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WATER DRIVE OF OIL WITH SEVERAL BOUNDARIES UNDER SPATIAL FILTRATION

Abstract

The work is devoted to spatial filtration process of two fluids with several mobile bounds. The mathematical formulation of the problem which is reduced to the system of integro-differential equations is given.

A spatial filtration process of two fluids with several boundaries are considered. Nowadays for more complete drive of oil from pools the water thickened by various polymers or other substances is used. For study the efficiency of their application it is necessary to carry out hydrodynamic investigations. In this case by using the frontal drive model the problem is reduced in mathematics plan to determination of several mobile interfaces.

The works [1-3] were devoted to the equation of motion of interface of two immiscible fluids in the porous medium in conditions of spatial filtration. In these papers the formula of surface S at any moment of time was given in un-explicit form.

Unlike from these works in the present work the formulas the interface $S_i(t)$ of the fluids are given in the parametric form and the process of spatial filtration of two fluids with several mobile bounds is reduced to the solution of the system of equations.

Let at initial moment of time $t=0$ in an homogeneous stratum with constant thickness H, of penetrability k and porosity m there are two fluids: driving and being drived. The drived fluid occupies the one connected areas $D_i(0)$ bounded upper and below by non-penetrable horizontal planes - the roofing and foundation of the pool, but on the periphery by smooth surfaces $S_i(t)$ $(i=1,...,N)$, and the driving fluid has the remaining part of the pool. Viscosity of the being drived fluid arranged in area $D_i(t)$ is denoted by μ_i and viscosity of the driving fluid - by μ_0. The production wells can be arranged in areas $D_i(t)$ and the injection wells - in area D_0. We denote the general number of wells at moment of time t by $M_2(t)$ $(M_2(t)>0)$. The coordinates of the injection well by (x_j, y_j, z_j) and its production rate by $Q_j(t)$ $(j=1,2,...,M_j(t))$. Let's denote the coordinates of the production wells arranged in area $D_i(t)$ by (x_{ij}, y_{ij}, z_{ij}), and the production rate by $Q_{ij}(t)\,(i=1,2,...,N;\ j=1,2,...,l_i(t))$.

If to consider the rock and its saturating fluids to be non-compressible, the filtration laminar and subjected to Darcy's law, then system of equations of isothermal filtration of the fluids without taking into account capillary and gravitation forces is reduced to Laplace's equation for determination of $P(x,y,z)$ in each of areas $D_i(t)$ and D_0:

$$\Delta P_i = 0;\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \tag{1}$$

$(i=1,2,...,N)$; (x,y,z) are Cartesian coordinates. This equation holds in all filtration area except the particular points (wells), the interface $S_i(t)$ and generally in an infinitely distant point.

Let $\forall t \in [0,T]$ the bounds of separation of phases $S_i(t)$ have not general points and each of them represent a closed orientated non-selfintersecting smooth surface given by the parametric equation

$$x_i = x_i(u,v),\ y_i = y_i(u,v),\ z_i = z_i(u,v) \tag{2}$$

moreover parameters u,v change in some bounded area Δ on the plane oxy.

For any $t \in [0,T]$ on interface of phases $S_i(t)$ the following conditions are fulfilled.

1. By passing through the surface the pressure $S_i(t)$ changes continuously

$$P^+(x,y,z,t) = P^-(x,y,z,t) \tag{3}$$

P^+ is the limit pressure at approach to $S_i(t)$ inside, and P^- is the limit pressure at approach to $S_i(t)$ outside.

2. The flow changes continuously by passing through the surface $S_i(t)$

$$c_i \frac{\partial P^+(x,y,z,t)}{\partial n} = c_0 \frac{\partial P^-(x,y,z,t)}{\partial n}. \tag{4}$$

Here n is the normal to the interface of phases by passing from area $D_i(t)$ to area D_0, $c_i = k/\mu_i$, $c_0 = k/\mu_0$; μ_i is viscosity of the fluid in area $D_i(t)$; μ_0 is viscosity of the driving phase.

We seek distribution of pressure $P(x,y,z,t)$ in the form

$$P(x,y,z,t) = \varphi(x,y,z,t) + \sum_{i=1}^{N} \int_{S_i(t)} \rho_i(\xi_i(\sigma),\eta_i(\sigma),\zeta_i(\sigma),t) R_i^{-1} dS_{Q_i}, \tag{5}$$

where ρ_i is density of Newtonian potential of the simple layer at the point $\theta_i(\xi_i(\sigma),\eta_i(\sigma),\zeta_i(\sigma),t)$ continuously spread along $S_i(t)$

$$R_i\left((x-\xi_i)^2 + (y-\eta_i)^2 + (z-\zeta_i)^2\right)^{1/2};$$

$$\varphi(x,y,z,t) = (2\pi H c_0)^{-1} \sum_{j=1}^{M_i} Q_j(t) R_j^{-1} + (2\pi H)^{-1} \sum_{i=1}^{N} c_i^{-1} \sum_{j=1}^{L_i(t)} Q_{ij}(t) R_{ij}^{-1};$$

$$R_{ij}\left((x-x_{ij})^2 + (y-y_{ij})^2 + (z-z_{ij})^2\right)^{1/2}$$

(x_{ij}, y_{ij}) are the coordinates of the j-th well arranged in area $D_i(t)$.

It is obvious, that the function $P(x,y,z,t)$ satisfies Laplacian equation and on the bound $S_i(t)$ it is continuous.

We can write Darcy's law and condition (4), the velocity of displacement of surface $S_i(t)$ by the normal, in the form:

$$v_{in} = -\frac{c_i}{m}\frac{\partial P^+}{\partial n} = -\frac{c_0}{m}\frac{\partial P^-}{\partial n}.$$

On the base of the property of the potential of the simple layer it follows from (5)

$$p = \left(\frac{1}{c_i} - \frac{1}{c_0}\right)\frac{m}{2\pi} v_{in}. \tag{6}$$

The limit value of the derivative of pressure with respect to normal at approaching to $S_i(t)$ inside is equal to:

$$\frac{\partial P^+}{\partial n}=\frac{\partial \varphi}{\partial n}-\pi\rho+\sum_{i=1}^{N}\int_{S_i(t)}\rho\frac{\partial}{\partial n}R_i^{-1}dS_Q\,. \tag{7}$$

Then using (6), (7) for determination of v_{in} we obtain the following integral equation

$$v_{in}+\frac{\lambda}{2\pi}\sum_{i=1}^{N}\int_{S_i(t)}v_{in}\frac{\partial}{\partial n}R_i^{-1}dS_Q=-\frac{c_0}{m}\frac{\partial \varphi}{\partial n}, \tag{8}$$

where $\lambda=\dfrac{c_0-c_i}{c_i}$.

Motion $S_i(t)$ of interface of two fluids is determined from the system

$$\frac{\partial x_i}{\partial t}=\pm v_{in}\frac{\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial y_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial y_i}{\partial v}\end{vmatrix}}{W},$$

$$\frac{\partial y_i}{\partial t}=\pm v_{in}\frac{\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}}{W},$$

$$\frac{\partial z_i}{\partial t}=\mp v_{in}\frac{\begin{vmatrix}\frac{\partial y_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial y_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}}{W},$$

where

$$W=\left(\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial y_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial y_i}{\partial v}\end{vmatrix}^2+\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial y_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial y_i}{\partial v}\end{vmatrix}^2+\begin{vmatrix}\frac{\partial y_i}{\partial u} & \frac{\partial x_i}{\partial u}\\ \frac{\partial y_i}{\partial v} & \frac{\partial x_i}{\partial v}\end{vmatrix}^2\right)^{1/2}.$$

More detailed dedication of the equation of motion of surface S was given in [4].

Thus, the problem on motion of several interface of fluids in the non-bounded space is reduced to the solution of the system of equations (9)-(12)

$$v_{in}+\frac{\lambda}{2\pi}\sum_{i=1}^{N}\int_{S_i(t)}v_{in}\frac{\partial}{\partial n}R_i^{-1}dS_Q=-\frac{c_0}{m}\frac{\partial \varphi}{\partial n}, \tag{9}$$

$$\frac{\partial x_i}{\partial t}=\pm v_{in}\frac{\begin{vmatrix}\frac{\partial z_i}{\partial u} & \frac{\partial y_i}{\partial u}\\ \frac{\partial z_i}{\partial v} & \frac{\partial y_i}{\partial v}\end{vmatrix}}{W}, \tag{10}$$

$$\frac{\partial y_i}{\partial t} = \pm v_{in} \frac{\begin{vmatrix} \frac{\partial z_i}{\partial u} & \frac{\partial x_i}{\partial u} \\ \frac{\partial z_i}{\partial v} & \frac{\partial x_i}{\partial v} \end{vmatrix}}{W}, \tag{11}$$

$$\frac{\partial z_i}{\partial t} = \mp v_{in} \frac{\begin{vmatrix} \frac{\partial y_i}{\partial u} & \frac{\partial x_i}{\partial u} \\ \frac{\partial y_i}{\partial v} & \frac{\partial x_i}{\partial v} \end{vmatrix}}{W}, \tag{12}$$

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