MECHANICS

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BENDING OSCILLATIONS OF THE NON-LINEAR BAR WITH THE BIOFACTOR

Abstract

On the base of the model of biofactor the process of bending oscillations of the non-linear elastic bar with a finite length for the reaction of the material to the external action is investigation. For the first approximation the analytical expressions of solutions is obtained and the amplitude-frequency characteristics versus the biofactor (the reaction delay parameter) are constructed.

Investigation of the problem on the mechanical action to the different parts of the human organism causes the working out of the mechanical models of these organs. In [2] the same model of the mouse fibers was constructed. For the first time the concept of the biofactor was introduced in it as the quantity which influences on deformations and displacements in the material at the same time with the mechanical actions. In [1] the other method of consideration of the biologic activity of the material was suggested which does not contradict to the approach given in [2]. However, in investigations for the biosystems the consideration of the non-linearity of elastic and non-elastic mechanical properties of the material and the biofactor is absent.

In this work on the base of the model of biofactor the process of bending oscillations of the non-linear elastic bar with the finite length for the reaction of the material to the external action is investigated [1].

The equation of the bending motion of the bar has the form:

$$(1-A)\frac{\partial^2 M(x,t)}{\partial x^2}+A\tau\frac{\partial^3 M(x,t)}{\partial x^2\partial t}=-\rho F\frac{\partial^2\eta}{\partial t^2}+q(x,t), \tag{1}$$

where $M(x,t)$ is the bending moment, $\eta(x,t)$ is the deflection of the bar, ρ is the density, F is the square of the cross-section of the bar, $0\le A\le 1$, $0\le\tau<1$ are the parameters of the biofactor, moreover τ represents the lag of the bioreaction to the external influence, that is, the corresponding shear by the time.

According to Cauderer non-linear law the relation between the bending moment and the deflection is taken as:

$$M(x,t)=EJ_0\left[\frac{\partial^2\eta}{\partial x^2}-a_3E^2\frac{J_2}{J_0}\left(\frac{\partial^2\eta}{\partial x^2}\right)^3\right], \tag{2}$$

where E is the Young's modulus, $E=\frac{9KC_t}{3K+C_t}$; C_t is the shear modulus, $C_t=\frac{E}{2(1+\nu)}$; K is the modulus of the volume compression, $K=E/3(1-2\nu)$.

For one's turn

$$J_0=\iint_S y^2dydz,\qquad J_2=\iint_S y^4dydz,$$

$$a_3=-\frac{2}{9}\frac{3K}{3K+C_t}\cdot\frac{\gamma_2}{G^2},\qquad \gamma_2=-g_2,$$

where γ_2 is the non-linearity parameter, and J_0 and J_2 are the inertion moments of the axial cross.

Taking into account (2) in (1) we obtain the solvable equation of the non-linear motion of the bioelastic bar of the form:

$$(1-A)EJ_0\frac{\partial^2}{\partial x^2}\left[\frac{\partial^2\eta}{\partial x^2}-a_3E^2\frac{J_2}{J_0}\left(\frac{\partial^2\eta}{\partial x^2}\right)^3\right]+$$
$$+A\tau EJ_0\frac{\partial^2}{\partial x\partial t}\left[\frac{\partial^3\eta}{\partial x^3}-3a_3E^2\frac{J_2}{J_0}\left(\frac{\partial^2\eta}{\partial x^2}\right)^2\cdot\frac{\partial^3\eta}{\partial x^3}\right]=-\rho F\frac{\partial^2\eta}{\partial t^2}+q(x,t). \quad (3)$$

By (3) the problem on bending oscillations of the bar with the finite length whose one edge is free off the forces, and the other is subjected to the across oscillations with the given amplitude and the frequency. The boundary conditions have the form:

$$\eta(0,t)=U_0\cos\omega t; \qquad \frac{\partial^2\eta(0,t)}{\partial x^2}=0;$$
$$\frac{\partial^2\eta(l,t)}{\partial x^2}=0; \qquad \frac{\partial^3\eta(l,t)}{\partial x^3}=0. \quad (4)$$

Here U_0 is the given amplitude of the across oscillations of the left edge, ω is the given frequency of this oscillations.

As the experiments show the non-linearity parameter γ_2 or its aggregate combination is the number less than unit and can be considered as the small parameter that let represent $a_3=a_3'\lambda$, where λ is the small parameter. The form of the law (2) obtained by the experiments confirms that. It gives the possibility for search of the solution of the problem to use the small parameter method.

Disintegrate the sought function $\eta(x,t)$ into the series by the non-linearity small parameter

$$\eta(x,t)=\sum_m\lambda^m\eta_m(x,t). \quad (5)$$

Considering decomposition of (5) in (3) and comparing the terms of the same order infinitesimal for the elastic material of the bar we will obtain the system of the reccurent differential equations of the form:

$$L\eta_m(x,t)=f_m(\eta_0,\dots,\eta_{m-1}) \quad m=0,1,2,\dots, \quad (6)$$

where operator L has the following representation:

$$L=(1-A)EJ_0\frac{\partial^4}{\partial x^4}+A\tau EJ_0\frac{\partial^5}{\partial x^4\partial t}+\rho E\frac{\partial^2}{\partial t^2}-q(x,t). \quad (7)$$

Then for the first two approximations:

$$f_0=0,$$
$$f_1=-6a_3'E^3J_2(1-A)\frac{\partial^2\eta_0}{\partial x^2}\left(\frac{\partial^3\eta_0}{\partial x^3}\right)^2-3a_3'E^3J_2(1-A)\frac{\partial^4\eta_0}{\partial x^4}\left(\frac{\partial^2\eta_0}{\partial x^2}\right)^2-$$
$$-6a_3'E^3J_2A\tau\frac{\partial^2\eta_0}{\partial x^2\partial t}\left(\frac{\partial^3\eta_0}{\partial x^3}\right)^2-12a_3'E^3J_2A\tau\frac{\partial^2\eta_0}{\partial x^2}\cdot\frac{\partial^3\eta_0}{\partial x^3}\cdot\frac{\partial^4\eta_0}{\partial x^3\partial t}-$$
$$-6a_3'E^3J_2A\tau\frac{\partial^2\eta_0}{\partial x^2}\cdot\frac{\partial^3\eta_0}{\partial x^2\partial t}\cdot\frac{\partial^4\eta_0}{\partial x^4}-6a_3'E^3J_2A\tau\left(\frac{\partial^2\eta_0}{\partial x2}\right)^2\cdot\frac{\partial^5\eta_0}{\partial x^4\partial t}. \quad (8)$$

Decomposition (5) allowed to decompose the boundary conditions (4):

$$\begin{cases} \eta_0(0,t)=\vartheta_0\cos\omega t\,; & \dfrac{\partial^2\eta_0(0,t)}{\partial x^2}=0 \\ \dfrac{\partial^2\eta_0(l,t)}{\partial x^2}=0\,; & \dfrac{\partial^3\eta_0(l,t)}{\partial x^3}=0 \end{cases}, \tag{9}$$

$$\begin{cases} \eta_m(0,t)=0\,; & \dfrac{\partial^2\eta_m(0,t)}{\partial x^2}=0 \\ \partial^2\eta_m(l,t)=0\,; & \dfrac{\partial^3\eta_m(l,t)}{\partial x^3}=0 \end{cases}. \tag{10}$$

Therefore, the system of boundary-value problems (6)-(9), (10) has been obtained.

The zero and first approximations are determined in the form:

$$\eta_m(x,t)=\sum_k\left[V_{mk}(x,\omega)e^{-ik\omega t}+V_{mk}\overline{(x,\omega)}e^{ik\omega t}\right]. \tag{11}$$

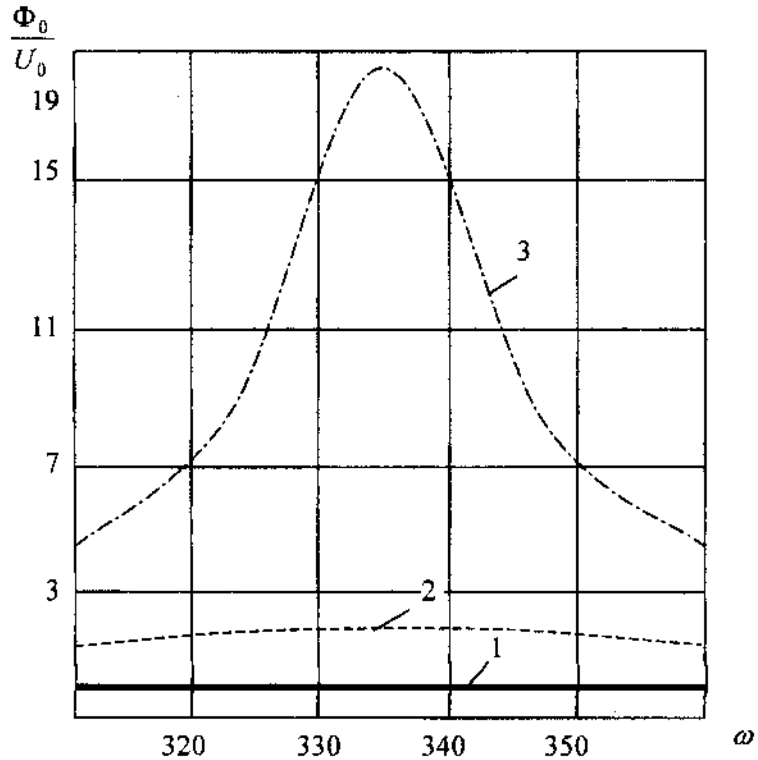

Fig. 1. Influence of the biofactor (the zeroth approximation).
$\eta=10^{-3}$, $1-\tau=10^{-2}$, $2-\tau=10^{-3}$, $3-\tau=10^{-4}$.

Let us reduce the obtained form of the solution for the first two approximations

$$\eta_0(x,t)=\Phi_0(x,\omega)\cos[\omega t-\psi_0(x,\omega)],$$

$$\eta_1(x,t)=\Phi_1(x,\omega)\cos[\omega t-\psi_1(x,\omega)]+\Phi_2(x,\omega)\cos[3\omega t-\psi_2(x,\omega)], \tag{12}$$

where the amplitude $\Phi_m(x,\omega)$ and the initial phase of oscillations $\psi_m(x,\omega)$ represent the analytical closed expressions determined by the parameters of the biofactor (A and τ) whose forms are not reduced here because of their awkwardness.

The representations (24) show that the non-linearity reduces to the excitation of higher harmonics and to the change of the primary sinusoidal profile of the wave.

With purpose to make out the influence of the biofactor and viscosity of the material on the process of bending oscillations the functions of amplitude $\Phi_m(x,\omega)$ have been calculated in dependence on frequency ω for the free end of the bar $x=l$ for different values of the parameter of the biofactor- lag time of reaction τ. These numerical data are represented in the form of the corresponding amplitude-frequency curve. In fig. 1, 2 the amplitude-frequency curve for the zeroth and second approximations in elastic case for three values of the parameter of lag of reaction $\tau\left(1-\tau=10^{-2};\,2-\tau=10^{-3};\,3-\tau=10^{-4}\right)$. It is discovered, that with decrease of parameter of lag of the reaction of amplitude-frequency the curves sharply rinse right up to disappearance of the resonance peaks, moreover the shift of main resonance frequency to the least value happens.

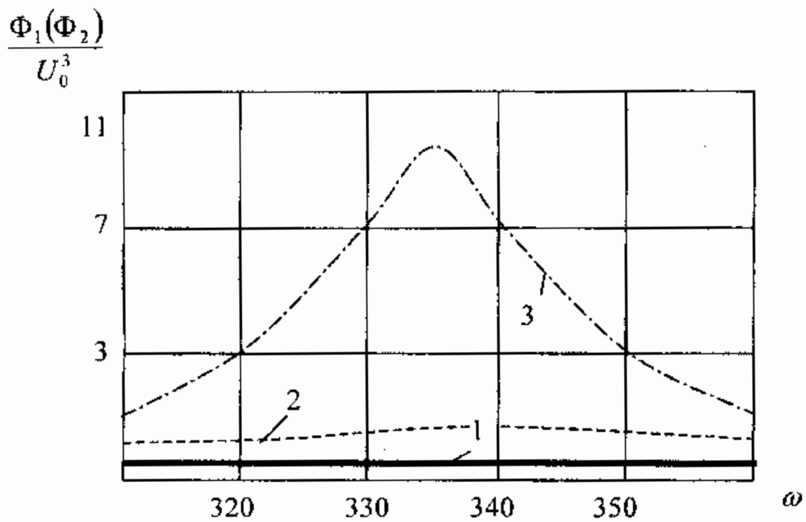

Fig. 2. Influence of the biofactor (the first approximation).

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