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# SCATTERED FAILURE OF THE THICK PIPE EXTERNAL ARMOURED WITH ELASTIC SHELL

**Abstract**

*On the base of conception of damage accumulation in the body at its loading the scattered failure process of the cylindrical transversal isotrop thick pipe covered with thin elastic shell is investigated. The mathematical model of the process is constructed on the base of which is the system of non-linear integral equations with respect to the variable of radial coordinate of failure front and pressure on the contact of the pipe with the shell. The numerical analogues are constructed as for incubation as for main period. The carried out calculations let make out the influence on the parameters of failure process of the relation of rigidity of the pipe in the circle and radial directions and also the parameters of singularity of the kernel of damaging operator.*

The considered problem is related to the known problem on failure of Lee and Radock solid rocket fuel [1]. The thick-walled cylinder fabricated from the material is investigated which is exposed to creeping and arranged in a thin metallic shell. Pressure $P_t$ is applied in the inner surface and changes by time. The inner radius is the given function of time- material burns out. In assumption that the pipe and the shell are in the conditions of plane deformation and the material of the pipe is non-compressible the solution is reduced to the integral equation for the function connected with radial displacement of the pipe. In [2] this problem was solved on the base of the author model of damaging body. It was reduced to the pair of integral equations and also allowed to determine the principle of change of the inner radius of the pipe. In the present work the similar problem is investigated for cylindrical ortotrop material of the pipe.

We will suppose that in the material of the pipe the damage accumulation process happens and the deformation correlations are determined by the formulas of [3]. As the failure criterion we take the criterion by maximal tensile stress which is the circle stress:

$$\sigma_\theta + M^*\sigma_\theta = \sigma_0 , \qquad (1)$$

where $M^*$ damage operator of hereditary type, $\sigma_0$ strength of the defectless material.

The picture of failure represents following. At moment of time $t_0$ determined from condition (1) in the inner surface of the cylinder where the value of circle stress $\sigma_\theta$ achieves maximum the failure process begins which will occupy further other new layers. Finally the circle failure zone is formed (by virtue of axial symmetry of the problem) expanding to the external bound of the pipe sealed with elastic shell (fig.1). Then on the bound of separation of failed and imfailed areas on the failure front the condition is always fulfilled

$$\sigma_r = -P(t) \qquad (2)$$

on the contact surface of the pipe with shell

$$\varepsilon_\theta(b) = -\gamma\sigma_r(b), \qquad (3)$$

where $b$ is the external radius of the pipe. And

$$\varepsilon_\theta(b) = \frac{u_r(b)}{b}; \qquad (4)$$

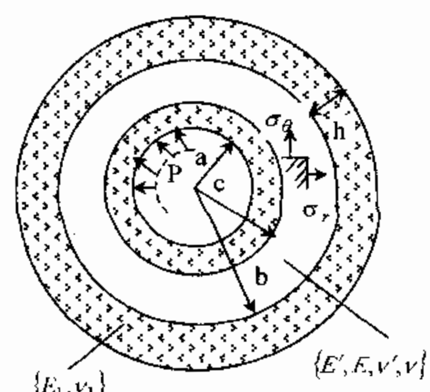


Fig.1.

$$\varepsilon_\theta(b) = \frac{u_r(b)}{b}; \tag{5}$$

$$\gamma = \frac{b}{h}\frac{1-\nu_1^2}{E_1}. \tag{6}$$

Here $u_r$ is radial displacement of points of the pipe, $\sigma_r$ is radial stress; $q$ is contact pressure on the joint of the pipe with shell, $h$ is thickness of the shell; $E_1$ and $\nu_1$ are Young's modulus and Poisson's coefficient of the material of shell. Using the representations for stress and displacement for cylindrical ortotrop elastic pipe [4] and taking that the damaging process in contrast to deformation process is isotrop we have from conditions (2) and (3) for incubation $0 \le t \le t_0$ the following system of one non-linear integral equation and one algebraic equation with respect to contact pressure $q(t)$ and continuation $t_0$ of incubation:

$$\begin{cases} \psi(\beta_0, q(t)) + \int\limits_0^t M(t-\tau)\psi(\beta_0, q(\tau))d\tau = \gamma q(t) \\ f(\beta_0, q(t)) + \int\limits_0^{t_0} M(t_0-\tau)\varphi(\beta_0, \beta_0, q(t_0)q(\tau))d\tau = \sigma_0 \end{cases} \tag{7}$$

Here $\beta_0 = \frac{a}{b}$, where $a$ is the initial inner radius of the pipe; $M(t-\tau)$ is kernel of damaging operator.

Expansion process of failure zone connected with motion of failure front is described by following system of two non-linear integral equations with respect to contact pressure $q(t)$ and dimensionless radius of failure front $\beta(t) = c(t)/b$:

$$\begin{cases} \psi(\beta(t), q(t)) + \int\limits_0^t M(t-\tau)\psi(\beta(t), q(\tau))d\tau = \gamma q(\tau) \\ f(\beta(t), q(t)) + \int\limits_0^{t_0} M(t-\tau)\varphi(\beta(t), \beta(\tau), q(t)q(\tau))d\tau = \sigma_0 \end{cases} \tag{8}$$

Here $\beta(t) = \beta_0$ for $0 \le t \le t_0$.

In system (7) and (8):

$$\psi(\beta(\tau), q(\tau)) = (\beta_{12} + k\beta_{22})\frac{P(\tau)\beta^{k+1}(\tau) - q(\tau)}{1-\beta^{2k}(\tau)} + \\ + (\beta_{12} - k\beta_{22})\frac{q(\tau)\beta^{k-1}(\tau) - P(\tau)}{1-\beta^{2k}(\tau)}\beta^{k+1}(r), \tag{9}$$

$$\varphi(\beta(t), \beta(\tau), q(t), q(\tau)) = \\ = \frac{k}{1-\beta^{2k}(\tau)}\left\{\left[P(\tau)\beta^{k+1}(\tau) - q(\tau)\right]\beta^{k-1}(t) - \left[q(\tau)\beta^{k-1}(\tau) - P(\tau)\right]\left(\frac{\beta(\tau)}{\beta(t)}\right)^{k-1}\right\}. \tag{10}$$

For the considered form of cylindrical transversal isotropy:

$$k^2 = \frac{\beta_{11}}{\beta_{22}};\ \beta_{11} = \frac{1}{E'}\left(1 - \nu'^2\frac{E}{E'}\right);\ \beta_{22} = \frac{1-\nu^2}{E}; \\ \beta_{12} = -\frac{\nu'(1+\nu)}{E'};\ E' = E_r;\ E = E_\theta = E_z \tag{11}$$

For solution of systems (7) and (8) numerical method was used. For that system (7) was substituted by the numerical analogue:

$$Q_n = \frac{(\chi-1)\beta_0^{k+1} + R_n\left(1-\beta_0^{2k}\right)}{\eta+\chi-(1+\eta)\beta_0^{2k}}, \tag{12}$$

$$R_n = h\sum_{i=0}^{n-1} M(t_n - t_i)\frac{(\chi-1)\beta_0^{k+1} + Q_i\left(\beta_0^{2k}-\chi\right)}{1-\beta_0^{2k}};\ R_0 = 0\ , \tag{13}$$

$$Q_n > 0 \tag{14}$$

$$\left|\frac{1+\beta_0^{2k}}{1-\beta_0^{2k}} - \frac{2\beta_0^{k-1}}{1-\beta_0^{2k}}Q_n + L_n - G\right| < \varepsilon, \tag{15}$$

$$L_n = h\sum_{i=0}^{n-1} M(t_n - t_i)\frac{1+\beta_0^{2k} - 2\beta_0^{k-1}Q_i}{1-\beta_0^{2k}}, \tag{16}$$

where the dimensionless quantities were used:

$$Q = \frac{q}{p};\ G = \frac{\sigma_0}{kp};\ \tilde{T} = \frac{t}{T}\ \ \text{(sign of tilde in (13) and (16) is omitted)}$$

$$\eta = \frac{\gamma}{\beta_{12} - k\beta_{22}};\ \chi = \frac{\beta_{12} + k\beta_{22}}{\beta_{12} - k\beta_{22}}; \tag{17}$$

First, contact pressure $Q_n$ is determined on each step $t_n = nh$ by the given initial $\beta_0$ and calculated on the foregoing stages $Q_i$, (13). Then conditions (14), (15) are checked. If condition (14) is not fulfilled then calculation process is stopped or the condition $Q_n < 0$ means peeling of the pipe from the shell. If condition (14) is fulfilled then calculation process is continued until condition (5) is satisfied, number $n_*$, which just determines the continuity of incubation $t_0 = t_{n_*} = n_* h$. After that it should pass to solution of system (8) whose numerical analogue is following:

$$Q_n = \frac{(\chi-1)\beta_n^{k+1} + R_{n,i}\left(1-\beta_n^{2k}\right)}{\eta+\chi-(1+\eta)\beta_n^{2k}}, \tag{18}$$

$$R_{n,i} = h\sum_{i=0}^{n-1} M(t_n - t_i)\frac{(\chi-1)\beta_i^{k+1} + Q_i\left(\beta_i^{2k}-\chi\right)}{1-\beta_i^{2k}}, \tag{19}$$

$$\varphi(\beta_n,\beta_i) = \frac{1}{\beta_n^{k+1}\left(1-\beta_i^{2k}\right)}\times$$
$$\times\left\{\beta_n^{k+1}\left(1+\beta_n^{2k}\right) - \left(\beta_n^{2k}+\beta_i^{2k}\right)\frac{(\chi-1)\beta_i^{k+1} + R_{i,j}\left(1-\beta_i^{2k}\right)}{\eta+\chi-(1+\eta)\beta_i^{2k}}\right\}, \tag{20}$$

$$\varphi(\beta_n,\beta_i) + h\sum_{i=0}^{n-1} M(t_n - t_i)\varphi(\beta_n,\beta_i) = G. \tag{21}$$

Here $n > n_*$, $\beta_i = \beta_0$; $i < n_*$.

Algorithm of solution consists of following stages:

First contact pressure $Q_n$ is determined by (18) using (19) and values $Q_i$ of the foregoing steps of calculations; then by (20) on each step the non-linear algebraic equation (21) is solved with respect to current coordinate of failure front $\beta_n$. At the same time two conditions $Q_n > 0$ and $\beta_n > \beta_{n-1}$ are checked. At imsatisfaction of any of them calculation process is stopped. By the first condition that means peeling of the pipe from the shell and by the second condition that means achievement of an infinite large value of the velocity of motion of failure front, i.e. finish of failure process.

For incubation the calculations were carried out for the cases $\chi = 0$ (non-compressibility of the material of the pipe) and $\chi = -1$ (equality to zero of Poisson's coefficient $\nu'$). It is taken $P = P_0 = const$; $\beta_0 = 0{,}3$; $\eta = -5/k$; $k = 0{,}5;1;2$. For the parameter of singularity of damaging kernel $M(t_n - t_i) = (t - t_i)^{-\alpha}$ the values are taken: $\alpha = 0;\ 0{,}5$. The results of calculations by incubation $t_0$ are reduced in Table 1 and fig.2

**Table 1**

| | $\chi=0;\ \beta=0$ | $\chi=-1;\ \beta=0$ | $\chi=0;\ \beta=0{,}5$ | $\chi=-1;\ \beta=0{,}5$ |
|---|---|---|---|---|
| $K=0{,}5$ | 2,69 | 3,0025 | 2,11 | 3,04 |
| $k=1$ | 1,545 | 1,64 | 2,653 | 0,729 |
| $k=2$ | 0,4037 | 0,4075 | 0,05 | 0,054 |

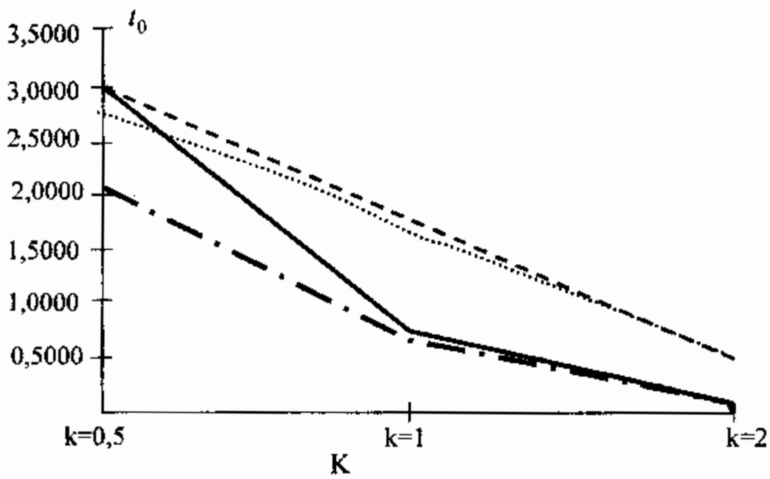


**Fig.2.** ............ $\chi = 0;\ \beta = 0;$

— · — · — · · $\chi = 0;\ \beta = 0{,}5;$

- - - - - - - - $\chi = -1;\ \beta = 0;$

———— $\chi = -1;\ \beta = 0{,}5;$

From them it follows the significant increase of incubation with decrease of "$k$", i.e., by (11) with decrease of circle rigidity of the pipe. Also it is remarked sufficient noticeable influence of singularity of damaging kernel. Singularity of the kernel leads to decrease of incubation from 5% to two-multiple dimension.

The main period is calculated for the non-compressible case $\chi = -1$

***Table 2***

| $\alpha$ / $k$ | | 0 | 0,25 |
|---|---|---|---|
| 0,5 | $t_0$ | 3,2275 | 3,285 |
| | $t_{kp}$ | 0,4974 | 0,5207 |
| 1 | $t_0$ | 1,64 | 1,3475 |
| | $t_{kp}$ | 0,5441 | 0,5721 |
| 2 | $t_0$ | 0,4075 | 0,2088 |
| | | 0,5215 | 0,4582 |

for values $k = 0{,}5;1;2$ and $\alpha = 0; 0{,}25$. The results of calculations are given in Table 2 and in figures 3 and 4. Also the corresponding values of incubation are pointed there. The reduced data certify

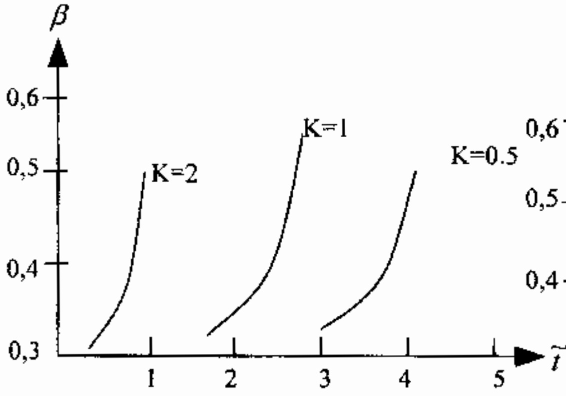


**Fig.3** $\alpha = 0;\ M = \cos t;\ \chi = -1$

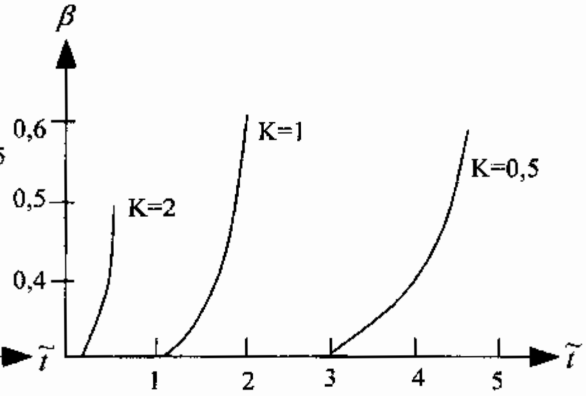


**Fig.4** $\alpha = 0{,}25;\ M = mt^{-\alpha};\ \chi = -1$

on decrease of limit critic coordinates of failure front with decrease of "$k$", i.e. with decrease of circle rigidity of the pipe. Singularity of the kernel of damaging operator reduces to increase of this parameter.

**References**

[1]. Работнов Ю.Н. *Элементы наследственной механики твердых тел.* М., 1977, 384 с.

[2]. Ахундов М.Б. *Разрушение наследственно упругой толстостенной трубы, заключенной в тонкую упругую оболочку.* Изв. АН Аз.ССР, 1987, №4, с.138-143.

[3]. Ахундов М.Б. *Механизм деформирования и рассеянного разрушения композитных структур.* Изв. АН СССР, МТТ, 1991, №4, с.173-179.

[4]. Лехницкий С.Г. *Теория упругости анизотропного тела.* М., Наука, 1977, 416 с.

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