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INFLUENCE OF RIGIDITY OF THE SPRING ON MOTION OF A SPHERICAL INCLUSION IN THE COMPRESSIBLE FLUID

Abstract

Motion inside of a spherical spring mass in a compressible fluid in acoustic formulation is investigated in the paper. In dependence on the parameters of the problem the graph of pressure is constructed.

In [1] the problem on the spherical inclusion with springed mass moving in the compressible medium under action of waves was considered. The problem was solved in acoustic formulation. In present work the quality and quantity investigation of character of motion of the inclusion is carried out. Vortex-free motion of the medium in acoustic formulation is described by the equation

$$\Delta\varphi = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}, \tag{1}$$

where a is velocity of sound propagation, φ is velocity potential, $\bar{\vartheta} = grad\varphi$, $\bar{\vartheta}$ is vector of velocity, Δ is Laplacian operator.

After wave going if we neglect the passing phenomena (diffraction), the medium moving in one direction with known velocity of initial moment surrounds the moveless inclusion. According to the relativity principle we can consider the medium moveless and give the velocity of the fluid to the inclusion.

The inclusion moves by the law:

$$\begin{aligned} M_1\frac{d^2x_1}{dt^2} &= P + L(x_2 - x_1)\,, \\ M_2\frac{d^2x_2}{dt^2} &= -L(x_2 - x_1)\,, \end{aligned} \tag{2}$$

where M_1 is mass of the holder, M_2 mass of the springed body, x_2 is displacement of the springed body, x_1 is displacement of the holder, L is rigidity of the spring, P is force of action of the fluid to the inclusion.

Therefore, the problem for equation (1) is considered with boundary conditions (2) and initial conditions

$$t = 0\,;\;\; \varphi = 0\,;\;\; \frac{\partial\varphi}{\partial t} = 0\,. \tag{3}$$

After Laplace-Karlson transformation in representations it will be

$$r^2\bar{\varphi}_1'' + 2r\bar{\varphi}_1' - \left(2 + \frac{p^2}{a^2}r^2\right)\bar{\varphi}_1 = 0\,. \tag{4}$$

Solution of (4) under the condition of restriction at infinity has the form

$$\bar{\varphi}_1 = c\frac{1}{r^2}\left(\frac{p}{a}r + 1\right)e^{-\frac{p}{a}r}, \tag{5}$$

where

$$c = \dot{x}_0\left[M_1L + M_2L + M_1M_2p^2\right] \Bigg/ \frac{e^{-\frac{p}{a}r_0}}{r_0}\left[\left(M_1L + M_1M_2p^2 + M_2L\right)\times\right.$$
$$\left.\times\left(-\frac{p^2}{a^2} - \frac{2p}{ar_0} - \frac{2}{r_0}\right) + \left(L + M_2p^2\right)\left(\frac{4\pi}{3}p r_0^2\frac{p}{a} + \frac{4\pi}{3}pr_0^2\right)\right]. \tag{5'}$$

For finding out the quality picture of the process let's consider the case when mass of the shell is much less than mass of the inclusion and we can neglect it in (5'). Then by the first of equations (2) the internal and external forces with respect to the shell will be balanced.

Omitting the components with M_1 in (5') we obtain:

$$c = \frac{r_0\dot{x}_0Le^{\frac{p}{a}r_0}}{\frac{M_*}{ar_0}\left[p^3 + \left(\frac{3a}{r_0} - \frac{Lr_0}{M_*a}\right)p^2 + L\left(\frac{1}{M_2} - \frac{2}{M_*}\right)p + \frac{La}{r_0}\left(\frac{1}{M_2} - \frac{1}{M_*}\right)\right]}, \tag{6}$$

where $M_* = \frac{4}{3}\pi\rho r_0^3$.

Introduce the denotations:

$$\begin{cases} Q = -\frac{1}{27}\left(\frac{\left(\frac{3a}{r_0} - \frac{Lr_0}{M_*a}\right)^2}{3} + L\left(\frac{1}{M_2} - \frac{1}{M_*}\right)\right)^3 + \frac{1}{4}\left(\frac{2}{27}\left(\frac{3a}{r_0} - \frac{Lr_0}{M_*a}\right)^3 - \right. \\ \left. -\frac{L}{3}\left(\frac{3a}{r_0} - \frac{Lr_0}{M_*a}\right)\left(\frac{1}{M_2} - \frac{1}{M_*}\right) + \frac{La}{r_0}\left(\frac{1}{M_2} - \frac{1}{M_*}\right)\right)^2, \\ \Phi = -\frac{\left(\frac{3a}{r_0} - \frac{Lr_0}{M_*a}\right)^2}{3} + L\left(\frac{1}{M_2} - \frac{1}{M_*}\right). \end{cases} \tag{7}$$

Analysis shows that for conditions $Q \ge 0$, $\Phi > 0$ and $Q \ge 0$, $\Phi < 0$, *motion* of the inclusion has the character of damping oscillations and for condition $Q > 0$, $\Phi < 0$ motion of the inclusion is not oscillation.

Let's introduce following parameters of the problem:

$$\omega_1 = \frac{a}{r_0},\ \omega = \sqrt{\frac{L}{M_2}},\ \omega_2 = \sqrt{\frac{L}{M_*}}.$$

Taking into account (6) in (5) and using the formula for motion $\mathcal{P}$ the following approximate expression for its representation is found:

$$\frac{p\left(p + \frac{a}{r}\right)}{\left[\left(p + c_1 + 0{,}5F\right)^2 + 0{,}75F^2\right]\left(p + c_1 - F\right)}. \tag{8}$$

Its original has the form

$$\rightarrow \frac{c_1 - F - \frac{a}{r}}{3F^2} e^{-(c_1+0,5F)t} \cos\sqrt{0,75}F \cdot t + \frac{1}{\sqrt{0,75}F}\left[1 + \frac{c_1 - F - \frac{a}{r}}{2F}\right] \times$$

$$\times e^{-(c_1+0,5F)t} \sin\sqrt{0,75}F \cdot t - \frac{c_1 - F - \frac{a}{r}}{3F^2} e^{-(c_1-F)t}, \tag{9}$$

where $c_1 = \omega_1 - \frac{\omega_2^2}{3\omega_1}$, $F = \sqrt[3]{\left(\omega_1 - \frac{\omega_2^2}{3\omega_1}\right)^3 - \omega_1\left(\left(\omega^2 + 2\omega_2^2\right)\right)}$.

In figures the graphs of pressure p change are demonstrated as on the surface of the shell $r = r_0$, as on some distance from it. Calculations have been carried out for following values of the parameters

$\dot{x}_0 = 10$ m/sec $\qquad \omega_2 = 10$ hs

$a = 1400$ m/sec^2 $\qquad r_0 = 5$ m

The curve corresponds to in fig.1: $r = r_0$; in fig.2: $r = 2r_0$; in fig.3: $r = 3r_0$.

As it is seen from the graphs, motion of the inclusion has damping character. With increase of r oscillations damp quicklier.

$r = r_0$

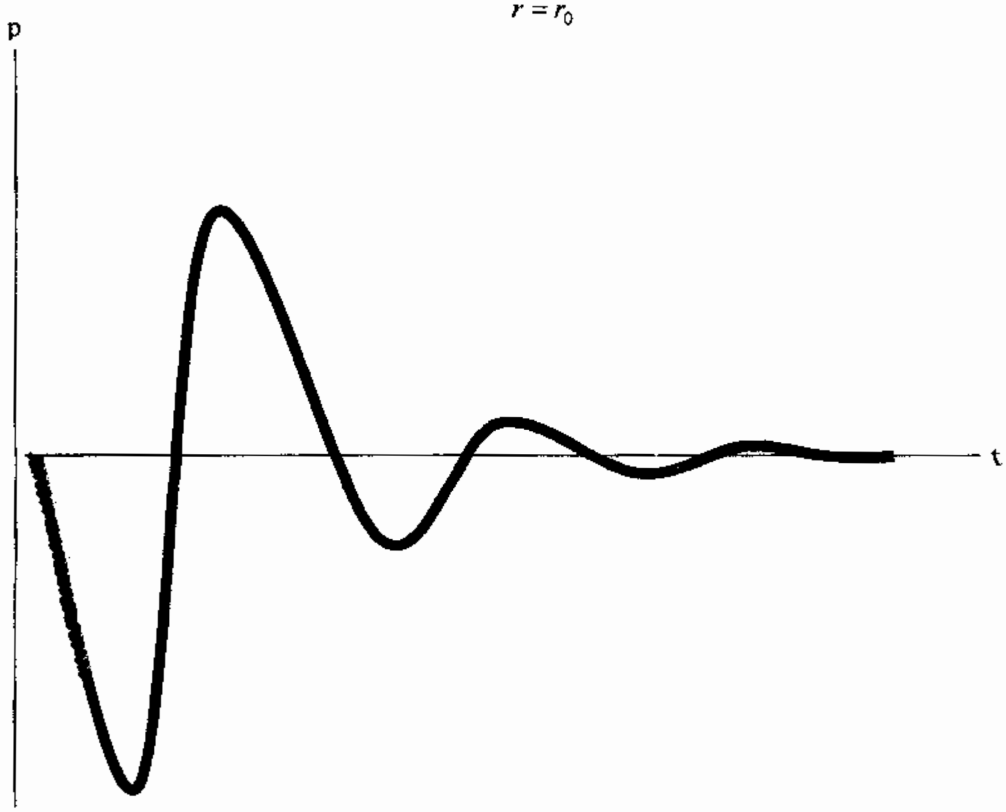

Fig. 1.

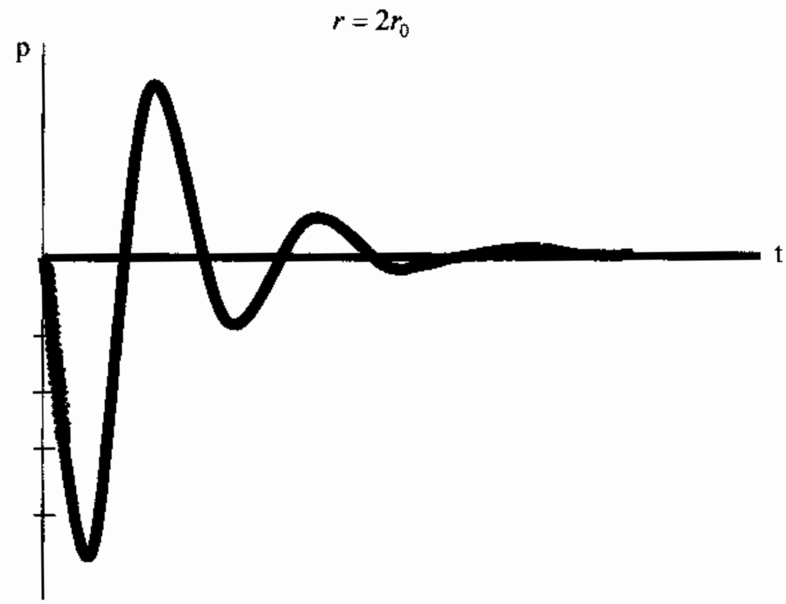

Fig. 2.

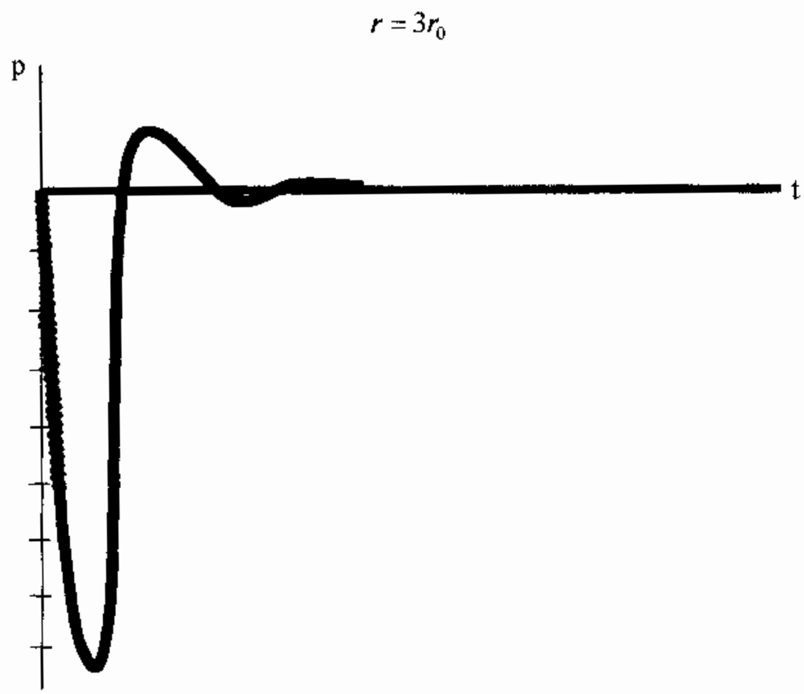

Fig. 3.

Reference

[1]. Сейфуллаев А.И., Агаева Н.А. *Решение задачи о движении сферического включения с подпружиненной массой в акустической среде.* Изв. ИММ АН Азербайджана, т.XVIII, №2, Баку, «Элм», 1998, с.222-224.

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