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THE IMBEDDING THEOREMS FOR THE SPACE OF BESOV-MORREY TYPE WITH DOMINANT MIXED DERIVATIVES

Abstract

In the work the new functional space $S^l_{p,\theta,a,\chi,\tau}B$ *is constructed, the new integral presentation is received and the imbedding theorems for functions from the constructed space are proved.*

Sobolev's and Nicolsky's spaces with dominant mixed derivative (difference) S^r_pW and S^r_pH were included and studied by S.M. Nicolsky [1], Besov space with dominant mixed derivative $S^r_{p,q}B$ - by A.D.Djabrailov [2] and T.I. Amanov [3] by different methods, Sobolev-Luiville space with dominant mixed derivative $S^r_{p,q}L$ - by P.I. Lizorkin and S.M. Nicolsky [4].

Further Sobolev-Morrey space $W^l_{p,a,\chi}$ were included and studied by V.P. Ilyin [6], Nicolsky-Morrey spaces $H^l_{p,\lambda}$ were included and studied by J. Ross [7], Besov-Morrey space $B^l_{p,\theta,a,\chi}$ were included and studied by U.V. Netrusov [8], the space $L^l_{p,\theta,a,\chi}$ was included by V.S. Guliyev and studied in [9]. Note that, the space of Besov-Morrey type $B^l_{p,\theta,a,\chi,\tau}(G)\left(B^l_{p,\theta,a,\chi,\infty}=B^l_{p,\theta,a,\chi}\right)$ and the space of Triebel-Morrey type $L^l_{p,\theta,a,\chi,\tau}\left(L^l_{p,\theta,a,\chi}=L^l_{p,\theta,a,\chi}\right)$ were included by V.S.Guliev and studied in [9], and in [10] Sobolev-Morrey space with dominant mixed derivatives $S^l_{p,a,\chi}W$ was includied and studied.

In this work the space $S^l_{p,\theta,a,\chi,\tau}B(G)$, $p\in[1,\infty)^n$; $a\in[0,1]^n$; $\chi, t, l\in(0,\infty)^n$; $\theta,\tau\in[1,\infty]$ of Besov-Morrey type with dominant mixed derivatives has been constructed. The new integral representations have been obtained and from the point of view of the imbedding theory of some properties of function from this space studied in the case, when domain $G\subset R^n$ satisfies the condition of flexible horn.

Let R^n be n dimensional Euclidean space of points $x=(x_1,...,x_n)$, $G\subset R^n$, $e_n=\{1,2,...,n\}$, $e\subseteq e_n$. The number of all possible subsets e from e_n is equal to 2^n. Let also $k=(k_1,...,k_n)$, $k^e=(k^e_1,...,k^e_n)$, $k^e_j=k_j$ for $j\in e$; $k^e_j=0$ at $j\in e_n\setminus e$;

$$\Delta^{k^e}(t)f(x)=\left(\prod_{j\in e}\Delta^{k_j}_j(t_j)\right)f(x);\quad \Delta^{k_j}_j(t_j)f(x)=\sum_{i=1}^{k_j-i}(-1)^{k_j-i}C^i_{k_j}f(x+it_je^j);$$

$$D^{k^e}f(x)=D_1^{k_1^e}D_2^{k_2^e}...D_n^{k_n^e}f(x);\quad \int_{a^e}^{b^e}f(x)dx^e=\left(\prod_{j\in e}\int_{a_j}^{b_j}dx_j\right)f(x);$$

$$I_{t^\chi}(x)=\left\{y:|y_j-x_j|<t_j^{\chi_j},\ j=1,...,n\right\};$$

$$G_{t^\chi}(x)=G\cap I_{t^\chi}(x);\quad \lfloor t_j\rfloor_1=\min\{1,t_j\},\ j=1,2,...,n.$$

Let's consider for $T \in (0,\infty)^n$ for every $x \in G$ the trajectory

$$\rho(t)=\rho(t,x)=(\rho_1(t_1,x),\rho_2(t_2,x),...,\rho_n(t_n,x)),\quad 0\le t_j\le T_j,\ \ 1\le j\le n,$$

where for all j, $1 \le j \le n$, $\rho_j(0,x)=0$, the functions $\rho_j(u_j,x)$ are absolutely continuous with respect to u_j on $[0,T_j]$ and $|\rho_j'(u_j,x)|\le 1$ for almost all $u_j \in [0,T_j]$, where $\rho_j'(u_j,x)=\frac{\partial}{\partial u_j}\rho_j(u_j,x)$. For $\theta \in (0,1]^n$ we'll call each of the sets $V(x,\theta)=\bigcup_{0\le t_j\le T_j,\, j=1,...,n}[\rho(t,x)+t\theta I]$, $x+V(x,\theta)\subset G$ as flexible horn and point x as top $x+V(x,\theta)$, where $t\theta I=\{(t_1\theta_1 y_1),...,(t_n\theta_n y_n): y\in I\}$. We'll suppose that $x+V(x,\theta)\subset G$. In the case, $t_1=\dots=t_n=t$, $\rho(t,x)=\rho(t^\lambda,x)$, $\theta=(\theta^{\lambda_1},...,\theta^{\lambda_n})$, $\theta\in(0,1]$, $V(x,\theta)=V(\lambda,x,\theta)=\bigcup_{0\le t\le T}[\rho(t^\lambda,x)+t^\lambda\theta^\lambda I]$ is flexible horn λ, included by O.V. Besov [5]. Let $m=(m_1,...,m_n)$, m_j be natural, $k=(k_1,...,k_n)$, k_j be integer non-negative numbers, $m_j > l_j - k_j > 0$, $j = 1,2,...,n$, h, h_0 $t_0 \in (0,\infty)^n$; h_0, t_0 - fixed positive vector.

We denote by $S^l_{p,\theta,a,\chi,\tau}B(G,1)$ Banach space of locally summable functions on G with finite norm

$$\|f\|_{S^l_{p,\theta,a,\chi,\tau}B(G,1)}=\sum_{e\subseteq e_n}\left\{\int_{0^e}^{h_0^e}\left[\frac{\left\|\Delta^{m^e}(h,G_h)D^{k^e}f\right\|_{p,a,\chi,\tau}}{\prod_{j\in e}h_j^{l_j-k_j}}\right]^\theta\prod_{j\in e}\frac{dh_j}{h_j}\right\}^{\frac{1}{\theta}}, \tag{1}$$

where

$$\|f\|_{L_{p,a,\chi,\tau}(G)}=\sup_{x\in G}\left\{\int_0^{t_1^0}\dots\int_0^{t_n^0}\left[\frac{1}{\prod_{j=1}^n[t_j]_1^{\sum_{j=1}^n\chi_j\frac{a_j}{p_j}}}\times\left\{\int_{G_{t_n^{\chi_n}}(x_n)}\left[\dots\left\{\int_{G_{t_2^{\chi_2}}(t_2)}\times\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\times\left(\int_{G_{t_1^{\chi_1}}(x_1)}|f|^{p_1}dy_1\right)^{p_2/p_1}dy_2\right\}^{p_3/p_2}\dots\right]^{p_n/p_{n-1}}dy_n\right\}^{1/p_n}\right]^\tau\prod_{j=1}^n\frac{dt_j}{t_j}\right\}^{1/\tau}. \tag{2}$$

If instead of $\Delta^{m^e}(h,G_h)$ we take $\Delta^{m^e}(h,G)$, then

$$S^l_{p,\theta,a,\chi,\infty}B(G,1)\equiv S^l_{p,\theta,a,\chi,\infty}B(G),$$

$$S^l_{p,\theta,a,\chi,\infty}B(G)\equiv S^l_{p,\theta,a,\chi}B(G);\quad S^l_{p,\infty,a,\chi,\tau}B(G)\equiv S^l_{p,a,\chi,\tau}H(G),$$

$1<p=p_1=...=p_n=\theta<\infty$, l is a non-integer vector,

$$S^l_{p,p,a,\chi,\tau}B(G)\equiv S^l_{p,a,\chi,\tau}W(G),$$

for every l and $p=p_1=\ldots=p_n=\theta=2$

$$S^l_{2,2,a,\chi,\tau}B(G)\equiv S^l_{2,a,\chi,\tau}W(G).$$

Properties of space $S^l_{p,\theta,a,\chi,\tau}B(G)$:

1. $\|f\|_{S^l_{p,\theta}B(G)} \le \|f\|_{S^l_{p,\theta,a,\chi}B(G)} \le C\|f\|_{S^l_{p,\theta,a,\chi,\tau}B(G)}$;
2. normed space $S^l_{p,\theta,a,\chi,\tau}B(G)$ are complete;
3. for $c > 0$ the norms

$$\|f\|_{S^l_{p,\theta,a,c\chi,\tau}B(G)} \quad \text{and} \quad \|f\|_{S^l_{p,\theta,a,\chi,\tau}B(G)}$$

are equivalent;

4. a) $\|f\|_{S^l_{p,\theta,0,\chi,\infty}B(G)} = \|f\|_{S^l_{p,\theta}B(G)}$, b) $\|f\|_{S^l_{\infty,\theta}B(G)} \le \|f\|_{S^l_{p,\theta,1,\chi,\tau}B(G)}$.

Assume

$$\varepsilon_j = l_j - \alpha_j - (1-\chi_j a_j)\left(\frac{1}{p_j} - \frac{1}{q_j}\right), \quad j \in e_n,$$

$$\varepsilon_j^0 = l_j - \alpha_j - (1-\chi_j a_j)\frac{1}{p_j}, \quad j \in e_n.$$

Lemma 1. *Let* $1 \le p \le q \le r \le \infty$, $0 < \chi \le 1$, $0 < t_j \le T_j \le 1$, $0 < \rho_j < \infty$, $j = 1,2,\dots,n$, $f \in L_{p,a,\chi,\tau}(G)$, $1 < \tau < \infty$

$$I_e\left(x, t^e + T^{e_n/e}\right) = \int_{R_n}\int_{-\infty}^{\infty} \Phi_e\left(\frac{y}{t^e + T^{e_n/e}}, \frac{\rho\left(t^e + T^{e_n/e}, x\right)}{t^e + T^{e_n/e}}\right)\xi_e\left(\frac{u}{t^e}, \frac{\rho\left(t^e, x\right)}{2t^e}, \frac{1}{2}p'\left(t^e, x\right)\right) \times$$

$$\times \Delta^{m^e}(\delta u) f\left(x + y + u^e\right) du^e dy,$$

where $\Phi_e(y,z) \in C^\infty(R^n, R^n)$, $\xi_e \in C_0^\infty\left(R^{|e_i|}\right)\Phi_e(\cdot, z)$ *are infinite differentiable functions and finitary uniform with respect to* z *from arbitrary compact. Then the inequality has place:*

$$\sup_{x\in U}\left\|I_e\left(\cdot, t^e + T^{e_n/e}\right)\right\|_{q,U_{\rho^\chi}(\bar{x})} \le C\left\|\prod_{j\in e} t_j^{-l_j}\Delta^{m^e}(t) f\right\|_{p,a,\chi,\tau,G_{T^\chi}(U)} \prod_{j\in e_n/e} T_j^{1-(1-\chi_j a_j)\left(\frac{1}{p_j}-\frac{1}{r_j}\right)} \times$$

$$\times \prod_{j\in e} t_j^{l_j+2-(1-\chi_j a_j)\left(\frac{1}{p_j}-\frac{1}{r_j}\right)} \prod_{j=1}^{n}\left[\rho_j\right]_1^{\chi_j \frac{a_j}{r_j}} \prod_{j=1}^{n} \rho_j^{\chi_j\left(\frac{1}{q_j}-\frac{1}{r_j}\right)}, \tag{3}$$

$U_{\rho^\chi}(\bar{x}) = \left\{x : \left|x_j - \bar{x}_j\right| < \rho_j^{\chi_j},\ j = 1,2,\dots,n\right\}$.

Lemma 2. *Let all conditions of lemma 1 be satisfied* $\eta = (\eta_1,\dots,\eta_n)$, $0 < \eta_j \le T_j$, $\alpha = (\alpha_1,\dots,\alpha_n)$, $\alpha_j \ge 0$ *are integer,* $j = 1,2,\dots,n$

$$K_{\eta e}(x) = \prod_{j\in e_n/e} T_j^{-1-\alpha_j} \int_{0^e}^{\eta^e} \prod_{j\in e} t_j^{-3-\alpha_j} I_e\left(x, t^e + T^{e_n/e}\right) dt^e,$$

$$K_{\eta Te}(x) = \prod_{j\in e_n/e} T_j^{-1-\alpha_j} \int_{\eta^e}^{T^e} \prod_{j\in e_n/e} t_j^{-3-\alpha_j} I_e\left(x, t^e + T^{e_n/e}\right) dt^e.$$

Then the inequalities have place:

$$\sup_{x\in U}\left\|K_{\eta e}(\cdot)\right\|_{q,U_{\rho^\chi}(\bar{x})} \le C_1\left\|\prod_{j\in e} t_j^{-l_j}\Delta^{m^e}(t) f\right\|_{p,a,\chi,\tau,G_{T^\chi}(U)} \prod_{j\in e_n/e} T_j^{-\alpha_j-(1-\chi_j a_j)\left(\frac{1}{p_j}-\frac{1}{q_j}\right)} \times$$

$$\times \prod_{j=1}^{n} \left[\rho_j\right]_1^{\chi_j \frac{a_j}{q_j}} \prod_{j\in e} \eta_j^{\varepsilon_j}, \quad (\varepsilon_j > 0), \tag{4}$$

$$\sup_{x\in U} \left\| K_{\eta T e}(\cdot) \right\|_{q,U_{\rho^\chi}(\bar{x})} \le C_2 \left\| \prod_{j\in e} t_j^{-l_j} \Delta^{m^e}(t) f \right\|_{p,a,\chi,\tau,G_{T\chi}(U)} \prod_{j\in e_n/e} T_j^{-\alpha_j - (1-\chi_j a_j)\left(\frac{1}{p_j}-\frac{1}{q_j}\right)} \times$$

$$\times \prod_{j=1}^{n} \left[\rho_j\right]_1^{\chi_j \frac{a_j}{q_j}} \begin{cases} \prod_{j\in e} T_j^{\varepsilon_j}, \ \varepsilon_j > 0 \\ \prod_{j\in e} \ln \dfrac{T_j}{\eta_j}, \ \varepsilon_j = 0 \\ \prod_{j\in e} \eta_j^{\varepsilon_j}, \ \varepsilon_j < 0 \end{cases} \tag{5}$$

Lemma 3. *Let* $1 \le p \le q \le \infty,\ 0 < \chi \le 1,\ 0 < t_j \le T_j \le 1,\ \alpha = (\alpha_1, \dots, \alpha_n),\ \alpha_j \ge 0$ *be integer,* $j = 1,2,\dots,n;\ 1 \le \tau_1 < \tau_2 \le \infty$ *and let* $\varepsilon_j > 0$. *Then the inequality has place:*

$$\left\| K_{T^e} \right\|_{q,b,\chi,\tau_2;U} \le C \left\| \prod_{j\in e} t_j^{-l_j} \Delta^{m^e}(t) f \right\|_{p,a,\chi,\tau_1,G}. \tag{6}$$

Theorem 1. *Let* $G \subset R^n$ *be the domain with condition of flexible horn,* $1 \le p \le q \le \infty,\ \alpha = (\alpha_1, \dots, \alpha_n), \alpha_j \ge 0$ *be integer* $j = 1,2,\dots,n;\ \tau_1, \tau_2, \theta, \theta_1 \in [1,\infty],\ 1 \le \tau_1 < \tau_2 \le \infty,\ f \in S^l_{p,\theta,a,\chi,\tau_1} B(G),\ \varepsilon_j > 0$, *and let* $j = 1,2,\dots,n$; *then the imbedding has place:*

$$D^\alpha : S^l_{p,\theta,a,\chi,\tau_1} B(G,1) \subset_{\succ} L_{q,b,\chi,\tau_2}(G),$$

more exactly

$$\left\| D^\alpha f \right\|_{q,G} \le C \sum_{e\subseteq e_n} \prod_{j=1}^{n} T_j^{s_{e,j}} \left\{ \int_{0^e}^{t_0^e} \left[\frac{\left\| \Delta^{m^e}(t,G_t) D^{k^e} f \right\|_{p,a,\chi,\tau}}{\prod_{j\in e} t_j^{l_j - k_j}} \right]^\theta \prod_{j\in e} \frac{dt_j}{t_j} \right\}^{\frac{1}{\theta}}, \tag{7}$$

$$\left\| D^\alpha f \right\|_{q,b,\chi,\tau_2;G} \le C_1 \left\| f \right\|_{S^l_{p,\theta,a,\chi,\tau_1} B(G,1)},\ (p \le q < \infty), \tag{8}$$

$$s_{e,j} = \begin{cases} \varepsilon_j,\ j \in e \\ -\alpha_j - (-1 - \chi_j a_j)\left(\dfrac{1}{p_j} - \dfrac{1}{q_j}\right),\ j \in (e_n/e), \end{cases}$$

and, if $\varepsilon_j - l_j^1 > 0$, $j = 1,2,\dots,n;\ \theta < \theta_1$

$$D^\alpha : S^l_{p,\theta,a,\chi,\tau_1} B(G,1) \subset_{\succ} S^{l^1}_{q,\theta_1,b,\chi,\tau_2} B(G,1),$$

$$\left\| D^\alpha f \right\|_{S^{l^1}_{q,\theta_1} B(G,1)} \le C_2 \sum_{e\subseteq e_n} \prod_{j=1}^{n} T_j^{s_{e,j} - l_j^1} \left\{ \int_{0^e}^{t_0^e} \left[\frac{\left\| \Delta^{m^e}(t,G_t) D^{k^e} f \right\|_{p,a,\chi,\tau}}{\prod_{j\in e} t_j^{l_j - k_j}} \right]^\theta \prod_{j\in e} \frac{dt_j}{t_j} \right\}^{\frac{1}{\theta}}, \tag{9}$$

$$\left\| D^\alpha f \right\|_{S^{l^1}_{q,\theta_1,b,\chi,\tau_2} B(G,1)} \le C_2 \left\| f \right\|_{S^l_{p,\theta,a,\chi,\tau_1} B(G,1)},\ (p \le q < \infty), \tag{10}$$

moreover $T \le \min\{1, T_0\}$, C, C_1, C_2, C_3 *are the constants independent on* f, *and* C, C_2 *independent on* T.

Particularly, if $\varepsilon_j^0 > 0$, *then* $D^\alpha f$ *is continuous on* G *and*

$$\sup_{x\in G}\left|D^\alpha f\right| \le C_4 \sum_{e\subseteq e_n}\prod_{j=1}^{n} T_j^{s_{e,j}^0}\left\{\int_{0^e}^{t_0^e}\left[\frac{\left\|\Delta^{m^e}(t,G_t)D^{k^e}f\right\|_{p,a,\chi,\tau}}{\prod_{j\in e} t_j^{l_j-k_j}}\right]^\theta \prod_{j\in e}\frac{dt_j}{t_j}\right\}^{\frac{1}{\theta}}, \tag{11}$$

$$s_{e,j}^0 = \begin{cases}\varepsilon_j^0, \; j\in e\\ -\alpha_j - (1-\chi_j a_j)\dfrac{1}{p_j}, \;\; j\in e_n/e\end{cases}$$

Proof. Let $f \in S_{p,\theta,a,\chi,\tau_1}^l B(G)$. If $\varepsilon_j > 0$, then $l_j - \alpha_j > 0$, $j = 1,2,\dots,n$; since $p \le q$, $0 \le a \le 1$.

$f \in S_{p,\theta,a,\chi,\tau_1}^l B(G) \to S_{p,\theta,a,\chi}^l B(G) \to S_{p,\theta}^l B(G)$, then $D^\alpha f$ exists and $D^\alpha f \in L_p(G)$. Then for almost every point $x \in G$ the integral representation is valid:

$$D^\alpha f = \sum_{e\subseteq e_n}(-1)^{|\alpha|}\prod_{j\in e_n/e}T_j^{-1-\alpha_j}\int_{0^e}^{T^e}\frac{dt^e}{\prod_{j\in e}t_j^{3+\alpha_j}}\int_{R_n}\int_{-\infty}^{\infty}\Psi_e^{(\alpha)}\left(\frac{y}{t^e+T^{e_n/e}},\frac{\rho\left(t^e+T^{e_n/e},x\right)}{t^e+T^{e_n/e}}\right)\times$$

$$\times\zeta_e\left(\frac{u}{t},\frac{\rho(t,x)}{2t},\frac{1}{2}\rho_t'(t,x)\right)\Delta^{m^e}(\delta u)f\left(x+y+u^e\right)du^e dy, \tag{12}$$

where $\Psi_e^{(\alpha)}(\cdot,z) \in C_0^\infty(R^n)$, $\zeta_e \in C_0^\infty\left(R^{|e|}\right)$ their carries are constrained in I^1 and such that, the carrier of representation (12) is contained in $x + V(x,\theta) \subset G$. The parameter of representation $\delta > 0$ is considered sufficient small, so $\Delta^{m^e}(\delta u, G_\delta)f = \Delta^{m^e}(\delta u)f$. Then

$$\left\|D^\alpha f\right\|_{q,G} \le C\sum_{e\subseteq e_n}\left\|K_{eT}(\cdot)\right\|_{q,G}, \tag{13}$$

where

$$K_{eT} = \prod_{j\in e_n/e}T_j^{-1-\alpha_j}\int_{0^e}^{T^e}\frac{dt^e}{\prod_{j\in e}t_j^{3+\alpha_j}}\int_{R^n}\int_{-\infty}^{\infty}\Psi_e^{(\alpha)}\xi_e\Delta^{m^e}(\delta u, G_\delta)f\left(x+y+u^e\right)du^e dy.$$

From inequality (4) $U = G$, $\eta_j = T_j$, $\rho_j \to \infty$, $r_j = q_j$, $j = 1,2,\dots,n$;

$$\begin{aligned}\left\|K_{eT}(\cdot)\right\|_{q,U_{\rho^\chi}} &\le C_1\left\|\prod_{j\in e}t_j^{-l_j}\Delta^{m^e}(t,G_t)f\right\|_{p,a,\chi,\tau,G_{\gamma\chi}(U)}\times\\ &\times\prod_{j\in e_n/e}T_j^{-\alpha_j-(1-\chi_j a_j)\left(\frac{1}{p_j}-\frac{1}{q_j}\right)}\prod_{j\in e}T_j^{\varepsilon_j}\prod_{j=1}^{n}\left[\rho_j\right]_1^{\chi_j\frac{a_j}{q_j}}.\end{aligned} \tag{14}$$

Consequently,

$$\left\|D^\alpha f\right\|_{q,G} \le C_1\sum_{e\subseteq e_n}\prod_{j=1}^{n}T_j^{s_{e,j}}\left\|\prod_{j\in e}t_j^{-l_j}\Delta^{m^e}(t,G_t)f\right\|_{p,a,\chi,\tau,G}.$$

With help of the inequality

$$\left\|\Delta^m(h,G_h)f\right\|_{p,G} \le \frac{c}{h}\int_0^h \left\|\Delta^m(\eta)f\right\|_{p,G} d\eta ,$$

for $1\le\theta\le\infty$, we obtain that

$$\left\|D^\alpha f\right\|_{q,G} \le C_2 \sum_{e\subseteq e_n}\prod_{j=1}^n T_j^{s_{e,j}} \left\{\int_{0^e}^{t_0^e}\left\|\prod_{j\in e} t_j^{-l_j}\Delta^{m^e}(t,G_t)f\right\|_{p,a,\chi,\tau;G}^\theta \prod_{j\in e}\frac{dt_j}{t_j}\right\}^{\frac{1}{\theta}} .$$

For proof of other inequalities we estimate

$$\left\|\Delta^{M^e}(h,G_h)D^\alpha f\right\|_{q,G} .$$

We divide equality (12) into two integrals from 0 to H and from H to T. In the first integral we transfer difference to $\Delta^{m^e} f$, in the second integral we transfer the taking of the difference to the kernel. Then we write the taking of the difference as the integral on $[0,1]^{M^e}$ and we make substitution of variable in it, reduce it to $|e|$-dimensional; after that substitution of variable we transfer the integration from the kernel to function f. After these transformations we obtain the following inequality:

$$\left|\Delta^{M^e}(h,G_h)D^\alpha f\right| \le C_3\sum_{e\subseteq e_n}\prod_{j\in e_n/e}T_j^{-1-\alpha_j}\int_{0^e}^{H^e}\frac{dt^e}{\prod_{j\in e}t_j^{3+\alpha_j}}\times$$

$$\times\int_{R^n}\int_{-\infty}^{\infty}\left|\Psi_e^{(\alpha)}\left(\frac{y}{t^e+T^{e_n/e}},\frac{\rho\left(t^e+T^{e_n/e},x\right)}{t^e+T^{e_n/e}}\right)\right|\left|\zeta_e\left(\frac{u}{t},\frac{\rho(t,x)}{2t},\frac{1}{2}\rho_t'(t,x)\right)\right|\times$$

$$\times\left|\Delta^{M^e}(h\xi)\Delta^{m^e}(\delta u,G_\delta)f\left(x+y+u^e\right)\right|du^e dy + C_4\sum_{e\subseteq e_n}\prod_{j\in e}h_j^{M_j}\prod_{j\in e_n/e}T_j^{-1-\alpha_j}\times$$

$$\times\int_{H^e}^{T^e}\frac{dt^e}{\prod_{j\in e}t_j^{3+\alpha_j+M_j}}\int_{R^n}\int_{-\infty}^{\infty}\left|\Psi_e^{(\alpha+M^e)}\left(\frac{y}{t^e+T^{e_n/e}},\frac{\rho\left(t^e+T^{e_n/e},x\right)}{t^e+T^{e_n/e}}\right)\right|\left|\zeta_e\left(\frac{u}{t},\frac{\rho(t,x)}{2t},\frac{1}{2}\rho_t'(t,x)\right)\right|\times$$

$$\times\int_{0^e}^{1^e}\left|\Delta^{m^e}(\delta u,G_\delta)f\left(x+y+u+Mh\xi^e\right)\right|d\xi^e du^e dy = C_2\sum_{e\subseteq e_n}\left(B_e^1(\cdot,x)+B_e^2(\cdot,x)\right),$$

$$\left\|\Delta^{M^e}(h,G_h)D^\alpha f(\cdot)\right\|_{q,G_{\rho\chi}(\bar{x})} \le C_3\sum_{e\subseteq e_n}\left(\left\|B_e^1(\cdot)\right\|_{q,G_{\rho\chi}(\bar{x})}+\left\|B_e^2(\cdot)\right\|_{q,G_{\rho\chi}(\bar{x})}\right).$$

From inequality (4) it follows, that for $\rho\to\infty$, $H=T$,

$$\left\|B_e^1(\cdot)\right\|_{q,G} \le C_3\prod_{j\in e_n/e}T_j^{-\alpha_j-(1-\chi_j a_j)\left(\frac{1}{p_j}-\frac{1}{q_j}\right)}\prod_{j\in e}T_j^{\varepsilon_j}\left\|\prod_{j\in e}t_j^{-l_j}\Delta^{M^e}(h)\Delta^{m^e}(t,G_t)f\right\|_{p,a,\chi,\tau_1;G} \le$$

$$\le C_4\prod_{j=1}^n T_j^{s_{e,j}}\left\|\prod_{j\in e}t_j^{-l_j}\Delta^{m^e}(t,G_t)f\right\|_{p,a,\chi,\tau_1;G} .$$

From inequality (5) for $\rho\to\infty$ $\left(l_j^1\le M_j,\, j\in e\right)$

$$\left\|B_e^2(\cdot)\right\|_{q,G} \le C_5\prod_{j\in e_n}h_j^{M_j}\prod_{j=1}^n T_j^{s_{e,j}-M_j}\left\|\prod_{j\in e}t_j^{-l_j}\Delta^{m^e}(t,G_t)f\right\|_{p,a,\chi,\tau_1;G} \le$$

$$\leq C_6 \prod_{j\in e} h_j^{l_j^1} \prod_{j=1}^{n} T_j^{s_{e,j}-l_j^1} \left\| \prod_{j\in e} t_j^{-l_j} \Delta^{m^e}(t,G_t) f \right\|_{p,a,\chi,\tau_1;G}, \quad s_{e,j} - l_j^1 > 0$$

hence, for $\theta < \theta_1$

$$\left\{ \int_{0^e}^{h_0^e} \left[\frac{\left\| \Delta^{M^e}(h,G_h) D^\alpha f \right\|_q}{\prod_{j\in e} h_j^{l_j^1}} \right]^{\theta_1} \prod_{j\in e} \frac{dh_j}{h_j} \right\}^{\frac{1}{\theta_1}} \leq$$

$$\leq C_8 \sum_{e\subseteq e_n} \prod_{j=1}^{n} T_j^{s_{e,j}-l_j^1} \left\{ \int_{0^e}^{h_0^e} \left\| \prod_{j\in e} t_j^{-l_j} \Delta^{m^e}(t,G_t) f \right\|_{p,a,\chi,\tau_1}^{\theta} \prod_{j\in e} \frac{dt_j}{t_j} \right\}^{\frac{1}{\theta}} \leq$$

$$\leq C_9 \sum_{e\subseteq e_n} \prod_{j=1}^{n} T_j^{s_{e,j}-l_j^1} \left\{ \int_{0^e}^{h_0^e} \left\| \prod_{j\in e} t_j^{k_j-l_j} \Delta^{m^e-k^e}(t,G_t) D^{k^e} f \right\|_{p,a,\chi,\tau_1}^{\theta} \prod_{j\in e} \frac{dt_j}{t_j} \right\}^{\frac{1}{\theta}}.$$

Taking into account the inequality

$$\left\| K_{eT}(\cdot) \right\|_{q,b,\chi,\tau_2;U} \leq c \left\| \prod_{j\in e} t_j^{-l_j} \Delta^{m^e}(t,G_t) f \right\|_{p,a,\chi,\tau_1;G}$$

inequalities (8) and (10) are proved analogically.

Now, let $\varepsilon_j^0 > 0$. Let's show that then $D^\alpha f$ is continuous on G. On the base of identity (12) and inequality (7) for $q=\infty$, $\varepsilon_j = \varepsilon_j^0 > 0$ we have

$$\left\| D^\alpha f - D^\alpha f_T \right\|_{\infty,G} \leq \sum_{e\subseteq e_n} \left\| K_{eT} \right\|_{\infty,G} \leq$$

$$\leq \sum_{0\neq e\subseteq e_n} \prod_{j=1}^{n} T_j^{s_{e,j}^0} \left\{ \int_{0^e}^{t_0^e} \left\| \prod_{j\in e} t_j^{-l_j} \Delta^{m^e}(t,G_t) f \right\|_{p,a,\chi,\tau}^{\theta} \prod_{j\in e} \frac{dt_j}{t_j} \right\}^{\frac{1}{\theta}},$$

$$\lim_{T\to 0} \left\| D^\alpha f - D^\alpha f_T \right\|_{\infty,G} = 0$$

Since $D^\alpha f_T$ is continuous on G, convergence of $L_\infty(G)$ concides with the uniformity, in the given case, and consequently, $D^\alpha f$ is continuous on G. Also it has been proved that generalized derivative $D^\alpha f$ satisfies Hölder's multiple condition in L_q metrics for f of the constructed space.

Author expresses his sincere gratitude to Prof. A.D. Djabrailov and Prof. V.S. Guliyev for attention to the work.

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Received November 11, 1999; Revised June 21, 2000.
Translated by Nazirova S.H.