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MOTION OF THE ELASTIC HOLD UP INCLUSION IN THE FLUID

Abstract

In the paper the twodimensional problem on the motion of the elastic hold up against the lateral motion of the circle inclusion in the fluid acting the wave resistance is investigated.

Study of joint motion of constructions and environment is important for calculation of constructions for rigidity and strength under action of seismic waves, gust, see waves and others.

With purpose of simplification of the problem we can model the construction as a discrete medium. For example, if the object is arranged on a pile, then the pile can be considered as elastic lateral strut and etc.

In this paper the twodimensional problem on motion of the elastic hold up circle inclusion against lateral motion in the fluid acting wave resistance is consider.

1. Equation of motion and boundary conditions. Formulation of the problem.

Vortex-free motion of the medium is described by the equation

$$\Delta\varphi = \frac{1}{a^2}\frac{\partial^2}{\partial}\frac{\varphi}{t^2}, \tag{1}$$

where $a = \sqrt{\frac{B}{\varphi}}$.

B is coefficient of compression, φ is velocity potential, $V = grad\varphi$, V is vector of velocity, Δ is Laplacian operator.

After wave going if we neglect the passing phenomena (diffraction) then the moveless at initial moment inclusion is surrounded by the medium moving in one direction with the known velocity. By principle of relativity we can consider the medium moveless and give the velocity of the fluid to the inclusion.

The inclusion moves by the law

$$\begin{aligned} M_1\frac{d^2x_1}{dt^2} &= P + L(x_2 - x_1), \\ M_2\frac{d^2x_2}{dt^2} &= -L(x_2 - x_2), \end{aligned} \tag{2}$$

where M_1 is mass of yoke; M_2 is mass of the springed body; x_1 is displacement of the yoke; x_2 is displacement of the springed body; L is rigidity of the spring; P is force of action of the fluid to the inclusion.

For the circle inclusion with radius r_0

$$P = -r_0\int_0^{2\pi} P_2\cos\theta_0 d\theta_0, \tag{3}$$

where ρ is density, θ_0 is polar angle.

Velocity of the yoke $\frac{dx_1}{dt}$ can be expressed by radial V_r and circle V_θ components of velocity of the fluid on the bound of the fluid with the yoke. The last ones are connected with potential $V_r = \frac{\partial\varphi}{\partial r}$

$$V_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta_0}. \tag{4}$$

At the same time

$$V_r = V_x \cos\theta_0, \tag{5}$$

where $V_x = \frac{dx_1}{dt}$.

From (4) and (5) it follows:

$$\frac{\partial\varphi}{\partial r} = \frac{dx_1}{dt}\cos\theta_0. \tag{6}$$

Condition (6) expresses equality of normal to the surface of the yoke components of velocities of the fluid and the yoke.

It is obvious, tangential components of velocities won't coincide. Taking into account that the tangential component of velocity of the fluid $V_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta_0}$ and the inclusion $\frac{dx_1}{dt}\sin\theta$, it is possible to find the velocity of slippage of the fluid on the surface of the inclusion

$$\frac{dx_1}{dt}\sin\theta - V_\theta = \frac{\sin\theta_0}{\cos\theta_0}\frac{\partial\varphi}{\partial r} - \frac{1}{r}\frac{\partial\varphi}{\partial\theta_0} = tg\theta_0\frac{\partial\varphi}{\partial r} - \frac{1}{r}\frac{\partial\varphi}{\partial\theta_0}. \tag{7}$$

Thus, the problem for equation (1) with boundary conditions (2) is considered taking into account (3) and (6) and the initial condition $t = 0$

2. Solution of the problem in general case.

The solution is sought in the form:

$$\varphi(r,\theta_0,t) = \varphi_1(r,t)\cos\theta_0. \tag{8}$$

Taking into account that Laplacian operator in cylindric coordinates has the form

$$\Delta\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}$$

and also (8) equation (1) has the form:

$$\frac{\partial^2\varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_1}{\partial r} - \frac{\varphi_1}{r^2} = \frac{1}{a^2}\frac{\partial^2\varphi_1}{\partial t^2}. \tag{9}$$

After Laplace-Korson transformation equation (9) in representations will be:

$$\varphi_1'' + \frac{1}{r}\overline{\varphi}_1' - \frac{1}{r^2}\overline{\varphi}_1 = \frac{p^2}{a^2}\overline{\varphi}_1. \tag{10}$$

Solution of (10) under the condition of restriction at infinity is:

$$\overline{\varphi}_1 = ck_1\left(\frac{pr}{a}\right), \tag{11}$$

where v is McDonald's function of the first order with known original, i.e.

$$k_1\left(\frac{pr}{a}\right)\to\begin{cases}0 & for \quad 0\le t\le\frac{r}{a}\\ \frac{\sqrt{a^2t^2-r^2}}{r} & for \quad t\ge\frac{r}{a}\end{cases} \tag{12}$$

Further we determine force P from (2) with help of (3) . Taking into account the represented expression for φ_1 from (8), i.e. for

$$p=-\rho\frac{\partial\varphi}{\partial t}=-\rho\frac{\partial\varphi_1}{\partial t}\cos\theta_0 \quad and \quad \frac{dx_1}{dt}=\frac{\partial\varphi_1}{\partial r}.$$

From (3) we determine

$$P=-r_0\int_0^{2\pi}p\cos\theta_0 d\theta_0=r\rho\frac{\partial\varphi_1}{\partial t}\int_0^{2\pi}\cos^2\theta_0 d\theta_0=\rho r_0\pi\frac{\partial\varphi_1}{\partial t}. \tag{13}$$

Equation (2) taking into account (13), (6), (8) will have the form:

$$M_1\frac{\partial^2\varphi_1}{\partial r\partial t}+\rho r_0\pi\frac{\partial\varphi}{\partial t}=L(x_2-x_1) \tag{14}$$

$$M_2\frac{\partial^2 x_2}{\partial t^2}+L(x_2-x_1)=0.$$

In the case of elastic hold up inclusion in the first of equations (2), taking x_2 as the coordinates of the bear in relative motion which is $x_2=\dot{x}t$, we have:

$$M_1\frac{\partial^2\varphi_1}{\partial r\partial t}+\rho r_0\pi\frac{\partial\varphi}{\partial t}+L(x_1-\dot{x}_0t)=0, \tag{15}$$

where $\dot{x}_0$ is velocity of the fluid at minimal moment.

After Laplace-Karson transformation we obtain

$$PM(\overline{\varphi}_1'-\dot{x}_0)+\rho\pi r_0 p\overline{\varphi}_1+L\left(\overline{x}_1-\frac{\dot{x}_0}{p}\right)=0,$$

where $\overline{x}_1=\frac{\overline{\varphi}_1'}{p}$.

Putting expression of φ_1' from (11), $n\overline{\varphi}_1'=-c\left(\frac{p}{a}k_0+\frac{1}{r}k_1\right)$, the last equation can be reduced to the form:

$$C=\frac{-r_0\dot{x}_0}{\frac{pk_0}{\omega_1}+k_1+\frac{mp^2k_1}{p^2+\omega^2}}, \tag{16}$$

where $\omega^2=\frac{L}{M}$, $\omega_1=\frac{a}{r_0}$, $m=\frac{M}{\rho\pi r_0^2}$.

Taking into account, that $pk_0\to\frac{\omega_1}{\sqrt{\theta^2-1}}$; $\frac{p^2}{p^2+\omega^2}\to\cos\frac{\omega}{\omega_1}(\theta-1)$ where $\theta=\frac{at}{r_0+1}$ and time shift of the argument by time is obtained by multiplication and division of nominator and denominator in (16) by $\exp\left(\frac{pr_0}{a}\right)$, the denominator in the expression of c will be:

$$Z = \frac{\theta^2}{\sqrt{\theta^2 - 1}} + m\cos\frac{\omega}{\omega_1}(\theta - 1) * \sqrt{\theta^2 - 1}. \qquad (17)$$

Original of the boundary function having expression (16) for known original of the denominator (17) can be found by the method of Volterra integral equation (12).

Calculations have been carried out for $m = 0{,}3$, $\frac{\omega}{\omega_1} = 2$. In the figures the graphs of dependence of function S_1 which is the original C on dimensionless time are shown.

This function let determine the potential and parameters.

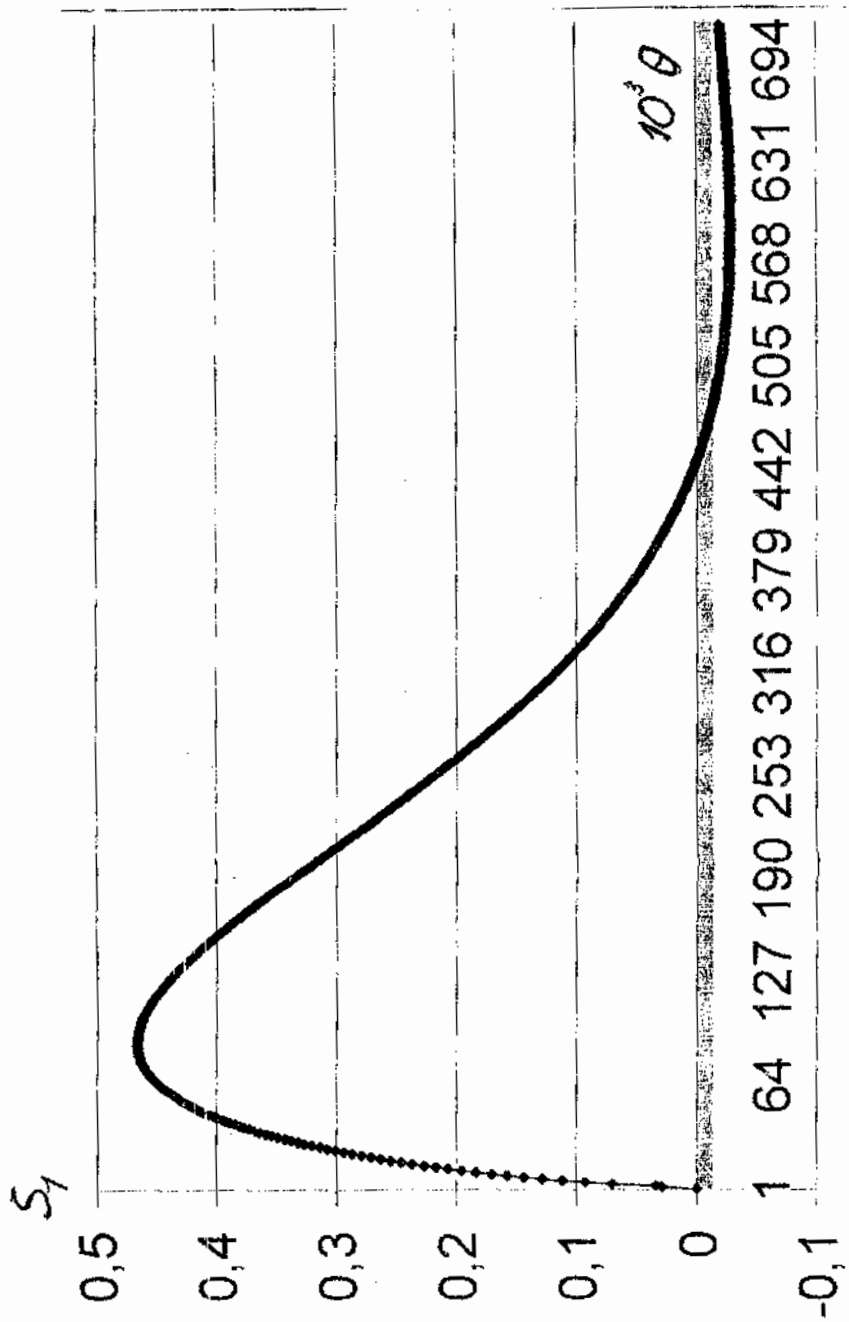
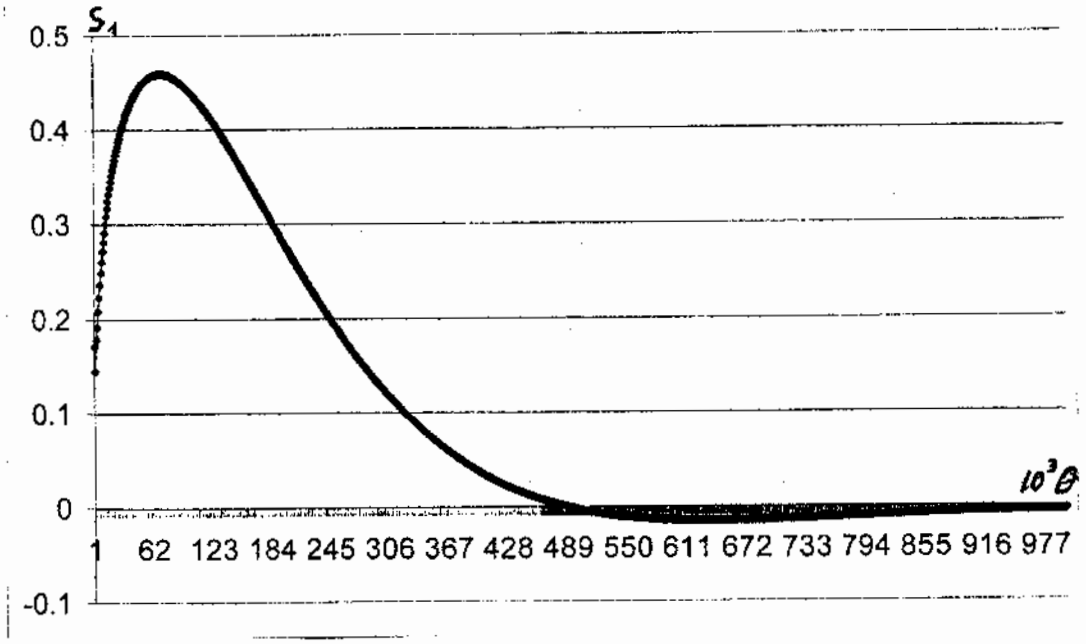


Fig. 1.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0.5
0.4
0.3
0.2
0.1
0
-0.1
$10^3\theta$
1 62 123 184 245 306 367 428 489 550 611 672 733 794 855 916 977

Fig.2.

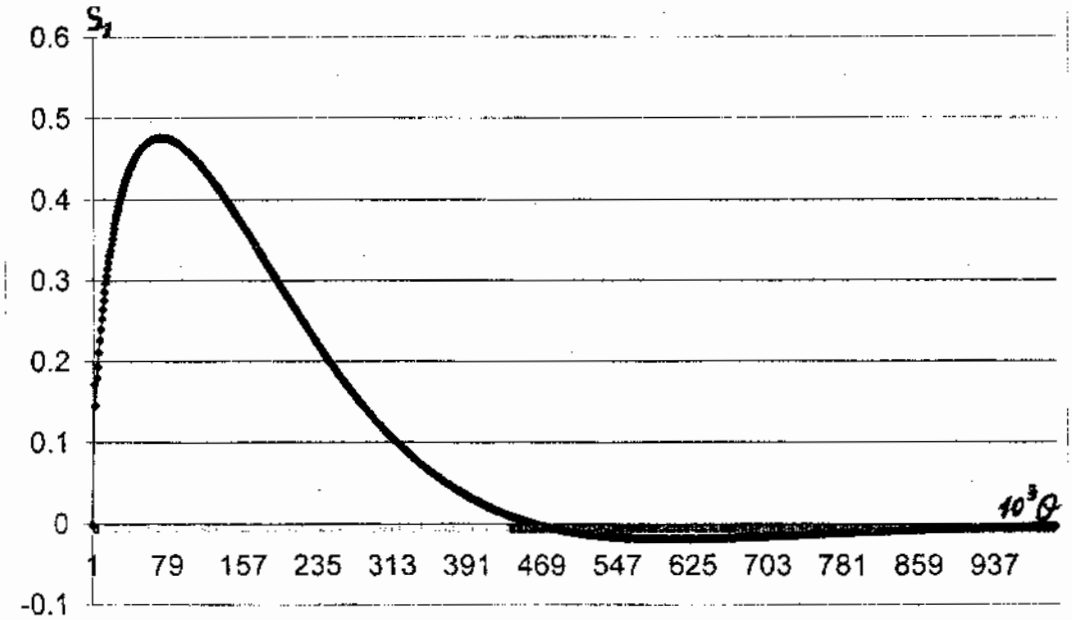
S_1
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
$10^3\theta$
1 79 157 235 313 391 469 547 625 703 781 859 937

Fig.3.

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Received February 16, 2000; Revised May 24, 2000.
Translated by Soltanova S.M.