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THE SCATTERING DATA FOR THE HYPERBOLIC SYSTEM OF THREE  
DIFFERENTIAL EQUATION ON SEMI-AXIS

## Abstract

*In the paper the minimal information is extracted for solution of inverse scattering problem for hyperbolic system of three equations on semi-axis.*

Let us consider on the semi-axis  $x \geq 0$  the hyperbolic system of equations of the first order of the form

$$\xi_i \frac{\partial u_i(x,t)}{\partial t} - \frac{\partial u_i(x,t)}{\partial x} = \sum_{j=1}^3 c_{ij}(x,t) u_j(x,t), \quad i=1,2,3, \quad (1)$$

where  $0 \leq x < +\infty$ ,  $-\infty < t < +\infty$ ,  $\xi_1 > \xi_2 > 0 > \xi_3$ ,  $c_{ii}(x,t) = 0$ ,  $i=1,2,3$ .

It is supposed that elements  $c_{ij}(x,t)$  are the complex-valued measurable by  $x$  and  $t$  functions admitting the estimation

$$\int_{-\infty}^{+\infty} \int_0^{+\infty} |c_{ij}(x,t)|^2 dx dt < +\infty, \quad i, j=1,2,3 \quad (2)$$

Direct and inverse problems of scattering for system (1) on semi-axis for the condition (2) were studied in [1]. But for solution of the inverse problem the data contain more variables than the sought coefficients of equation (1). Here it was managed for supplementary conditions to extract minimal information which is sufficient for solution of the inverse scattering problem.

For the system of equations (1) on the semi-axis two problems are considered. The first problem is the finding of the solution of system (1) by the given falling waves  $a = (a_1, a_2)$  which determine for  $x \rightarrow +\infty$  asymptotics of solutions  $u_1, u_2$  of the form:

$$u_i(x,t) = a_i(t + \xi_i x) + o(1), \quad i=1,2, \quad (3)$$

and which satisfies the boundary conditions:

$$u_3(0,t) = u_1(0,t). \quad (4)$$

The second problem is the finding of the solution of system (1) by the given falling waves  $a = (a_1, a_2)$  and the boundary condition

$$u_3(0,t) = u_2(0,t). \quad (5)$$

We will call the joint consideration of these two problems the scattering problem for system (1) on semi-axis.

**Theorem 1. [1].** *Let coefficients of system (1) satisfy the condition (2). Then there exists the only solution of the scattering problem on semi-axis for system (1) with arbitrary given falling waves  $a_1(s), a_2(s) \in L_2(-\infty, \infty)$ . Moreover,*

$$u_k^k(x,t) = b_k(t + \xi_k x) + o(1), \quad b_k \in L_2(-\infty, \infty), \quad k=1,2 \quad (6)$$

for the  $k$ -th problem.

On the base of Theorem 1 according to (6) two solutions of system (1) – the solutions of the first and second problems – correspond to every vector-function  $a = (a_1, a_2) \in L_2$ , which gives falling waves. These two solutions determine according to

(6) the profiles of two scattering waves  $b = (b_1, b_2) \in L_2$ . So in space  $L_2(R, R_2)$  the operator  $S = \|S_{ij}\|_{i,j=1}^2$  is determined which transforms  $a$  into  $b$ :

$$b = Sa. \quad (7)$$

**Theorem 2[1].** Let coefficients of system (1) satisfy the condition (2). Then the scattering operator of the problem on semi-axis has inverse  $S^{-1} = \|\gamma_{ij}\|_{i,j=1}^2$ . Then  $S = I + F$ ,  $S^{-1} = I + J$ , where  $I$  is unitary operator and  $F$  and  $J$  are Hilbert-Schmidt matrix integral operators. Operators  $(\gamma_{11} + \gamma_{12})^{-1}$ ,  $\gamma_{22}$ ,  $S^{-1}$  admit left factorization

$$(\gamma_{11} + \gamma_{12})^{-1} = (I + N_-)^{-1}(I + N_+), \quad (8)$$

$$\gamma_{22} = (I + M_-)^{-1}(I + M_+), \quad (9)$$

$$S_{11}^{-1} = (I + R_-)^{-1}(I + R_+), \quad (10)$$

Operator  $S_{22} - S_{12}$  has the structure

$$S_{22} - S_{12} = I + G_+. \quad (11)$$

In equalities (8)-(11) operators  $N_-, M_-, R_-, N_+, M_+, R_+, G_+$  are volterra integral operators of the corresponding polarity.

**Theorem 4. [1].** Let potential  $U(x, t) = \|u_{ij}(x, t)\|_{i,j=1}^3$  in system (1) satisfy the condition (2). Then by scattering operator  $S$  for system (1) on semi-axis the coefficients  $u_{ij}(x, t)$  are determined uniquely.

Using these results of [1] let us introduce the scattering data for system (1) on semi-axis.

**Definition.** We will name  $\{S_{11}, S_{12}, S_{22} - S_{12}, \gamma_{21}S_{11}\}$  the scattering data for system (1).

Let us note that from formulas (10) and (48) in [1] it follows that operator  $\gamma_{21}S_{11}$  has the form

$$\gamma_{21}S_{11} = b_-. \quad (12)$$

So operators  $S_{11}, S_{12}, S_{22} - S_{12}, \gamma_{21}S_{11}$  determine volterra operators  $R_-, R_+, (S_{12})_-, (S_{12})_+, G_+, b_-$ . Their quantity corresponds to the quantity of unknown coefficients of the system of equations (1).

Following theorem is valid.

**Theorem.** Let coefficients of system (1) satisfy the conditions (1) and

$$\|\gamma_{11} - I\| \leq q < 1. \quad (13)$$

Then by the scattering data coefficients  $u_{ij}(x, t)$  ( $i, j = 1, 2, 3; i \neq j$ ) are determined uniquely.

**Proof.** Let the scattering data be given

$$\{S_{11}, S_{12}, S_{22} - S_{12}, \gamma_{21}S_{11}\}.$$

Then from the equality

$$\gamma_{21} = (\gamma_{21}S_{11})S_{11}^{-1} \quad (14)$$

and

$$S_{22} = (S_{22} - S_{12}) + S_{12} \quad (15)$$

we find  $\gamma_{21}$  and  $S_{22}$ .

From equality  $SS^{-1} = I$  we obtain

$$S_{11}\gamma_{11} + S_{12}\gamma_{21} = I \quad (16)$$

$$S_{21}\gamma_{11} + S_{22}\gamma_{21} = 0. \quad (17)$$

Hence we have

$$\gamma_{11} = S_{11}^{-1}[I - S_{12}\gamma_{21}], \quad (18)$$

and from conditions (13) and (17)

$$S_{21} = -\gamma_{11}^{-1} S_{22}\gamma_{21}. \quad (19)$$

All elements of operator  $S$  are determined uniquely.

Using theorem 4 [1] we come to the necessary result.

#### References

- [1]. Нижник Л.П., Искендеров Н.Ш. *Обратная нестационарная задача рассеяния для гиперболической системы трех уравнений первого порядка на полупоси*. Укр. мат. журн., 1990, т.42, №7, с.931-938.

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