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OPTIMAL DISTRIBUTION OF THE LOADING IN THE RADIAL ROLLING-CONTACT BEARING TAKING INTO ACCOUNT LIMITATION OF MAXIMAL CONTACT FORCES

Abstract

It is considered the variation problem on determination of the law of optimal distribution of the loading in the radial rolling-contact bearing. It is established the purpose to enhance the durability of the latter. The problem is solved on the base of the Euiler equation with consideration of limitation of maximum contact forces and the condition of equilibrium of the integral ring. Necessary calculation dependencies are obtained.

Investigations of mechanisms of integration of bodies in units of friction of machines and equipment form a special group of the problems of mechanics. Such problems were considered in some works, for example, in [1-5]. One of the interesting and important in practice sense problems of that group is the problem on optimal distribution of the loading between working elements of the rotation bearing and particularly the radial rolling-contact bearing. These bearings are typical units of friction of machines and they are widely used in modern techniques. Their any improve has great significance for national economy.

By many investigations it is known that the character of distribution of forces «in» the bearing and the law of distribution of loading between the rolling bodies have great influence on the stress condition and durability of rolling contact bearings [6-8 and others]. Because of that the question appears on the best distribution of the loading between the working elements of the bearing. The variation problem on optimal distribution of the loading in the radial bearing was formulated for the first time and considered in [8]. There the problem was solved not taking into account the condition of limitation of value of the forces acting between the elements of the bearing. In many cases to avoid plastic deformations and beforehand fatigue failure the limitation for maximal value of the distributed «inside» the bearing loading. In present paper the pointed problem taking into account the pointed limitation is considered.

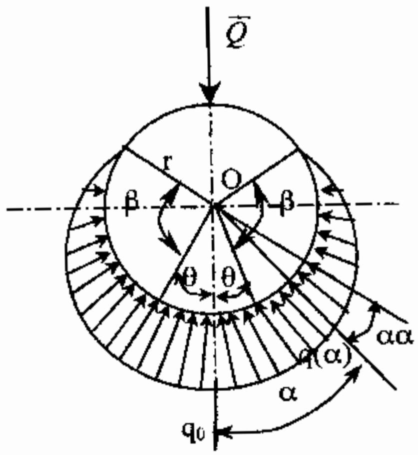
For simplification of the problem we consider that the number of the rolling bodies in the bearing is sufficient large and so discrete distribution of contact forces is designated as continuous. For necessity the passage in calculations to discrete loading is not principal difficult. The method of such passage was given in [8].

Let the redial force $\overline{Q}$ act to the bearing (see fig.). Under action of this force the distributed forces of reaction arise with intensivity $q(\alpha)$, where α is an angular coordinate of an arbitrary point of contact.

Taking into account that durability of the rolling-contact bearing is proportional to the loading, the functional to be minimized can be represented in the form

$$J_1 = 2r\left[q_\theta^m \theta + \int_\theta^{\theta+\beta} q^m(\alpha) d\alpha \right], \tag{1}$$

where r = radius of the inner running way; 2θ = central angle within of which the intensivity of the distributed loading is constant and equal to q_0 (q_0 is the admissible by technical reasoning value of the intensivity of the distributed loading); m = constant characterizing the degree of influence of the loading on the durability of the rolling-contact bearing, β = angle, fixing the states of boundary points of contact. Value of m is determined on the base of experimental investigations. Usually $m>1$. According to reference data for the first approximation we can take: for ball - bearings $m=3$, for roller bearing $m=3{,}33$. By calculations of rolling-contact bearings it is supposed, that minimization of functional J_1 is equivalent to maximization of durability.



Scheme of loading of radial rolling-contact bearing

For any law of distribution of loading the condition of equilibrium of the inner ring of the bearing:

$$2r\left[\int_0^{\theta} q_0 \cos\alpha\, d\alpha + \int_{\theta}^{\theta+\beta} q(\alpha)\cos\alpha\, d\alpha\right] - Q = 0 \tag{2}$$

or

$$q_0 \sin\theta + \int_{\theta}^{\theta+\beta} q(\alpha)\cos\alpha\, d\alpha - \frac{Q}{2r} = 0. \tag{3}$$

must be fulfilled.

Considering θ given value the problem can be formulated in the form:

$$J_2 = \int_{\theta}^{\theta+\beta} q^m(\alpha)d\alpha \to \min \tag{4}$$

taking into account the condition obtained from (3):

$$\int_{\theta}^{\theta+\beta} q(\alpha)\cos\alpha\, d\alpha = \frac{Q}{2r} - q_0 \sin\theta = const. \tag{5}$$

For determination of the sought function $q(\alpha)$ we use Euler equation [9]

$$\frac{\partial F}{\partial q} - \frac{d}{d\alpha}\frac{\partial F}{\partial q'} = 0, \tag{6}$$

where F is the integrand function in (4). Substitution of the integrand function $F=q^m(\alpha)$ in (6) reduces to the trivial solution $q(\alpha)=0$, which does not satisfy (2). Taking into account the above mentioned let's consider the functional

$$J^* = A - J_2, \tag{7}$$

where A is some constant. In this case by virtue of (5), it is advisable to represent this constant in the form

$$A = mB^{m-1}\int_{\theta}^{\theta+\beta} q(\alpha)\cos\alpha \, d\alpha \,, \tag{8}$$

where B is a new constant subjected to determination.

From (7) it follows that the extremum of functional J_2 corresponds to the extremum of functional J^*.

Substituting (4) and (8) in (7), we obtain

$$J^* = \int_{\theta}^{\theta+\beta}\left[mB^{m-1}q(\alpha)\cos\alpha - q^m(\alpha)\right]d\alpha \,. \tag{9}$$

Taking now the integrand according to (9) from the above written Euler equation we find

$$q(\alpha) = B\sqrt[m-1]{\cos\alpha} \,. \tag{10}$$

Position of boundary points is determined from the condition of transversality

$$\left[mB^{m-1}q(\alpha)\cos\alpha - q^m(\alpha)\right]\Big|_{\alpha=\theta+\beta} = 0 \,. \tag{11}$$

Hence taking into account (10) we determine:

$$\theta + \beta = \frac{\pi}{2} . \tag{12}$$

Taking into account (10) and (12) in (3), we have

$$q_0 \sin\theta + B\int_{\theta}^{\pi/2} \sqrt[m-1]{\cos^m\alpha}\, d\alpha - \frac{Q}{2r} = 0 \,. \tag{13}$$

Hence

$$B = \left(\frac{Q}{2r} - q_0\sin\theta\right) \Big/ \int_{\theta}^{\pi/2} \sqrt[m-1]{\cos^m\alpha}\, d\alpha \,. \tag{14}$$

Substituting (14) in (10) we obtain the main calculation formula for determination of the law of distribution of intensivity of the loading:

$$q(\alpha) = \left(\frac{Q}{2r} - q_0\sin\theta\right)\sqrt[m-1]{\cos\alpha} \Big/ \int_{\theta}^{\pi/2} \sqrt[m-1]{\cos^m\alpha}\, d\alpha \,. \tag{15}$$

Angle θ is found from the condition

$$q(\alpha)\big|_{\alpha=\theta} = q_0 \,. \tag{16}$$

Taking (15) into account from this condition we have

$$\sin\theta + \frac{\int_{\theta}^{\pi/2} \sqrt[m-1]{\cos^m\alpha}\, d\alpha}{\sqrt[m-1]{\cos\theta}} - \frac{Q}{2rq_0} = 0 \,. \tag{17}$$

θ can be determined from (17) by numerical methods. From (17) it is possible to find also the value Q for which $\theta = 0$. Let $Q\big|_{\theta=0} = Q^*$. Then according to (17)

$$Q^* = 2rq_0\int_{\theta}^{\pi/2} \sqrt[m-1]{\cos^m\alpha}\, d\alpha \,. \tag{18}$$

For $Q \le Q^*$ it should take $\theta = 0$.

Practical aspect of the considered problem and possible ways of realization of the obtained law of distribution of forces in radial rolling-contact bearings and also the method of passage in calculations from $q(\alpha)$ to discrete loading by rolling bodies were considered in detail in [8] and so they are not reduced here.

References

[1]. Опейко Ф.А. *Математическая теория трения.* Минск, «Наука и техника», 1971, 150 с.

[2]. Галин Л.А. *Контактные задачи теории упругости и вязкоупругости .* М., «Наука», 1980, 304 с.

[3]. Ишлинский А.Ю. *Прикладные задачи механики. Кн. 2. Механика упругих и абсолютно твердых тел.* М., «Наука», 1986, 410 с.

[4]. Горячева И.Г., Добычин М.Н. *Контактные задачи в трибологии.* М., «Машиностроение», 1988, 253 с.

[5]. Галахов М.А., Усов П.П. *Дифференциальные и интегральные уравнения математической теории трения.* М., «Наука», 1990, 278 с.

[6]. Ковалев М.П., Народецкий М.З. *Расчет высокоточных шарикоподшипников.* М., «Машиностроение», 1980, 373 с.

[7]. Рубин А.М. *Контактные задачи для шара и эллиптического желоба при двухточечном начальном касании.* Проблемы машиностроения и надежности машин, №1, 1996, с. 51-54.

[8]. Насибов Н.А. *О законе оптимального распределения нагрузки в радиальной опоре качения.* Ученые записки Азербайджанского института нефти и химии им. М.Азизбекова, №4, 1976, с. 58-62.

[9]. Лернер А.Я., Розенман Е.А. *Оптимальные управления.* М., «Энергия» , 1970, 359 с.

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