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TO THE SOLUTION OF THE EQUILIBRIUM PROBLEM OF THE NET

Abstract

The net deformation on a plane is investigated. It is carried out a method for the solution of the equilibrium problem of net and its applied to the concrete task.

According to the continual theory in [1,2,3] some aspects of dynamics of the net were considered.

Problems of mechanics of net are essentially non-linear. Lineriarization of differential equations does not give necessary results since in this case the equations are driven into independent equations of every family of threads.

Here we attempt to solve the problem by numerical method and that is connected with some difficulties taking into account that deformation is finite.

1. Equations of equilibrium.

In the case of plane deformation of the net the equations of equilibrium can be taken from [1] omitting inertia forces, i.e.

$$\begin{aligned}\frac{\partial}{\partial \xi}(\lambda \cos \alpha) + \frac{\partial}{\partial \eta}(\mu \sin \beta) &= 0, \\ \frac{\partial}{\partial \xi}(\lambda \sin \alpha) + \frac{\partial}{\partial \eta}(\mu \cos \beta) &= 0, \\ \frac{\partial}{\partial \eta}[(1 + \lambda) \cos \alpha] &= \frac{\partial}{\partial \xi}[(1 + \mu) \sin \beta], \\ \frac{\partial}{\partial \eta}[(1 + \lambda) \sin \alpha] &= \frac{\partial}{\partial \xi}[(1 + \mu) \cos \beta],\end{aligned}\tag{1}$$

where ξ and η are Lagrange coordinates of particles of the net counted along the corresponding family of threads in the initial non-deformed condition; α and β are the angles formed by the threads of the family with the corresponding coordinates axes; λ and μ are the relative elongation of threads of the family.

Solution of boundary-value problems for system (1) by numerical method is connected with significant difficulties, since it is necessary to solve the system of nonlinear algebraic equations.

With purpose to reduce to minimum the number of nonlinear algebraic equations and maximal affectivity of results instead of the known approaches it is considered the equilibrium of the net with least number of cells. That let reduce to minimum the calculation process.

Instead of formal application of the method of finite differences the precise equations of equilibrium in nodes of the net are written which are obtained from the first two equations of system (1) and the precise geometric correlations for cells are obtained which are obtained from the remained two equations of system (1).

Let's denote horizontal sides of cells by $l_{i,j}$ and vertical sides by $n_{i,j}$ (fig.1 and 2). Let's rewrite system (1) in the form:

$$\lambda_{i+1,j+1} \cos \alpha_{i+1,j+1} - \lambda_{i,j+1} \cos \alpha_{i,j+1} + \mu_{i+1,j+1} \sin \beta_{i+1,j+1} - \mu_{i+1,j} \sin \beta_{i+1,j} = 0,$$

$$\begin{aligned} \lambda_{i+1,j+1} \sin \alpha_{i+1,j+1} - \lambda_{i,j+1} \sin \alpha_{i,j+1} + \mu_{i+1,j+1} \cos \beta_{i+1,j+1} - \mu_{i+1,j} \cos \beta_{i+1,j} &= 0, \\ (1 + \lambda_{i,j+1}) \cos \alpha_{i,j+1} - (1 + \lambda_{i,j}) \cos \alpha_{ij} &= (1 + \mu_{i+1,j}) \sin \beta_{i+1,j} - (1 + \mu_{i,j}) \sin \beta_{i,j}, \\ (1 + \lambda_{i,j+1}) \sin \alpha_{i,j+1} - (1 + \lambda_{i,j}) \sin \alpha_{ij} &= (1 + \mu_{i+1,j}) \cos \beta_{i+1,j} - (1 + \mu_{i,j}) \cos \beta_{i,j}. \end{aligned} \quad (2)$$

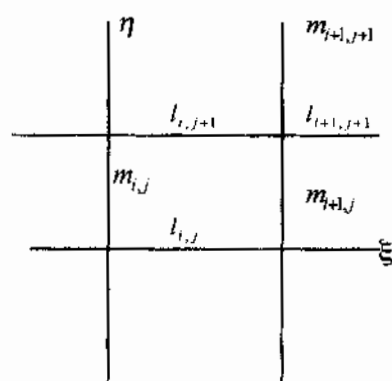


Fig.1

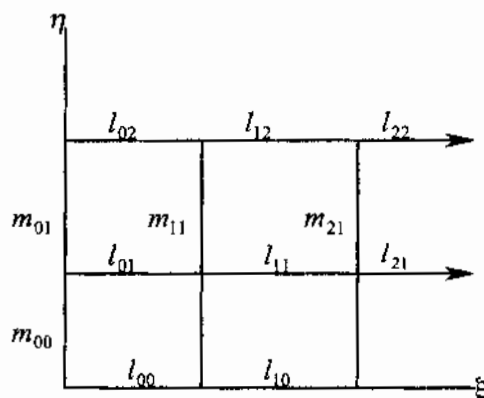


Fig.2

For working out the method let's consider the net of four cells (fig.2). using system (2) with the above pointed way we obtain:

1) $i=0, j=0$

$$\begin{aligned} \lambda_{1,1} \cos \alpha_{11} - \lambda_{0,1} \cos \alpha_{01} + \mu_{1,1} \sin \beta_{11} - \mu_{1,0} \sin \beta_{10} &= 0, \\ \lambda_{1,1} \sin \alpha_{11} - \lambda_{0,1} \sin \alpha_{01} + \mu_{1,1} \cos \beta_{11} - \mu_{1,0} \cos \beta_{10} &= 0, \\ (1 + \lambda_{0,1}) \cos \alpha_{01} - (1 + \lambda_{0,0}) \cos \alpha_{00} &= (1 + \mu_{1,0}) \sin \beta_{10} - (1 + \mu_{0,0}) \sin \beta_{00}, \\ (1 + \lambda_{0,1}) \sin \alpha_{01} - (1 + \lambda_{0,0}) \sin \alpha_{00} &= (1 + \mu_{1,0}) \cos \beta_{10} - (1 + \mu_{0,0}) \cos \beta_{00}; \end{aligned}$$

2) $i=1, j=0$

$$\begin{aligned} \lambda_{2,1} \cos \alpha_{21} - \lambda_{1,1} \cos \alpha_{11} + \mu_{2,1} \sin \beta_{21} - \mu_{2,0} \sin \beta_{20} &= 0, \\ \lambda_{2,1} \sin \alpha_{21} - \lambda_{1,1} \sin \alpha_{11} + \mu_{2,1} \cos \beta_{21} - \mu_{2,0} \cos \beta_{20} &= 0, \\ (1 + \lambda_{1,1}) \cos \alpha_{11} - (1 + \lambda_{1,0}) \cos \alpha_{10} &= (1 + \mu_{2,0}) \sin \beta_{20} - (1 + \mu_{1,0}) \sin \beta_{10}, \\ (1 + \lambda_{1,1}) \sin \alpha_{11} - (1 + \lambda_{1,0}) \sin \alpha_{10} &= (1 + \mu_{2,0}) \cos \beta_{20} - (1 + \mu_{1,0}) \cos \beta_{10}; \end{aligned}$$

3) $i=0, j=1$

$$\begin{aligned} \lambda_{1,2} \cos \alpha_{12} - \lambda_{0,2} \cos \alpha_{02} + \mu_{1,2} \sin \beta_{12} - \mu_{1,1} \sin \beta_{11} &= 0, \\ \lambda_{1,2} \sin \alpha_{12} - \lambda_{0,2} \sin \alpha_{02} + \mu_{1,2} \cos \beta_{12} - \mu_{1,1} \cos \beta_{11} &= 0, \\ (1 + \lambda_{0,2}) \cos \alpha_{02} - (1 + \lambda_{0,1}) \cos \alpha_{01} &= (1 + \mu_{1,1}) \sin \beta_{11} - (1 + \mu_{0,1}) \sin \beta_{01}, \\ (1 + \lambda_{0,2}) \sin \alpha_{02} - (1 + \lambda_{0,1}) \sin \alpha_{01} &= (1 + \mu_{1,1}) \cos \beta_{11} - (1 + \mu_{0,2}) \cos \beta_{02}; \end{aligned}$$

4) $i=1, j=1$

$$\begin{aligned} \lambda_{2,2} \cos \alpha_{22} - \lambda_{1,2} \cos \alpha_{12} + \mu_{2,2} \sin \beta_{22} - \mu_{2,1} \sin \beta_{21} &= 0, \\ \lambda_{2,2} \sin \alpha_{22} - \lambda_{1,2} \sin \alpha_{12} + \mu_{2,2} \cos \beta_{22} - \mu_{2,1} \cos \beta_{21} &= 0, \\ (1 + \lambda_{1,2}) \cos \alpha_{12} - (1 + \lambda_{1,1}) \cos \alpha_{11} &= (1 + \mu_{2,1}) \sin \beta_{21} - (1 + \mu_{1,1}) \sin \beta_{11}, \\ (1 + \lambda_{1,2}) \sin \alpha_{12} - (1 + \lambda_{1,1}) \sin \alpha_{11} &= (1 + \mu_{2,1}) \cos \beta_{21} - (1 + \mu_{1,1}) \cos \beta_{11}. \end{aligned}$$

Hence we obtain

$$l_{11}^1 = \frac{l_{12}^1 \cos \gamma_{12}^1 + l_{21}^2 \sin \gamma_{21}^2 - l_{21}^1 \sin \gamma_{21}^1}{\cos \gamma_{11}^1},$$

$$\sin \gamma_{11}^1 = \frac{(1 + l_{21}^1) \cos \gamma_{21}^1 - 1}{(1 + l_{11}^1)}, \quad (3)$$

Further, it is considered the case of fixing from the left and down of the net, free upper and with the given boundary conditions from the right $l_1 = 0,1; \gamma_1 = 0$

Giving as zero approximation the following

$$l_{11}^1 = 0,1; \quad l_{11}^2 = 0,1; \quad l_{12}^1 = 0,1; \quad l_{12}^2 = 0,1$$

$$l_{21}^1 = 0,1; \quad l_{21}^2 = 0,1; \quad l_{22}^1 = 0,1; \quad l_{22}^2 = 0$$

and putting them into the right-hand sides of equations (3) we will obtain values of all sought quantities in the first approximation. Repeating the procedures it is obtained the solution in the desired approximation

$$\gamma_{11}^1 = -0,0052; \quad l_{11}^1 = 0,1$$

$$\gamma_{21}^1 = 0,0994; \quad l_{21}^1 = 0$$

$$\gamma_{22}^1 = 0,2006; \quad l_{22}^1 = 0,002$$

$$\gamma_{12}^1 = -0,0122; \quad l_{12}^1 = 0,1$$

$$\gamma_{21}^2 = 0; \quad l_{21}^2 = 0,001$$

$$\gamma_{11}^2 = -0,0105; \quad l_{11}^2 = 0,1$$

$$\gamma_{22}^2 = 0; \quad l_{22}^2 = -0,001$$

$$\gamma_{12}^2 = -0,0087; \quad l_{12}^2 = 0,1$$

For boundary data $l_1 = 0,2; \gamma_1 = 0$

$$l_{11}^1 = 0,2; \quad l_{11}^2 = 0,2; \quad l_{12}^1 = 0,2; \quad l_{12}^2 = 0,2$$

$$l_{21}^1 = 0; \quad l_{21}^2 = 0; \quad l_{22}^1 = 0; \quad l_{22}^2 = 0$$

$$\gamma_{11}^1 = -0,0227; \quad l_{11}^1 = 0,1999$$

$$\gamma_{21}^1 = 0,2058; \quad l_{21}^1 = -0,0041$$

$$\gamma_{22}^1 = 0,4135; \quad l_{22}^1 = 0,0087$$

$$\gamma_{12}^1 = -0,0336; \quad l_{12}^1 = 0,2002$$

$$\gamma_{21}^2 = 0,0052; \quad l_{21}^2 = -0,1398$$

$$\gamma_{11}^2 = 0,0035; \quad l_{11}^2 = 0,1995$$

$$\gamma_{22}^2 = -0,0035; \quad l_{22}^2 = -0,1398$$

$$\gamma_{12}^2 = -0,0733; \quad l_{12}^2 = 0,2005$$

for boundary data $l_1 = 0,3; \gamma_1 = 0$

$$l_{11}^1 = 0,3; \quad l_{11}^2 = 0,3; \quad l_{12}^1 = 0,3; \quad l_{12}^2 = 0,3$$

$$l_{21}^1 = 0; \quad l_{21}^2 = 0; \quad l_{22}^1 = 0; \quad l_{22}^2 = 0$$

$$\begin{aligned}
\gamma_{21}^1 &= 0,3088; & l_{21}^1 &= -0,0032 \\
\gamma_{11}^1 &= -0,0558; & l_{11}^1 &= 0,3 \\
\gamma_{22}^1 &= 0,607; & l_{22}^1 &= 0,0211 \\
\gamma_{12}^1 &= -0,0576; & l_{12}^1 &= 0,3005 \\
\gamma_{21}^2 &= -0,0018; & l_{21}^2 &= 0,003 \\
\gamma_{11}^2 &= -0,0785; & l_{11}^2 &= 0,3002 \\
\gamma_{22}^2 &= -0,0018; & l_{22}^2 &= -0,021 \\
\gamma_{12}^2 &= -0,0663; & l_{12}^2 &= 0,3007
\end{aligned}$$

The results of calculations show that on some parts of the net the relative elongations get negative values (in continuous flexible constructions on such parts the creases are formed). As far as in flexible connections (such ones are nets too) negative tension is impossible, then on these parts it should a priori accept the tensions equal to zero.

References

- [1]. Агаларов Д.Т. *Исследование движения сетей при ударе*. Изв. АН Аз.ССР, т. 3, №6, 1982, с. 38-41.
- [2]. Эфендиев А.Н. *Волны в сетях с предварительным напряжением*. Изв. АН Аз.ССР, т. 6, №1, 1985, с. 49-53.
- [3]. Максудов Ф.Г., Искендер-заде Ф.А., Касумов О.К., Эфендиев А.Н. *Исследование плоских волн в сетевых системах*. Докл. АН Азерб., т. 42, №6, 1986, с. 10-15.

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