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IMPACT BY CONE SLACKENED BY THE HOLE

Abstract

In the paper the non-stressed thin membrane with the hole with the given radius is investigated in condition of rest. To the contour of the hole at initial moment equal to zero the uniform distributed impact loading is applied.

The dependences characterising propagation of strong gap; the position of the membrane at three moments of time is described.

The non-stressed thin membrane is considered which is in condition of rest and taking in plane XOZ the area external to the circle hole of the membrane with radius R. At moment $t = 0$ uniform distributed impact loading is applied to the contour of the hole and that causes initial jump of velocity v_0 of contour. During some time this contour under action of loading moves translational in the direction of axis OY expansing by some given law.

In such formulation the problem has symmetry with respect to axis OY and that let consider motion of the membrane only in the meridional section XOY (fig.1).

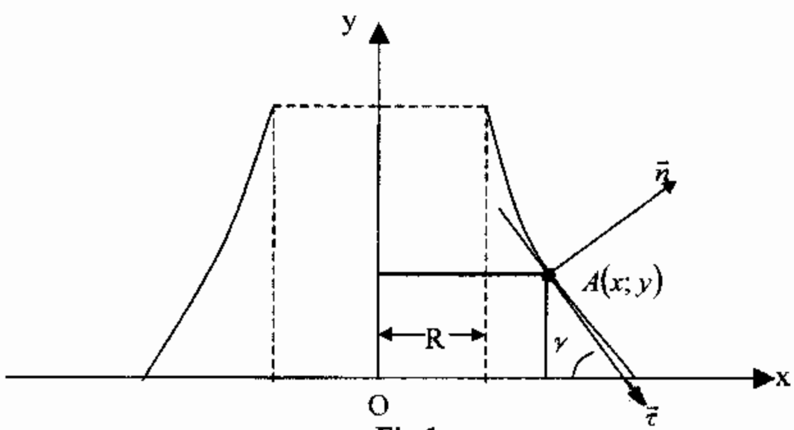

Fig.1

As independent variables we take time t and Lagrange coordinate r counted from the center of the hole; u, w are projections of vector of velocity on the direction of tangent $\vec{\tau}$ and normal $\vec{n}$ to the contour of the meridional section of the membrane in point $A(x;y)$; γ is angle between the tangent $\vec{\tau}$ in this point and axis Ox $\varepsilon_r(r,t)$ and $\varepsilon_\theta(r,t)$ are meridional and circle components of tensor of deformation; $\sigma_r(r,t)$ and $\sigma_\theta(r,t)$ are the corresponding stresses.

In general formulation of the problem the motion of the membrane happens in three different areas denoted as area I, area II, area III (fig.2).

In area I the membrane is in complete contact with the cone and in this area following equations have place [1]:

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(\delta_r \cdot r\right) - \frac{\sigma_\theta}{r}\sin\alpha , \qquad (1)$$

$$\frac{\partial \varepsilon_r}{\partial t} = \frac{\partial u}{\partial r} - 1, \tag{2}$$

$$\frac{\partial \varepsilon_\theta}{\partial t} = \frac{u}{r}\sin\alpha - 1. \tag{3}$$

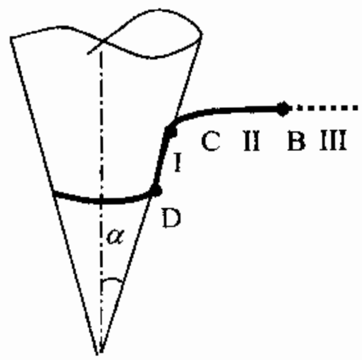

Fig.2.

Here and further ρ_0 is initial density of the membrane, α is angle of the halfmixture of the cone.

Taking into account (2) and (3) in (1) and introducing the new variable

$$\vartheta(r,t) = u(r,t) - \frac{1+\nu}{1+\sin\alpha} r$$

from (1) we obtain:

$$\frac{1}{a_0^2}\frac{\partial^2 \vartheta}{\partial t^2} = \frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r} - \frac{\vartheta}{r^2}\sin^2\alpha. \tag{1`}$$

In area II motion of the membrane is meridional-lateral and free off the impacting cone. In this area following equations have place [2]:

$$\rho_0\left(\frac{\partial u}{\partial t} - w\frac{\partial \gamma}{\partial t}\right) = \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta \cos\gamma}{r}, \tag{4}$$

$$\rho_0\left(\frac{\partial w}{\partial t} + u\frac{\partial \gamma}{\partial t}\right) = \sigma_r\frac{\partial \gamma}{\partial r} + \frac{\sigma_r \sin\gamma}{r}, \tag{5}$$

$$\frac{\partial u}{\partial r} - w\frac{\partial \gamma}{\partial r} = \frac{\partial \varepsilon_r}{\partial t}, \tag{6}$$

$$\frac{\partial w}{\partial r} + u\frac{\partial \gamma}{\partial r} = (1+\varepsilon_r)\frac{\partial \gamma}{\partial t}, \tag{7}$$

$$\frac{\partial \varepsilon_\theta}{\partial t} = \frac{1}{r}(u\cos\gamma - w\sin\gamma). \tag{8}$$

In area III motion of particles of the membrane is poor meridional and the equations of motion have the form:

$$\rho_0\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r}, \tag{9}$$

$$\frac{\partial \varepsilon_r}{\partial t} = \frac{\partial u}{\partial r}, \tag{10}$$

$$\frac{\partial \varepsilon_\theta}{\partial t} = \frac{u}{r}. \tag{11}$$

The systems of equations (1)-(3) and (9)-(11) must be closed by the equation of condition. In this work it is supposed that in all areas the motion of deformation of the membrane happens by Hook's linear-elastic law:

$$\sigma_r = \frac{E}{1-\nu^2}(\varepsilon_r + \nu\varepsilon_\theta),\ \sigma_\theta = \frac{E}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_r), \tag{12}$$

where E is Young's modulus, ν is Poisson's coefficient.

On the left hand side bond of area I, i.e. for $r = R$ free expensing membrane we have:

$$\sigma_r(R,t) = 0\,. \tag{13}$$

or

$$\varepsilon_r(R,t) + \nu\varepsilon_\theta(R,t) = 0\,. \tag{14}$$

Equation (14) can be written in the form:

$$\frac{\partial\vartheta(R,t)}{\partial r} = (1-\nu^2)\frac{\sin\alpha}{1+\sin\alpha} - \nu\sin\alpha\frac{\vartheta}{R}\,. \tag{15}$$

In [5] it was proved that the characteristics of equations (4)-(8) are:

$$\lambda_{1,2} = \pm a_0,\ \lambda_{3,4} = \pm\sqrt{\frac{\sigma_r}{\rho_0(1+\varepsilon_r)}},\ \lambda_5 = 0\,, \tag{16}$$

and the characteristic conditions equivalent to equations (4)-(8) have the form:

$$du - \frac{1}{\rho_0\lambda}d\sigma_r - wd\gamma = \frac{\sigma_r - \sigma_\theta\cos\gamma}{\rho_0 r}dt - \frac{\lambda\nu}{r}(u\cos\gamma - w\sin\gamma)dt,\ \lambda = \pm a_0\,, \tag{17}$$

$$dw + [u - (1+\varepsilon_r)\lambda]d\gamma = \frac{\sigma_\theta}{\rho_0 r}\sin\gamma dt, \quad \lambda = \pm b\,, \tag{18}$$

$$d\varepsilon_\theta = \frac{1}{r}(u\cos\gamma - w\sin\gamma)dt, \quad \lambda = 0\,. \tag{19}$$

In this work it has been established that the characteristics of systems (1)-(3) and (9)-(11) are only $\lambda = \pm a_0$ and $\lambda = 0$ and these systems pass correspondingly to following characteristic conditions:

$$d\vartheta_t = \pm a_0 d\vartheta_r + a dt, \quad \lambda = \pm a_0\,, \tag{20}$$

$$d\varepsilon_\theta = \left(\frac{\vartheta}{r}\sin\alpha + \frac{\nu\sin\alpha - 1}{1+\sin\alpha}\right)dt, \quad \lambda = 0\,, \tag{21}$$

where

$$a = a_0^2\left(\frac{1}{r}\vartheta_r - \frac{\vartheta}{r^2}\sin^2\alpha\right)$$

and

$$du - \frac{1}{\rho_0\lambda}d\sigma_r = \frac{\sigma_r - \sigma_\theta}{\rho_0 r}dt - \lambda\nu\frac{u}{r}dt,\ \lambda = \pm a_0\,, \tag{22}$$

$$d\varepsilon_\theta = \frac{u}{r}dt,\ \lambda = 0\,. \tag{23}$$

Moreover, the dimensionless quantities are used

$$\bar{t} = \frac{a_0 t}{R},\ \bar{r} = \frac{r}{R}, \bar{u} = \frac{u}{a_0}, \bar{w} = \frac{w}{a_0}, \bar{b} = \frac{b}{a_0}, \bar{c} = \frac{c}{a_0}\,.$$

Let's write out finally all correlations for determination of dimensionless quantities $u, w, \varepsilon_r, \varepsilon_\theta, \gamma$ (omit dash sign over the dimensionless quantities):

In area II

$$du \mp \left(d\varepsilon_r + \nu d\varepsilon_\theta\right) - w d\gamma =$$
$$= \frac{1}{r}\left[(1-\nu\cos\gamma)\varepsilon_r + (\nu-\cos\gamma)\varepsilon_\theta\right]dt \mp \frac{\nu}{r}(u\cos\gamma - w\sin\gamma)dt,\ dr = \pm dt\,, \tag{24}$$

$$dw + \left[u \mp \sqrt{(\varepsilon_r + \nu\varepsilon_\theta)(1+\varepsilon_r)}\right]d\gamma = \frac{1}{r}(\varepsilon_\theta + \nu\varepsilon_r)\sin\gamma dt,\ dr = \pm b dt\,, \tag{25}$$

$$d\varepsilon_\theta = \frac{1}{r}(u\cos\gamma - w\sin\gamma)dt\,. \tag{26}$$

In area I

$$du \mp \left(d\varepsilon_r + \nu d\varepsilon_\theta\right) = \frac{1}{r}\left[(1-\nu\sin\alpha)\varepsilon_r + (\nu - \sin\alpha)\varepsilon_\theta\right]dt \mp \frac{\nu u}{r}\sin\alpha dt,\ \ dr = \pm dt\,, \tag{27}$$

$$d\varepsilon_\theta = \frac{u}{r}\sin\alpha dt\,. \tag{28}$$

In area III

$$d\vartheta_t = \pm d\vartheta_r + \left(\frac{1}{r}\vartheta_r - \frac{\vartheta}{r^2}\sin^2\alpha\right)dt,\ \ dr = \pm dt\,, \tag{29}$$

$$d\varepsilon_\theta = \left(\frac{\vartheta}{r}\sin\alpha + \frac{\nu\sin\alpha - 1}{1+\sin\alpha}\right)dt\,. \tag{30}$$

At numerical solving the equations (17)-(23) and these equations along the characteristics (along $dr = \pm dt$ and $dr = \pm b dt$) are changed by finite differ equations on the net formed by the characteristics with step δ by the independent variables (fig.3).

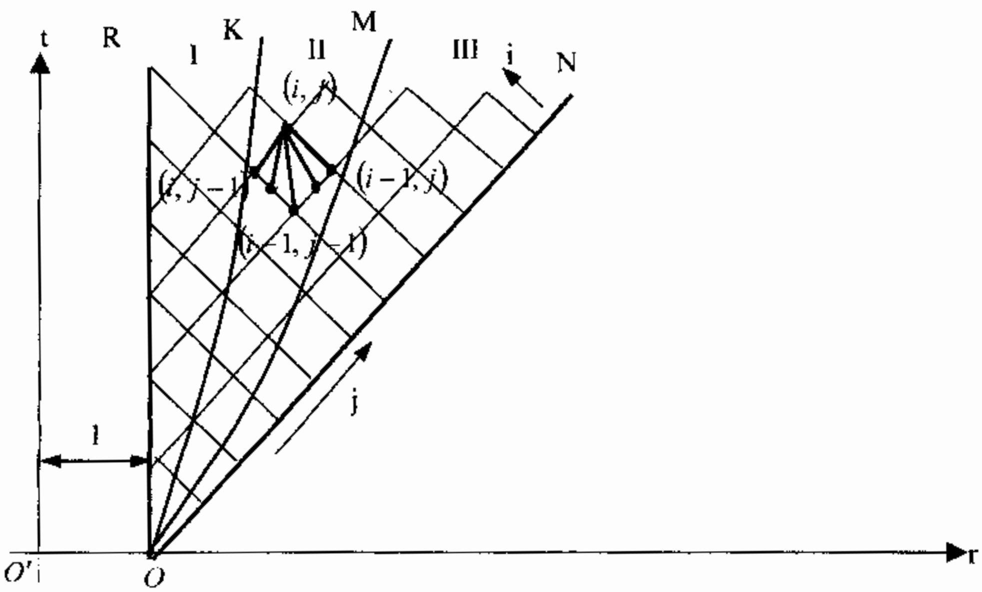

Fig.3.

Nodes of the net are determined by the coordinates:

$$r_{i,j} = 1 + (j-i)\delta,\ t_{i,j} = (i+j)\delta,\ i, j = 0,1,2,3,\ldots, i \le j\,.$$

On "IBM-486" calculation of one variant of the problem for $\nu = 0{,}25$ has been carried out.

Some results of calculation are represented in the form of graphs in fig.4-6.

In fig.4 the graphs are constructed which characterise propagation of main waves of strong gap. Both waves are propagated with variable velocity weak retarded. During some time intensivity of these waves decrease. The velocity of propagation of longitudinal waves are more than velocity of propagation of transversal waves for nearly for times.

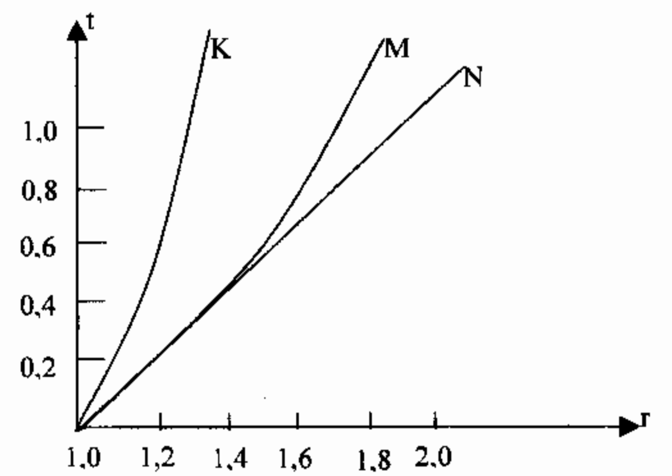

Fig.4.

In fig.5 the position of the membrane is represented at three moments of time.

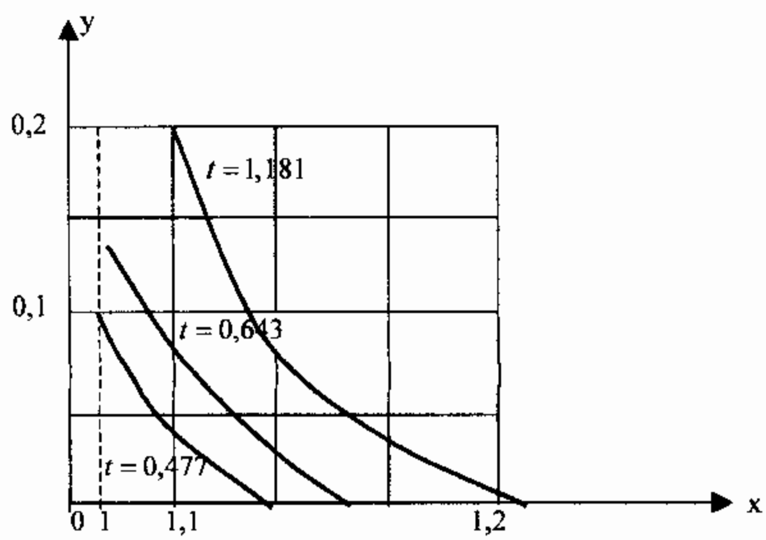

Fig.5.

In fig.6 the components of tensor of deformation $\varepsilon_r, \varepsilon_\theta$ and angle γ are represented for the point of the contour of the hole as function of time.

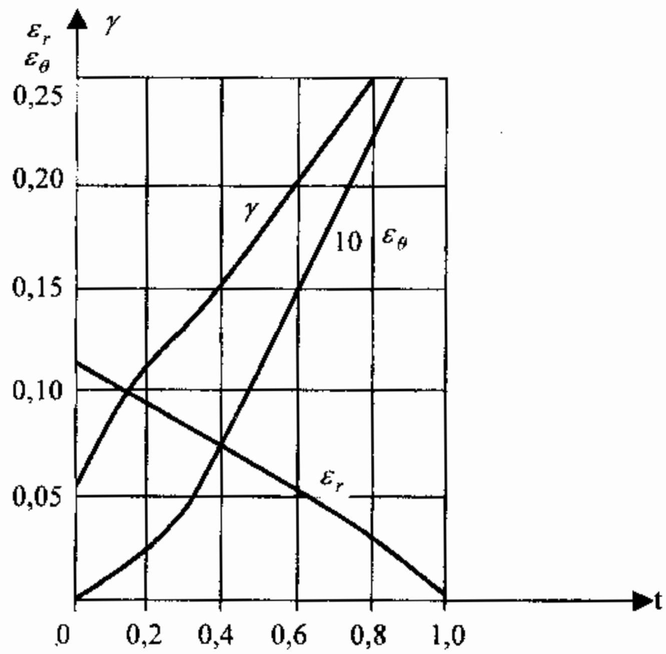

Fig.6

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