVABILITY OF BOUNDARY-VALUE PROBLEMS FOR A CLASS OF SECOND ORDER OPERATOR-DIFFERENTIAL EQUATIONS

Abstract

Sufficient conditions for the existence of the solution to initial boundary value problem are found for second order operator-differential equation whose principal part contains a normal operator. These conditions are expressed by the coefficients of a operator-differential equation.

A theorem on the existence of holomorphic solutions of the initial-boundary value problem is proved for a class of the second order operator-differential equations whose symbol contains a normal operator at the principal part.

In a separable Hilbert space H consider the boundary-value problem

$$P(d/dz)X' = -u'(z) + A^2 u(z) + A_1 u'(z) + A_2 u(z) = f(z), \quad z \in S_a, \tag{1}$$

$$w'(0) = 0, \tag{2}$$

where $S_a = \{z \mid \arg z \in (-a, a), 0 < a < \pi/2\}$, A, A_1, A_2 are linear operators in H , $f(z)$ and $u(z)$ are holomorphic at the sector S_a vector functions with values in H . Further, we assume the fulfillment of the following conditions:

- 1) A is a normal operator with compact continuous inverse A^{-1} , whose spectrum is contained in a cone sector
- 2) $B_j = A_j A^{-j}$ ($j = 1, 2$) are bounded operators in H ;
- 3) the members a and π satisfy the condition: $0 < a + \pi < \pi/2$.

It is obvious that by fulfilling the condition 1) the operator A has a polar expansion $A = UC$, where U is a unitary, and C is a positive-defined self-adjoint operator in H , moreover, $D(A) = D(A^*) = D(C)$ and for any $x \in D(A)$ $\|Ax\| = \|Cx\|$

Definition domain of the operator C^{-1} ($y \in H$) becomes the Hilbert space H_y with respect to the norm $\|x\|_y = \|C^{-1}x\|_H$. Further, denote by $L_2(\wedge; H)$ a Hilbert space of vector-functions $f(t)$ determined in $R_+ = (0, \infty)$ with values in H , for which

$$\|f\|_{L_2} = \left(\int_0^\infty \|f(t)\|^2 dt \right)^{1/2} < \infty$$

Denote by $H_2(a; H)$ a set of vector-functions $f(z)$ determined in S_a , with values in H , which are holomorphic in S_a and at each $pe[-a, a]$ of vector functions $f_v = f/(e^{i\theta})$ ($\theta \in (-a, a)$). This linear set is a Hilbert space with respect to the norm [1]

Determine the following Hilbert spaces