

LATIFOV F.S.

ASYMPTOTIC ANALYSIS OF THE PROBLEM ON FREE OSCILLATIONS OF THE CYLINDRICAL SHELL WITH THE HOLLOW FILLER IN THE INFINITE FLUID

Abstract

In the paper the frequencies of free non-axial symmetric oscillations of the circle cylindric shell in the infinite fluid and with the hollow filler are investigated. It is supposed that the change of the sought stress-strain condition in the circle direction is great and also the rigidity of the material of the shell is much more than the rigidity of the material of the filler. The formula is obtained for the frequency of oscillations of the considered system.

The problem on determination of lower frequencies of free oscillations of the cylindrical shell with the hollow elastic filler in the infinite ideal compressible fluid, is investigated. It is considered that the shell and the filler are joined rigidly and the inner surface of the filler is free from stress.

The system of equations of motion of the shell is represented with respect to displacements in the form [1]:

$$\sum_{j=1}^3 (L_{ij}(u_j) + h^2 N_{ij}(u_j)) = \frac{1-\nu^2}{Eh} q_i \quad (i=1,2,3). \quad (1)$$

Here L_{ij} and N_{ij} are known differential operators of the theory of shells

$$L_{11} = \frac{\partial^2}{\partial x^2} + \frac{1-\nu}{2} \frac{\partial^2}{\partial \varphi^2}; \quad L_{12} = L_{21} = \frac{1+\nu}{2} \frac{\partial^2}{\partial x \partial \varphi};$$

$$L_{13} = L_{31} = \nu \frac{\partial}{\partial x}; \quad L_{22} = \frac{1-\nu}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \varphi^2};$$

$$L_{23} = L_{32} = \frac{\partial}{\partial \varphi}; \quad L_{33} = 1 + b^2 \nabla^2 \nabla^2; \quad b^2 = \frac{h^2}{12K^2},$$

where ν is Poisson's coefficient of the material of the shell, E is Young's modulus, h is the thickness of the shell, R is the radius of the shell, q_i are external forces; x, φ are the coordinates.

The motion of the filler is described by Lamé's equations in displacements

$$a_e^2 \text{grad div } \vec{S} - a_l^2 \text{rot rot } \vec{S} + \omega^2 \vec{S} = 0. \quad (2)$$

Here a_e, a_l are velocities of propagation of lateral and longitudinal waves in the filler, \vec{S} is the vector of displacement of the filler, ω is the angular frequency of oscillations.

The equation of motion of the ideal compressible fluid in the cylindric system of coordinates has the form:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Phi}{\partial x^2} + \frac{\omega^2}{a^2} \Phi = 0, \quad (3)$$

where Φ is the potential of the fluid. The contact and the boundary conditions are added to the equation of motion of the shell (1), filler (2) and fluid (3).

The condition of equality of the components of displacements is given on the joint of the filler with the shell:

$$S_x = u_1, S_\varphi = u_2, S_r = u_3 \quad (r = R) \quad (4)$$

and the condition of equality of pressures:

$$q_{11} = -\sigma_{rx} - \rho h \frac{\partial^2 u_1}{\partial t^2}; \quad q_{22} = -\sigma_{r\varphi} = -\delta_{r\varphi} - \rho h \frac{\partial^2 u_2}{\partial t^2}. \quad (5)$$

Continuity of radial velocities and pressures is realised on the contact surface:

$$V_r = \frac{\partial u_3}{\partial t}; \quad \tilde{q}_{33} = -P; \quad \tilde{q}_{11} = \tilde{q}_{22} = 0 \quad (r = R), \quad (6)$$

where $\tilde{q}_{11}, \tilde{q}_{22}, q_{33}$ are pressures to the fluid from the shell. V_r and P are determined by the potential Φ by the formulas:

$$V_r = \frac{\partial \Phi}{\partial r}, \quad P = -\rho_0 \frac{\partial \Phi}{\partial t}, \quad (7)$$

where ρ_0 is the density of the fluid.

In the inner surface of the filler the condition of absence of stress is fulfilled:

$$\sigma_{rx} = \sigma_{r\varphi} = \sigma_{rr} = 0, \quad (r = R_1), \quad (8)$$

where R_1 is the radius of the canal of the filler.

Moreover, the potential of the fluid φ in the infinity satisfies Sommerfeld's conditions [2]:

$$\Phi = O(r^{-1}), \quad \frac{\partial \Phi}{\partial r} - i \frac{\omega}{a} \Phi = O(r^{-1}) \quad (9)$$

adding the contact condition (6)-(9) to the equations of motion of the shell (1), medium (2) and the fluid (3), we come to the contact problem on free oscillations of the shell with the hollow filler in the infinite deal compressible fluid. The problem on free shell with the filler in the fluid is reduced to the joint integration of the equations of the theory of shells, medium and fluid for fulfilment of the pointed conditions on the surface of their contact.

Let us take the solution of the equation of motion of the shell (1) in the form:

$$\begin{aligned} u_1 &= A \cos n\varphi \cos kx \sin \omega t, \\ u_2 &= B \sin n\varphi \sin kx \sin \omega t, \\ u_3 &= C \cos n\varphi \sin kx \sin \omega t. \end{aligned} \quad (10)$$

Here n is the number of halfwaves in the circle direction, ω is the angular frequency, π/k is the length of halfwaves along the element of the cylinder; A, B, C are the constants which we must determine.

The solution of the equations of motion of the elastic filler has the form [1]:

$$\begin{aligned} S_x &= \left[A_s k I_n(\gamma_e r) - \frac{c_s \gamma_e^2}{\mu} I_n(\gamma_e r) + \right. \\ &\quad \left. + \tilde{A}_s k K_n(\gamma_e r) - \frac{\tilde{c}_s \gamma_e^2}{\mu_i} K_n(\gamma_e r) \right] \cos n\varphi \cos kx \sin \omega t, \\ S_\varphi &= \left[-\frac{A_s n}{r} I_n(\gamma_e r) - \frac{C_s n k}{r \mu_e} I_n(\gamma_e r) - \frac{B_s}{n} \frac{\partial I_n(\gamma_e r)}{\partial r} - \right. \\ &\quad \left. - \frac{\tilde{A}_s n}{r} K_n(\gamma_e r) - \frac{\tilde{C}_s n k}{r \mu_i} K_n(\gamma_e r) - \frac{\tilde{B}_s}{n} \frac{\partial K_n(\gamma_e r)}{\partial r} \right] \times \end{aligned} \quad (11)$$

$$\begin{aligned}
& \times \sin n\varphi \sin kx \sin \omega t, \\
S_r = & \left[A_s \frac{\partial I_n(\gamma_e r)}{\partial r} - \frac{C_s k}{\mu_i} \frac{\partial I_n(\gamma_i r)}{\partial r} + \frac{B_s n}{r} I_n(\gamma_i r) + \right. \\
& \left. + \tilde{A}_s \frac{\partial K_n(\gamma_e r)}{\partial r} - \frac{\tilde{C}_s k}{\mu_i} \frac{\partial K_n(\gamma_i r)}{\partial r} + \frac{\tilde{B}_s n}{r} K_n(\gamma_i r) \right] \times \\
& \times \cos n\varphi \sin kx \sin \omega t,
\end{aligned}$$

where $\gamma_i^2 = k^2 - \mu_i^2$, $\gamma_e^2 = k^2 - \mu_e^2$, $\mu_i = \frac{\omega}{a_i}$, $\mu_e = \frac{\omega}{a_e}$; $A_s, \tilde{A}_s, B_s, \tilde{B}_s, C_s, \tilde{C}_s$ are the constants; $I_n(x)$, $K_n(x)$ are the modified Bessel's functions of the n -order of the first and second types, correspondingly.

Potential Φ of the fluid has the form [1]:

$$\Phi = \Phi_0 K_n(\gamma r) \cos n\varphi \sin kx \cos \omega t, \quad (12)$$

where $K_n(x)$ are Bessel's functions, $\gamma^2 = k^2 - \frac{\omega^2}{a^2}$, Φ_0 is the constant.

Components of the contact stress $\sigma_{rx}, \sigma_{r\varphi}, \sigma_{rr}$ (5) and (8) are determined in the form:

$$\begin{aligned}
\sigma_{rx} &= \mu_s \left(\frac{\partial S_x}{\partial r} + \frac{\partial S_r}{\partial x} \right); \\
\sigma_{r\varphi} &= \mu_s \left[r \frac{\partial}{\partial r} \left(\frac{S_\varphi}{r} \right) + \frac{1}{r} \frac{\partial S_r}{\partial \varphi} \right]; \\
\sigma_{rr} &= \lambda_s \theta + 2\mu_s \frac{\partial S_r}{\partial r}; \\
\theta &= \frac{\partial S_x}{\partial x} + \frac{1}{r} \frac{\partial (r S_r)}{\partial r} + \frac{1}{r} \frac{\partial S_\varphi}{\partial \varphi}.
\end{aligned} \quad (13)$$

Fulfilment of the contact conditions (5)-(8) with use of the expressions (10)-(12), (7), (13) gives the algebraic homogeneous system with respect to the constants $A_s, B_s, C_s, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s$. As far as this system is homogeneous then for existence of its non-trivial solution we equal the main determinant to zero and at the result we obtain the frequency equation

$$\det \|\alpha_{ij}\| = 0 \quad (i, j = 1, 2, \dots, 6) \quad (14)$$

Equation (14) is transcendental with respect to frequency ω . For simplicity of the frequency equation we use the following asymptotic formulas for the logarithmic derivatives of Bessel's functions $I_n(x)$ and $K_n(x)$ and also for $I_n(x)/I_n(x)$ and $K_n(x)/K_n(x)$ [3]:

$$\begin{aligned}
I'_n(x)/I_n(x) &\approx \frac{n}{x} + \frac{x}{2n}; \\
K'_n(x)/K_n(x) &\approx -\frac{n}{x} - \frac{x}{2n};
\end{aligned}$$

$$\begin{aligned} I_n(\varepsilon x)/I_n(x) &\approx \varepsilon^{2n+1/2} \left[1 + \frac{x^2}{2n} (1 - \varepsilon^2) \right]; \\ K_n(\varepsilon x)/K_n(x) &\approx \varepsilon^{-2n-3/2} \left[1 - \frac{x^2}{2n} (1 - \varepsilon^2) \right]. \end{aligned} \quad (15)$$

Let us note that formulas (15) are valid for $x \ll \sqrt{2n/(1 - \varepsilon^2)}$.

Frequency equation (14) which corresponds to the signification influence of inertial actions of the filler on the process of oscillations of the cylindrical shell with the hollow filler in the infinite fluid after simplification with help of asymptotical formulas

(15) is reduced to the algebraic equation with respect to λ $\left(\lambda = \frac{(1 - v^2) \rho \omega^2 R^2}{E} \right)$:

$$Q_1 \lambda + Q_2 = 0, \quad (16)$$

where

$$\begin{aligned} Q_1 &= \left\{ - \left[\frac{h_s^2 n^4}{6} + \frac{(1 - v^2) k^{*4}}{n^4} \right] S_0 - S_1 \gamma_1 \right\} \times \\ &\times \left(1 + (1 - \varepsilon) \chi^* \frac{\rho_s^*}{n} + \varepsilon^2 (1 - \varepsilon^2)^2 \chi_1 \frac{\rho_s^{*2}}{n^2} + \frac{\rho_0^*}{n h_s} \right); \\ Q_2 &= \left[\frac{h_s^2 n^4}{6} + \frac{(1 - v^2) k^{*4}}{n^4} + \chi (1 - \varepsilon) n E_s^* \right] S_1 \gamma_1; \\ S_0 &= 8(1 - \varepsilon)^2 n^3 k^* + n(3 - 4v_s)^2; \chi = \frac{1 - v^2}{2(1 + v_s)}; \\ \chi_1 &= \frac{v}{4} \chi; h_s = \frac{h}{R}; k^* = kR; E_s^* = \frac{E_s}{E h_s}; \\ \rho_0^* &= \rho_0 / \rho; \rho_s^* = \rho_s / \rho h_s; \varepsilon = R_1 / R. \end{aligned}$$

The root of (16) has the form:

$$\lambda = -Q_2 / Q_1. \quad (17)$$

From the expressions for Q_1 and Q_2 it is seen that for $\rho_0^* \rightarrow 0$ the formula (17) passes to the formula for the natural frequency of oscillations of the cylindric shell with the hollow filler without fluid. And for $\varepsilon \rightarrow 1$ it passes to the formula for the natural frequency of oscillations of the cylindric shell in the infinite fluid.

The obtained qualitative results are confirmed partially. The frequencies of natural oscillations of the cylindric shell with the hollow filler in the infinite fluid are found with help of the precise (14) and the approximate (16) frequency equations.

The results of calculations have been obtained for following values of the parameters characterising the materials of the shell, the filler and the fluid: $v = 0,3$, $v_s = 0,38$, $E_s^* = 0,01$, $\rho_s^* = 0,30$, $h/R = 0,01$, $\rho_0^* = 0,13$. In fig.1 the dependencies λ on number n of wave formation in the circle direction are represented. The dash line corresponds to the solution of the precise frequency equations (14) and the continual curve corresponds to the solution of the approximate frequency equation (16).

Analysis of calculation demonstrates that in the case of shell without the medium with increase of the number of waves n by the circumference the natural frequencies of oscillations of the cylindric shell with the hollow filler in the infinite fluid first decrease

and then they begin increasing and achieving the minimum. Moreover, the results show that the consideration of influence of the fluid reduces to decrease of natural frequency of oscillations of the system in comparison with the frequency of natural oscillations of the system "cylindric shell-filler".

The results of calculations show that with increase of n the difference between the precise and the approximate values λ is getting smaller. In fig.2 the dependences of λ on the radius of the canal of the filler is represented. It is seen from the figure that with increase of the diameter of the canal the frequencies increase.

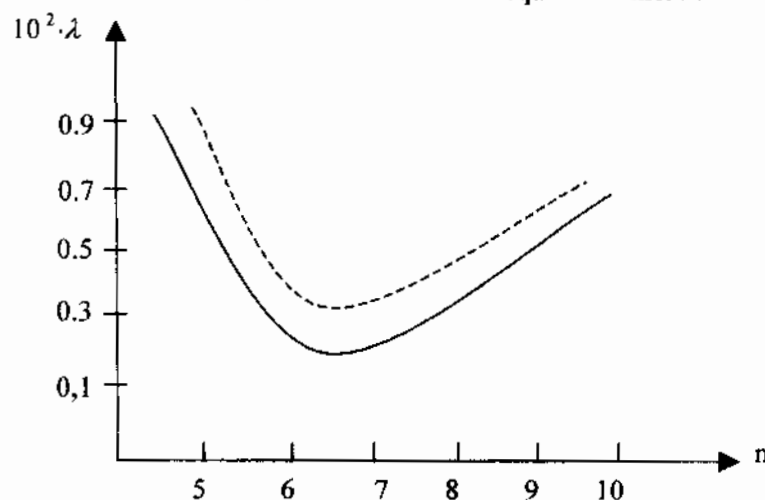


Fig.1. ($\varepsilon = 0,3$)

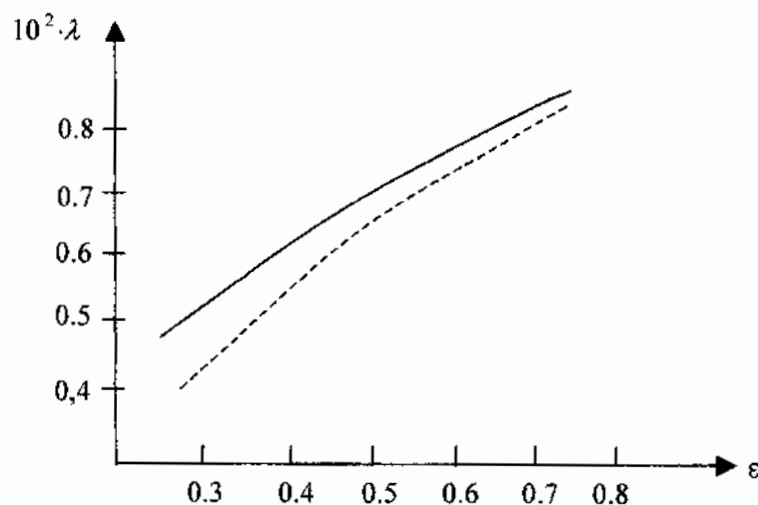


Fig.2. ($n = 5$)

References

- [1]. Ильгамов М.А. *Колебания упругих оболочек, содержащих жидкость и газ*. М.: Наука, 1969, 182с.
- [2]. Латифов Ф.С., Бергман Р.М. *Асимптотический анализ задачи о свободных колебаниях цилиндрической оболочки, контактирующей с упругой средой*. Известия АН СССР, Механика твердого тела, 1981, №1, с.185-191.

[3]. Смирнов В.Н. *Курс высшей математики*. Т.3, ч.(2), М., Наука, 672 с.

Latifov F.S.

Institute of Mathematics and Mechanics of AS Azerbaijan.

9, F.Agayeva str., 370141, Baku, Azerbaijan.

Tel.: 39-47-20(off.).

Received December 21, 1999; Revised June 21, 2000.

Translated by Soltanova S.M.