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STABILITY OF THE NON-HOMOGENEOUS CIRCLE PLATE ON RADIAL COMPRESSION

Abstract

The problem is solved under the condition that of elasticity and shear modulus depend on three space coordinates.

Detail analysis is carried out for the case of axially symmetric form of bending and when of elasticity and shear modulus depend on moving radius.

As it is known in different branches of machine building and instrument engineering, in civil engineering and other branches of techniques the circle plates fabricated of various non-homogeneous materials widely are used.

It is necessary to remark that ignoring the non-homogeneity at estimation of stability and analysis of frequency-amplitude characteristics of structural elements R can reduce to significant errors [1-2].

Depending on reasons of fabrication and on other aspects the Young modulus, Poisson's coefficient and specific density can be the functions of coordinates of points of the body [3,5]

$$E = E_0 f_1(r,\theta,z); \quad \nu = \nu_0 f_2(r,\theta,z); \quad \rho = \rho_0 f_3(z,\theta,z). \tag{1}$$

Here E_0, ν_0, ρ_0 are correspondingly Young modulus, Poisson coefficient and specific density of a homogeneous material.

It is supposed that functions f_1', f_2, f_3 with their derivatives are continuous functions.

Stress and strains are connected by the following correlations :

$$\begin{aligned} \sigma_1 &= A_1(r,\theta)b_1(z)\varepsilon_1 + A_2(r,\theta)b_2(z)\varepsilon_2, \\ \sigma_2 &= A_3(r,\theta)b_3(z)\varepsilon_1 + A_4(r,\theta)b_4(z)\varepsilon_2 . \end{aligned} \tag{2}$$

Taking that into account the hypothesis by Kirkhgoff and Lyav is valid for also continuous non-homogeneous elastic plate, we can write

$$\begin{aligned} \sigma_1 &= A_1(r,\theta)b_1(z)(l_1 - z\chi_1) + A_2(r,\theta)b_2(z)(l_2 - z\chi_2), \\ \sigma_2 &= A_3(r,\theta)b_1(z)(l_1 - z\chi_1) + A_4(r,\theta)b_4(z)(l_2 - z\chi_2). \end{aligned} \tag{3}$$

Here e_1, e_2, χ_1, χ_2 are correspondingly strains and curvatures of the mean surface:

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}; \quad \chi_2 = -\frac{\partial^2 w}{\partial y^2}, \tag{4}$$

where w is the deflection of the mean surface. Correlations between forces, moments with deformation and curvature of the central surface will have a view:

$$\begin{aligned} T_1 &= A_1(r,\theta)\int_{-h}^{+h} b_1(z)(e_1 - z\chi_1)dz + \\ &+ A_2(r,\theta)\int_{-h}^{+h} b_2(z)(e_2 - z\chi_2)dz, \end{aligned} \tag{5}$$

$$T_2 = A_3(r,\theta)\int_{-h}^{+h} b_3(z)(e_1 - z\chi_1)dz +$$

$$+A_4(r,\theta)\int_{-h}^{+h} b_4(z)(e_2 - z\chi_2)dz,$$

$$M_1 = A_1(r,\theta)\int_{-h}^{+h} zb_1(z)(e_1 - z\chi_1)dz + A_2(z,\theta)\int_{-h}^{+h} zb_2(z)(e_1 - z\chi_2)dz,$$

$$M_2 = A_3(r,\theta)\int_{-h}^{+h} zb_3(z)(e_1 - z\chi_1)dz + A_4(z,\theta)\int_{-h}^{+h} zb_4(z)(e_2 - z\chi_2)dz. \qquad (6)$$

We have the denotations:

$$\begin{array}{lll} a_1 = \int_{-h}^{+h} b_1(z)dz & k_1 = \int_{-h}^{+h} zb_1(z)dz & d_1 = \int_{-h}^{+h} z^2 b_1(z)dz \\ a_2 = \int_{-h}^{+h} b_2(z)dz & k_2 = \int_{-h}^{+h} zb_2(z)dz & d_2 = \int_{-h}^{+h} z^2 b_2(z)dz \\ a_3 = \int_{-h}^{+h} b_3(z)dz & k_3 = \int_{-h}^{+h} zb_3(z)dz & d_3 = \int_{-h}^{+h} z^2 b_3(z)dz \\ a_4 = \int_{-h}^{+h} b_4(z)dz & k_4 = \int_{-h}^{+h} zb_4(z)dz & d_4 = \int_{-h}^{+h} z^2 b_4(z)dz \end{array} \qquad (7)$$

Taking into account (7), the correlations (5,6) take a view:

$$\begin{array}{l} T_1 = A_1(r,\theta)[a_1e_1 - k_1\chi_1] + A_2(r,\theta)[a_2e_2 - k_2\chi_2], \\ T_2 = A_3(r,\theta)[a_3e_1 - k_3\chi_1] + A_4(r,\theta)[a_4e_2 - k_4\chi_2], \\ M_1 = A_1(r,\theta)[k_1e_1 - d_1\chi_1] + A_2(r,\theta)[k_2e_2 - d_2\chi_2], \\ M_2 = A_3(r,\theta)[k_3e_1 - d_3\chi_1] + A_4(r,\theta)[k_4e_1 - d_4\chi_2]. \end{array} \qquad (8)$$

Let us note that for axial symmetric form of loss stability the solution of the problem is rather simplified [1].

The equation of equilibrium and the condition of deformations compability in this case have a view:

$$\frac{d^2M_1}{dr^2} + \frac{2}{r}\frac{dM_1}{dr} - \frac{1}{r}\frac{dM_2}{dr} + 2Ph\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) -$$

$$k_0\left(1 + c_0\varphi(r)\right)w = 0, \qquad (9)$$

$$\frac{d^2 l_1}{dr^2} + \frac{2}{r}\frac{dl_1}{dr} - \frac{1}{r}\frac{dl_2}{dr} = 0.$$

Here $k_0[1 + c_0\varphi(r)]w$ is resistance of the external medium, k_0 and c_0 are positive constant quantities which characterize the properties of the external medium. Function $\varphi(r)$ is a continuous function.

For simplicity of analysis let us consider the case:

$$f_1' = f_1(r);\ f_2 = 1;\ f_3 = 1. \qquad (10)$$

In order to complete the equation of stability and the condition of compatibility of deformations it is necessary that l_1 and l_2 to be expressed by T_1, T_2, χ and χ. After simple transformations we can establish that between the pointed out quantities there is the following correlation:

$$M_1 = D_{11}(r)T_1 + D_{12}(r)T_2 + D_{13}(r)\chi_1 + D_{14}\chi. \qquad (11)$$

Taking off the elementary details let us note that in this case the equations system (8) is decomposed into two independent equations (here it is kept in mind the case $f(r) = 1 + \mu r R^{-1}\ f_2 = 1;\ f_3 = 1;\ R$ is the radius of the external contour of the plate). The first equation is for function of stress Φ and other one is for determination of the defection $\overline{W}$. Taking into account the above mentioned we can show that stability equation with respect to the defection $\overline{W}$ obtain a view:

$$(1+\mu\eta)\frac{d^4\overline{W}}{d\eta^4}+\frac{2+3\mu\eta}{\eta}\frac{d^3\overline{W}}{d\eta^3}+\frac{3\mu\eta-2}{2\eta^2}\frac{d^2\overline{W}}{d\eta^2}+$$
$$+\frac{1}{\eta^3}\frac{d\overline{W}}{d\eta}+9p^*\left(\frac{d^2\overline{W}}{d\eta^2}+\frac{1}{\eta}\frac{d\overline{W}}{d\eta}\right)-\widetilde{k}_0[1+c_0\varphi(\eta)]\overline{W}=0. \tag{12}$$

Here $\widetilde{k}_0=k_0E_0^{-1}$, c_0 are constant quantities

$$P^*=-\frac{\sigma_1R^2}{4E_0h^2},\ \overline{W}=W\cdot h^{-1}.$$

Let us consider the case when the plate is fixed in whole contour. In this case $\overline{W}$ must satisfy the conditions:

$$\overline{W}\Big|_{\eta=1}=0;\quad \frac{d\overline{W}}{d\eta}\Big|_{\eta=1}=0. \tag{13}$$

Denoting the left-hand side of (12) by $F(\eta)$ and choosing $\overline{W}$ in a view [1]:

$$\overline{W}=A(1-\eta^2)^2. \tag{14}$$

using the Bubnov-Galyorkin method we can write:

$$\int_0^1 F(\eta)(1-\eta^2)\eta\, d\eta=0. \tag{15}$$

Substituting (14) into (15) provided that resistance of the external medium is absent, after some transforms we can establish the formula

$$P^*=\frac{8}{3}\left(\frac{2}{3}+\frac{1}{5}\mu\right). \tag{16}$$

For $\mu=0$ from (16) we obtain the solution of the analogous problem for homogeneous medium ignoring the influence of resistance of external medium

$$P_0^*=\frac{16}{9}. \tag{17}$$

Let us note that value of the critic parameters significantly depends on resistance of the external medium. Let us consider the case when the function φ depends on η linearly, that is

$$\varphi(\eta)=1+c_0\eta. \tag{18}$$

Here c_0 is a positive constant quantity.

It is not difficult to show that in case (18) the critic loading is determined by the formula

$$P^*=\frac{8}{3}\left(\frac{2}{3}+\frac{\mu}{5}\right)-\frac{\widetilde{k}_0}{6}(1+1{,}66c_0). \tag{19}$$

From (19) for c_0 we obtain the solution of the analogous problem for the non-homogeneous plate of Winkler resistance. For $\mu=0$ we obtain the solution of the analogous problem for the homogeneous radial compressed plate taking into account the external resistance.

In the case when the function $\varphi(n)$ is quadratic, that is

$$\varphi(\eta) = 1 + c_0\eta^2 . \tag{20}$$

Formula (19) takes the following value.

$$P_0^* = \frac{8}{3}\left(\frac{2}{3} + \frac{M}{5}\right) - \frac{\widetilde{k}_0}{6}(1 + 0{,}42c_0). \tag{21}$$

From (19) and (21) we obtain the correlation between the critic loadings for the cases (18) and (20):

$$P = P_1^* - P_0^* = 1{,}16\widetilde{k}_0 c_0 . \tag{22}$$

For the concrete value of $\widetilde{k}_0$ and c_0 the numerical analysis had been carried out and the graphs were constructed (fig.1,2).

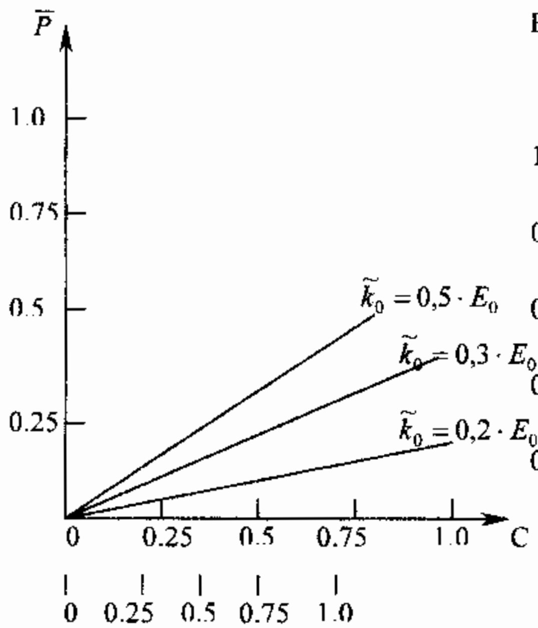

Fig.1.

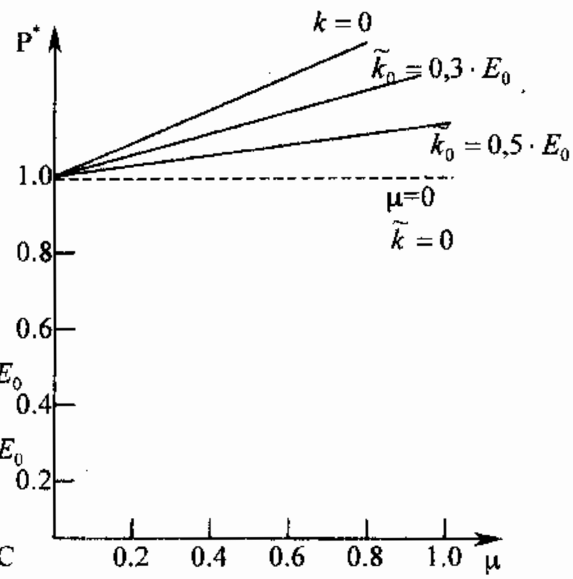

Fig.2.

As it is seen from the constructed graphics non-homogeneity and resistance of the external medium significantly influences on the values of the critic loading.

Let us note that the analogous results had been obtained also for the cases when the function $f(\eta)$ changes so:

$$f(\eta) = 1 + \mu_0 e^{\alpha\eta}; \;\; f(\eta) = 1 + a_1\cos 2\pi n\eta . \tag{23}$$

Here μ_0, α and a_1 are positive numbers and are determined by the experiment [4,5].

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