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DURABLE STRENGTH OF THE COMPOSITE PIPE UNDER ACTION OF INNER PRESSURE

Abstract

The equation of motion of failure front in the anisotropy pipe with cylindric symmetry is obtained. The scheme of its numerical realization is worked out. Influence of the correlation rigidity in the circle and radial directions and of the form of the kernel of damaging on failure process, beginning and the velocity of motion of failure front are found out.

Pipes used in industry and production are metallic as usual for which anticorrosion covers are applied. Nowadays the limit in increase of durability of such pipes has been achieved. That explains the great interest to investigation, projection and technology of production of composite pipes whose other advantage is their relative simplicity at mounting and heat stability. In present work on the base of continual principle the possible mechanism of failure process of the layer composite pipe which is under action of constant inner pressure is investigated.

As the determinative correlations we take the variant of the theory of deformation and failure [1]

$$\varepsilon_{ij} = \left(C_{ijkl} + L_{ijkl}^{*} + M_{ijkl}^{*}\right)\sigma_{kl} \; , \tag{1}$$

where C_{ijkl} is instantaneous modulus of elasticity; L_{ijkl}^{*} and M_{ijkl}^{*} are correspondingly operators of viscous flow and damaging of hereditary type. At active loading there is not difference between the viscosity operator and the damaging operator and they can be united. However at loading-off the representation for damaging operator is significant differ from the form of the operator of reversible creeping. Particularly, action of damaging operator is stopped when the realized in the body stress condition doesn't influence on process of damage accumulation. For the present investigation for concentration of attention namely on damaging process and taking account non-significance of deformation of viscous flow, we omit operator L_{ijkl}^{*} in correlation (1).

According to [2] for kernels of hereditary type operators the representations are valid:

$$M_{ijkl}(t) = \sum_{n=1}^{N} m_{ijkl}^{(N)} f_{N}(t). \tag{2}$$

At the same time according to the results of experimental investigations [3] in (2) we can be restricted with sufficient degree of precision for some types of composites only by one component supposing $N = 1$. That is, we can consider as the first approximation that the anisotropy of rheonomic properties is showed through the tensor of instantaneous modules of "damaging" m_{ijkl} which is proportional to the tensor of instataneous elasticity modules and that further will be accepted.

We'll consider that the pressure p given in the pipe at initial moment of time is remained constant further. Then in whole volume of the pipe the scheme of active loading is realized and damaging operator represents the usual continuous operator of hereditary type. Then correlations (1) will formal coincide with the correlations of theory of visco-elasticity of the anisotropy body to whose solving of problems the method of

correspondence by Volterr-Rabotnov is applied. That is the formulas for stress in the anisotropy damaging pipe are obtained from the formulas of the elastic anisotropy one by substitution of modules of elasticity in them by the corresponding operators of damaging.

There is cylindric symmetry for reeled-up composite pipe. We use for stress the known solution of the problem on anisotropy pipe with cylindric symmetry [4].

As failure criterion we take the criterion by maximal stress. So here we'll reduce only the form of maximal stress which is the circle stress σ_θ according [4]

$$\sigma_\theta = kp \frac{c^{k+1}}{1-c^{2k}} \left(\rho^{k-1} + \rho^{-k-1} \right), \tag{3}$$

where p is inner pressure, $\rho = r/b$, $c = a/b$, r is current radius of the pipe; a is inner radius, b is external radius. Parameter k characterizing the anisotropy degree has the form

$$k^2 = \frac{E_\theta}{E_r} \cdot \frac{1 - \nu_{zr}\nu_{rz}}{1 - \nu_{z\theta}\nu_{\theta z}}, \tag{4}$$

where E_θ and E_r are elasticity moduluses in the circle and radial directions, and $\nu_{zr}, \nu_{rz}, \nu_{z\theta}, \nu_{\theta z}$ are the corresponding Poisson's coefficients.

By principle of correspondence and taking into account the made assumptions with respect to kernels of damaging operators (2) it should change in (3) $E_\theta \to \widetilde{E}_\theta$, $E_r \to \widetilde{E}_r$ assuming constancy of Poisson's coefficients, where, for example, $1/\widetilde{E} = 1/E_\theta \left(1 + M^k\right)$. In this case, as it is seen from (4), the form of parameter k is remained unchanged, i.e. this parameter in the first approximation in the frame of abovemade assumptions can be accepted constant. But influence of time factor is realized through failure criterion, which as it has already been pointed, by the maximal stress will have the form:

$$\sigma_\theta + M^* \sigma_\theta = \sigma_0, \tag{5}$$

where s_0 of strength of the defectless material. For $r = a$ stress σ_θ takes the maximal value

$$\sigma_{\theta max} = kp \frac{1 + c^{2k}}{1 - c^{2k}}. \tag{6}$$

That means failure begins on the inner contour of the pipe. Then failure time t_0 is determined from condition (5)

$$\frac{1 + c^{2k}}{1 - c^{2k}} \left(1 + M^*\right) p = \frac{\sigma_\theta}{k}. \tag{7}$$

Let's reduce the explicit form for time of initial failure of the inner layer of the composite pipe for three forms of kernels of damaging operator

$$M(t) = mt^{-\alpha}; \qquad t_0 = \left[\frac{1-\alpha}{m} \left(\frac{\sigma_0}{kp} \frac{1 - c^{2k}}{1 + c^{2k}} - 1 \right) \right]^{\frac{1}{1-\alpha}}, \tag{8}$$

$$M(t) = me^{-\alpha t}; \qquad t_0 = \frac{1}{\alpha} \ln\left[1 + \frac{\alpha}{m} \left(1 - \frac{1 - c^{2k}}{1 + c^{2k}} \frac{\sigma_0}{kp} \right) \right]^{-1}, \tag{9}$$

$$M(t) = m = const; \qquad t_0 = \frac{1}{m} \left(\frac{1 - c^{2k}}{1 + c^{2k}} \frac{\sigma_0}{kp} - 1 \right). \tag{10}$$

According (8)-(10) for the value of pressure p which transforms the expression in the round brackets to zero failure of the inner layer happens instantly at application of inner pressure and further the failure front will propagate to the external surface.

In order to observe the motion of failure front [5], it is necessary to consider in (3) the inner radius dependent on time. Making the substitution in (3) $c = \beta(\tau)$; $\rho = \beta(t)$, we obtain

$$\sigma_\theta(t,\tau) = kp \frac{\beta^{k+1}(\tau)}{1 - \beta^{2k}(\tau)} \left\{ \beta^{k-1}(t) + \beta^{-k-1}(t) \right\}. \tag{11}$$

This formula for stress at moment of time τ, when the failure front has the coordinate $\beta(\tau)$ in the layer where the failure front will reach at moment of time $t > \tau$. In this layer at moment of time t, when the failure front will come to it, the circle stress will have the form

$$\sigma_\theta(t,t) = kp \frac{1 + \beta^{2k}(t)}{1 - \beta^{2k}(t)}. \tag{12}$$

Then according to the failure criterion (5) we'll obtain the following equation of failure front

$$\frac{1 + \beta^{2k}(t)}{1 - \beta^{2k}(t)} + \int_0^t M(t - \tau) \left[\frac{\beta(\tau)}{\beta(t)} \right]^{k+1} \frac{1 + \beta^{2k}(t)}{1 - \beta^{2k}(\tau)} d\tau = \frac{\sigma_0}{kp}, \tag{13}$$

which represents Volterra non-linear integral equation of the second type. Moreover, here $\beta(\tau) = \beta = a/b = const$ for $0 < \tau < t_0$, where t_0 is initial time of failure of the inner layer (incubation) determined by one of the formulas (8)-(10).

Because of complexity of immediately solving of (13) the numerical method had been applied to it. Let's introduce the denotations

$$\begin{cases} \varphi(\beta(t), \beta(\tau)) = \left[\frac{\beta(\tau)}{\beta(t)} \right]^{k+1} \frac{1 + \beta^{2k}(t)}{1 - \beta^{2k}(\tau)} \\ f(\beta(t)) = \varphi(\beta(t), \beta(\tau)); \ G = \frac{\sigma_0}{kp} \end{cases} \tag{14}$$

Then the integral equation (13) is written as

$$f(\beta(t)) + \int_0^t M(t - \tau)\varphi(\beta(t), \beta(\tau)) = G. \tag{15}$$

Further we'll consider the times t and τ dimensionless, for example, for the constant kernel as dimensionless times we can take $\tilde{t} = mt$, $\tilde{\tau} = mt$. For (15) its following analogue is used

$$f(\beta_n) + h \sum_{i=1}^{n-1} M(t_n - t_i)\varphi(\beta_n, \beta_i) = G, \tag{16}$$

where t_i are node points of time net $t_i = ih$. On each step the equation (16) represents the non-linear algebraic equation with respect to the coordinate of failure front β_n. For its further solving the following iteration process is used

$$\beta_n^{(k)} = \psi(\beta_n^{(k-1)}), \tag{17}$$

where

$$\psi(\beta_n) = \beta_n + \gamma \left[f(\beta_n) - h \sum_{i=1}^{n-1} M(t_n - t_i)\varphi(\beta_n, \beta_i) - G \right], \tag{18}$$

where parameter γ providing convergence of iteration process had been selected during the numerical experiment.

The numerical realization was carried out for three types of kernels of damaging operator: singular $M(t) = mt^{-\alpha}$, $0 < \alpha < 1$; exponential $M(t) = me^{-\alpha t}$ and constant $M = m = const$, for the following initial witdh of the pipe: $\beta_0 = 0{,}3$.

In fig.1 for Abel's singular kernel $M(t) = me^{-\alpha t}$ the curves are shown which characterize the law of motion of failure front for $\alpha = 0{,}5$, $\sigma_0/p = 3{,}2$ for three values of the parameter of anisotropy: $k = 0{,}9$ (rigidity in the circle direction is less than rigidity in the radial direction); $k = 1$ (the case of isotropy) and $k = 1{,}3$ (rigidity in the circle direction is more than rigidity in the radial direction). As it follows from the figure the increase of rigidity of the circle direction decreases significantly the velocity of motion of failure front, i.e. the process of failure is getting slow. The same happens for the exponential kernel (fig.2) and the constant kernel (fig.3).
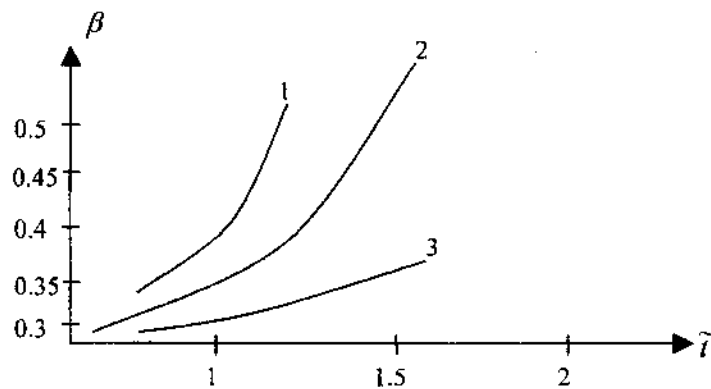
Fig.1. $M = mt^{-\alpha}$; $\tilde{t} = m^{\frac{1}{1-\alpha}}t$; $1 - k = 0.9$; $2 - k = 1$; $3 - k = 1.3$; $\sigma_0/p = 3.2$; $\alpha = 0{,}5$.
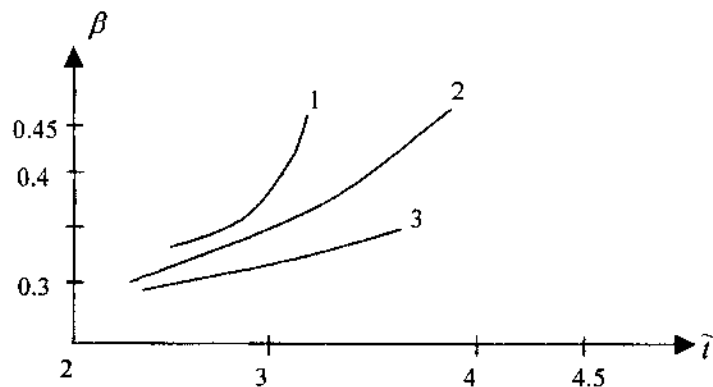
Fig.2. $M = me^{-\alpha t}$; $\tilde{t} = mt$; $1 - k = 0.9$; $2 - k = 1$; $3 - k = 1.3$; $\sigma_0/p = 2.828$; $\alpha = 0{,}5$.
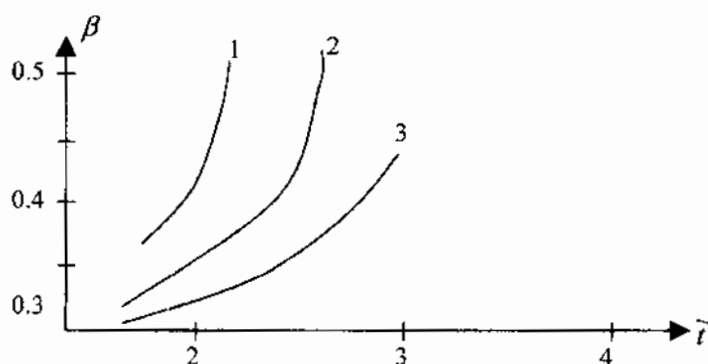
Fig.3. $M = m = const$; $\tilde{t} = mt$; $1 - k = 0.9$; $2 - k = 1$; $3 - k = 1.3$; $\sigma_0 / p = 3.2$.

Influence of the value of the inner pressure on the failure process is represented by the curves in fig. 4-12. From them it follows that with decrease of the value of the pressure the delay of failure process happens, moreover it may be significant even for relative not large decrease of the pressure.
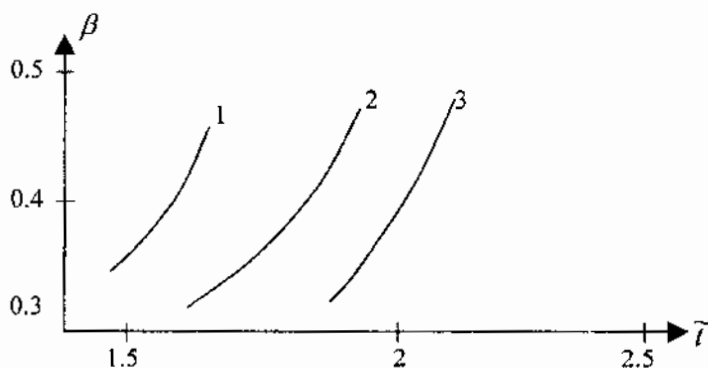

Fig.4. $M = m = const$; $\tilde{t} = mt$; $1 - \sigma_0 / p = 2.829$; $2 - \sigma_0 / p = 3$; $3 - \sigma_0 / p = 3.2$; $k = 3.2$.
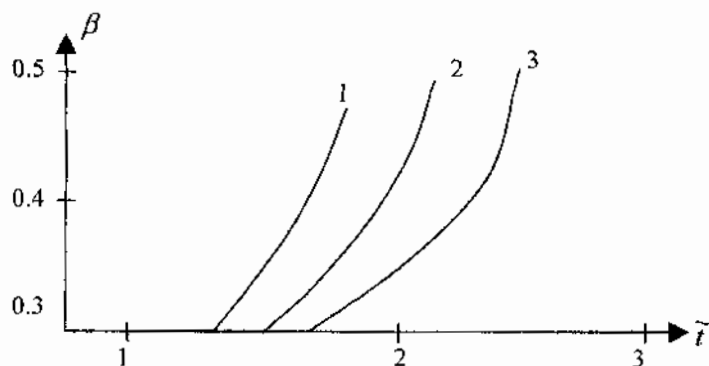
Fig.5. $M = m = const$; $\tilde{t} = mt$; $1 - \sigma_0 / p = 2.829$; $2 - \sigma_0 / p = 3$; $3 - \sigma_0 / p = 3.2$; $k = 1$.
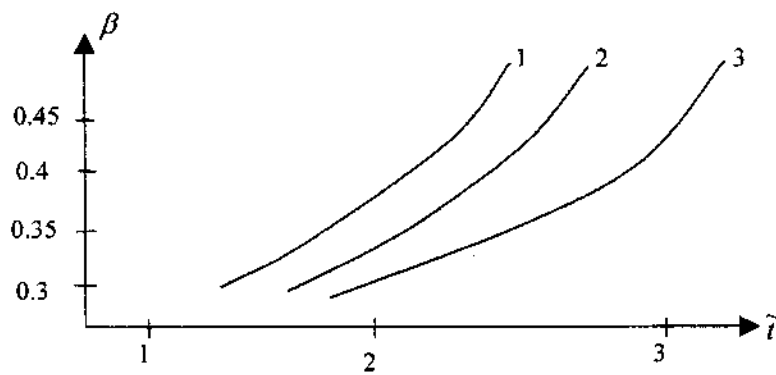
Fig.6. $M = m = const$; $\tilde{t} = mt$; $1 - \sigma_0 / p = 2.829$; $2 - \sigma_0 / p = 3$; $3 - \sigma_0 / p = 3.2$; $k = 1.3$.
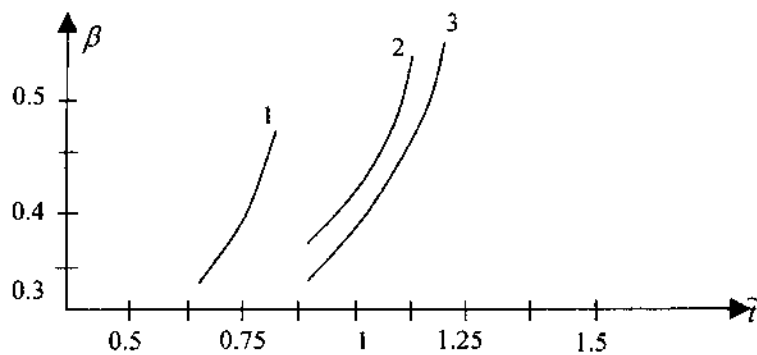

Fig.7. $M = mt^{-\alpha}$; $\tilde{t} = m^{\frac{1}{1-\alpha}}t$; $1 - \sigma_0 / p = 2.829$; $2 - \sigma_0 / p = 3$; $3 - \sigma_0 / p = 3.2$; $k = 0.9$; $\alpha = 0,5$.
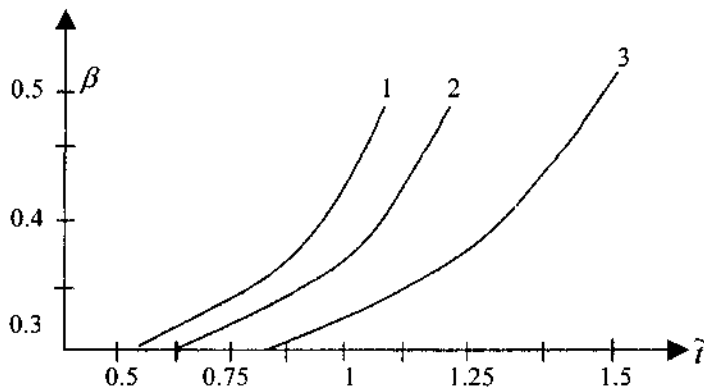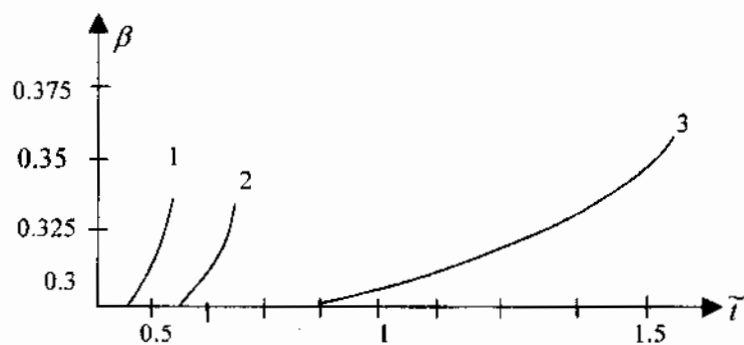

Fig.8. $M = mt^{-\alpha}$; $\tilde{t} = m^{\frac{1}{1-\alpha}}t$; $1 - \sigma_0 / p = 2.829$; $2 - \sigma_0 / p = 3$; $3 - \sigma_0 / p = 3.2$; $k = 1$; $\alpha = 0,5$.

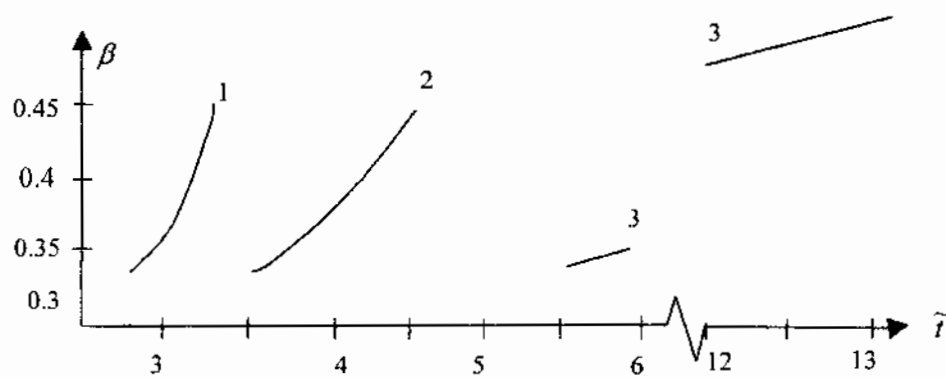**Fig.9.** $M = mt^{-\alpha}; \widetilde{t} = m^{\frac{1}{1-\alpha}} t; 1 -\sigma_0 / p = 2.829\ ; 2 -\sigma_0 / p = 3; 3 -\sigma_0 / p = 3.2; k = 1.3;$
$\alpha = 0,5$.

Fig.10. $M = me^{-\alpha t}; \widetilde{t} = mt; 1 -\sigma_0 / p = 2.829\ ; 2 -\sigma_0 / p = 3; 3 -\sigma_0 / p = 3.2; k = 0.9;$
$\alpha = 0,5$.

Fig.11. $M = me^{-\alpha t}; \widetilde{t} = mt; 1 -\sigma_0 / p = 2.829\ ; 2 -\sigma_0 / p = 3; 3 -\sigma_0 / p = 3.2; k = 1;$
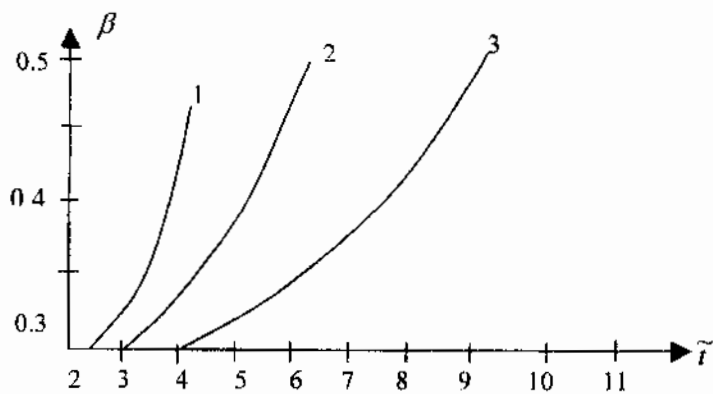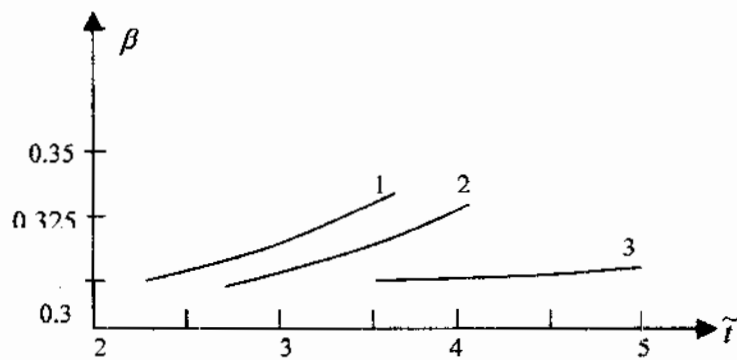$\alpha = 0,5$.

Fig.12. $M = me^{-\alpha t}; \tilde{t} = mt; 1 - \sigma_0 / p = 2.829 ; 2 - \sigma_0 / p = 3; 3 - \sigma_0 / p = 3.2; k = 1.3;$
$\alpha = 0,5$.

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