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ON CONTROLLABILITY AND REVERSIBILITY OF TWO-PARAMETER BILINEAR SEQUENTIAL MACHINES

Abstract

In the article the conceptions of controllability and reversibility of two-parameter bilinear sequential machine are introduced. The sufficient conditions of controllability and reversibility are found.

Let us consider the two-parameter bilinear sequential machine A (further BSM) described by the following equation of state and the initial conditions

$$\begin{cases} s(t+1,\vartheta+1)=[A+u(t,\vartheta)G]s(t,\vartheta)+Bs(t,\vartheta+1)+Ds(t+1,\vartheta), \\ s(t,0)=s(0,\vartheta)=w_0\neq 0, \end{cases} \tag{1}$$

where $A,G,B,D-(n\times n)$ are matrices with components from $GF(p)$, s is a $(n\times 1)$ matrix, i.e. n-dimensional vector-column with components from $GF(p)$, u is scalar.

Definition 1. *Two-parameter BSM A which is in the initial state w_0 is called quite controlled if there exists such natural number M that for any non-zero state s_1 the family of controls $\{u(k,l)\}_{\substack{0\le k\le M\\0\le l\le M}}$ exists which transfers BSM from state w_0 to state s_1.*

Assume that control $u(t,\vartheta)$ has range l with respect to parameter ϑ if $\vartheta=l$. Let $s_{w_0}(k,l)$ be the state of BSM A for $(t,\vartheta)=(k,l)$, initial state w_0 and zero controls upto $l-1$-th range with respect to ϑ if $l\ge 1$ and $s_{w_0}(k,0)=w_0$.

Theorem 1. *In order that two-parameter BSM A to be quite controlled it is sufficient that for any l, $0\le l\le n-1$*

$$rank\left[Gs_{w_0}(n-1,l),BGs_{w_0}(n-2,l),\dots,B^{n-2}Gs_{w_0}(1,l),B^{n-1}Gs_{w_0}(0,l)\right]=n. \tag{2}$$

Proof. Let us find $s(t,1)$ for $t\ge 1$

$$s(t,1)=[A+u(t-1,0)G]s(t-1,0)+Bs(t-1,1)+Ds(t,0)=$$
$$=[A+u(t-1,0)G]w_0+Bs(t-1,1)+Dw_0 .$$

Let's denote

$$\beta_{t-1}(w_0,0)=[A+u(t-1,0)G]w_0+Dw_0 .$$

We'll obtain

$$s(t,1)=\beta_{t-1}(w_0,0)+Bs(t-1,1)=\beta_{t-1}(w_0,0)+B[\beta_{t-2}(w_0,0)+Bs(t-2,1)]=$$
$$=\beta_{t-1}(w_0,0)+B\beta_{t-2}(w_0,0)+B^2s(t-2,1)=\beta_{t-1}(w_0,0)+B\beta_{t-2}(w_0,0)+$$
$$+B^2\beta_{t-3}(w_0,0)+\dots+B^{t-1}\beta_0(w_0,0)+B^tw_0=\sum_{\alpha=1}^{t}B^{\alpha-1}\beta_{t-\alpha}(w_0,0)+B^tw_0 .$$

Therefore,

$$s(n,1)=\sum_{\alpha=1}^{n}B^{\alpha-1}\beta_{n-\alpha}(w_0,0)+B^nw_0=\beta_{n-1}(w_0,0)+B\beta_{n-2}(w_0,0)+\dots+$$
$$+B^{n-1}\beta_0(w_0,0)+B^nw_0=[A+u(n-1,0)G]w_0+Dw_0+B[A+u(n-2,0)G]w_0+$$
$$+BDw_0+\dots+B^{n-1}[A+u(0,0)G]w_0+B^{n-1}Dw_0+B^nw_0=u(n-1,0)Gw_0+,$$
$$+u(n-2,0)BGw_0\cdots+u(0,0)B^{n-1}Gw_0+K(w_0),$$

where $K(w_0)$ does not depend on u.

Thus,

$$s(n,1)-K(w_0)=u(n-1,0)Gw_0+u(n-2,0)BGw_0+\cdots+u(0,0)B^{n-1}Gw_0. \qquad (3)$$

If

$$rank\left[Gw_0, BGw_0,\ldots,B^{n-1}Gw_0\right]=n, \qquad (4)$$

then system (3) is solvable for any $s(n,1)$, i.e. BSM A is quite controlled.

Let's compare (4) with (2). Since

$$s_{w_0}(n-1,0)=s_{w_0}(n-2,0)=\ldots=s_{w_0}(0,0)=w_0,$$

so we notice that (4) coincide with condition (2) for $l=0$.

Thus, if (2) is fulfilled for $l=0$, then the theorem has been proved.

Let (2) be not fulfilled for $l=0$. Let's find $s(t,2)$ for $t\geq 1$.

$$s(t,2)=[A+u(t-1,1)G]s(t-1,1)+Bs(t-1,2)+Ds(t,1)=[A+u(t-1,1)G]s(t-1,1)+$$
$$+D[\beta_{t-1}(w_0,0)+Bs(t-1,1)]+Bs(t-1,2)=[A+u(t-1,1)G+DB]s(t-1,1)+$$
$$+D\beta_{t-1}(w_0,0)+Bs(t-1,2).$$

Let's denote

$$[A+u(t-1,1)G+DB]s(t-1,1)+D\beta_{t-1}(w_0,0)=\beta_{t-1}(w_0,1).$$

We'll obtain

$$s(t,2)=\beta_{t-1}(w_0,1)+Bs(t-1,2)=\beta_{t-1}(w_0,1)+B[\beta_{t-2}(w_0,1)+Bs(t-2,2)]=$$
$$=\beta_{t-1}(w_0,1)+B\beta_{t-2}(w_0,1)+\cdots+B^{t-1}\beta_0(w_0,1)+B^t w_0=\sum_{\alpha=1}^{t}B^{\alpha-1}\beta_{t-\alpha}(w_0,1)+B^t w_0.$$

Then

$$s(n,2)=\sum_{\alpha=1}^{n}B^{\alpha-1}\beta_{n-\alpha}(w_0,1)+B^n w_0=\beta_{n-1}(w_0,1)+B\beta_{n-2}(w_0,1)+\cdots+B^{n-1}\beta_0(w_0,1)+B^n w_0.$$

Let's accept that the controls of zero range with respect to ϑ are equal to zero. Then $\beta_k(w_0,0)$ becomes the vector dependent on K and independent on controls of first range with respect to ϑ

$$s(n,2)=[A+u(n-1,1)G+DB]s_{w_0}(n-1,1)+B[A+u(n-2,1)G+DB]s_{w_0}(n-2,1)+\cdots+$$
$$+B^{n-1}[A+u(0,1)G+DB]s_{w_0}(0,1)+B^n w_0+K_1(w_0).$$

Here $K_1(w_0)$ does not depend on first range controls with respect to ϑ

$$s(n,2)=u(n-1,1)Gs_{w_0}(n-1,1)+u(n-2,1)BGs_{w_0}(n-2,1)+\cdots+$$
$$+u(0,1)B^{n-1}Gs_{w_0}(0,1)+K_2(w_0).$$

Then

$$s(n-2)-K_2(w_0)=u(n-1,1)Gs_{w_0}(n-1,1)+u(n-2,1)BGs_{w_0}(n-2,1)+$$
$$+\cdots+u(0,1)B^{n-1}Gs_{w_0}(0,1).$$

If

$$rank\left[Gs_{w_0}(n-1,1), BGs_{w_0}(n-2,1),\ldots,B^{n-1}Gs_{w_0}(0,1)\right]=n, \qquad (5)$$

then BSM A is quite controlled.

But (5) coincides with (2) for $l=1$. Thus, if condition (2) is fulfilled for $l=1$, then the theorem has been proved.

Now let (2) be fulfilled for some $l\leq n-1$. Let's find $s(t,l+1)$ for $t\geq 1$. We have

$$s(t,l+1)=[A+u(t-1,l)G]s(t-1,l)+Bs(t-1,l+1)+Ds(t,l).$$

Let's denote

$$[A+u(t-1,l)G]s(t-1,l)+Ds(t,l)=\beta_{t-1}(w_0,l).$$

We'll obtain

$$s(t,l+1)=\beta_{t-1}(w_0,l)+Bs(t-1,l+1)=\beta_{t-1}(w_0,l)+B\beta_{t-2}(w_0,l)+\cdots+ \\ +B^{t-1}\beta_0(w_0,l)+B^t w_0.$$

Further

$$s(n,l+1)=\beta_{n-1}(w_0,l)+B\beta_{n-2}(w_0,l)+\cdots+B^{n-1}\beta_0(w_0,l)+B^n w_0.$$

Suppose that all controls up to $l-1$-th range with respect to ϑ are equal to zero. Then

$$s(n,l+1)=u(n-1,l)Gs_{w_0}(n-1,l)+u(n-2,l)BGs_{w_0}(n-2,l)+\cdots+ \\ +u(0,l)B^{n-1}Gs_{w_0}(0,l)+\widetilde{K}(w_0),$$

where $\widetilde{K}(w_0)$ does not depend on controls of l-th range with respect to ϑ.

Further

$$s(n,l+1)-\widetilde{K}(w_0)=u(n-1,l)Gs_{w_0}(n-1,l)+u(n-2,l)BGs_{w_0}(n-2,l)+\cdots+ \\ +u(0,l)B^{n-1}Gs_{w_0}(0,l)$$

and it is clear that the condition

$$rank\left[Gs_{w_0}(n-1,l),BGs_{w_0}(n-2,l),\ldots,B^{n-1}Gs_{w_0}(0,l)\right]=n$$

provides quite controllability of BSM. The theorem has been proved $(M=n)$.

Remark 1. Let's give recursion relations for determination of $s_{w_0}(k,l)$.

For $l=0$ $s_{w_0}(k,0)=w_0$. If $l\geq 1$, then

$$s_{w_0}(k,l)=\sum_{\alpha=1}^{k}B^{\alpha-1}\gamma_{k-\alpha}(w_0,l-1)+B^k w_0,$$

where $\gamma_{k-1}(w_0,l-1)=As_{w_0}(k-1,l-1)+Ds_{w_0}(k,l-1)$.

Remark 2. For $l=0$ condition (2) has the simple form

$$rank\left[Gw_0,BGw_0,\ldots,B^{n-1}Gw_0\right]=n. \tag{6}$$

For $l=1$ supposing in addition commutativity of matrices B and G, we'll obtain

$$s_{w_0}(k,1)=\sum_{\alpha=1}^{k}B^{\alpha-1}\gamma_{k-\alpha}(w_0,0)+B^k w_0.$$

But

$$\gamma_{k-1}(w_0,0)=As_{w_0}(k-1,0)+Ds_{w_0}(k,0)=(A+D)w_0.$$

So

$$s_{w_0}(k,1)=\sum_{\alpha=1}^{k}B^{\alpha-1}(A+D)w_0+B^k w_0.$$

Substituting in (2), we have

$$rank\left[G\left(I+B+\cdots+B^{n-2}\right)(A+D)w_0+GB^{n-1}w_0,BG\left(I+B+\cdots+B^{n-3}\right)(A+D)w_0+\right. \\ \left.+BGB^{n-2}w_0,\ldots B^{n-2}G(A+D)w_0+B^{n-2}GBw_0,B^{n-1}Gw_0\right].$$

Let's subtract the second column from the first, the third one from the second, ..., the n-th column from the $n-1$-th one and we'll obtain

$$rank\left[G(A+D)w_0,BG(A+D)w_0,\ldots,B^{n-2}G(A+D)w_0,B^{n-1}Gw_0\right]=n. \tag{7}$$

Therefore, (6) and (7) are simple, easily verifiable sufficient conditions of controllability.

Remark 3. Let at some moment of time (k_0,l_0) BSM A be in the state $w_0 \neq 0$ and the matter is in that whether to transfer BSM to state $s_1 \neq 0$ will the help of the chain of controls $\{u(k,l)\}_{\substack{k_0\le k\le k_0+M \\ l_0\le l\le l_0+M}}$.

In this case the problem will be formulated correctly if in addition to the state $s(k_0,l_0)=w_0$ the states $s(k_0,l)$ and $s(k,l_0)$ $(k_0 \le k \le k_0+M,\ l_0 \le l \le l_0+M)$ are known, moreover

$s(k,l_0)=w_1(k)$,

$s(k_0,l)=w_2(l)$,

$w_1(k_0)=w_2(l_0)=w_0$ (condition of concordance).

Then this is equivalent to the following:

$$\begin{cases} s(t+1,\vartheta+1)=[A+u(t,\vartheta)G]s(t,\vartheta)+Bs(t,\vartheta+1)+Ds(t+1,\vartheta), \\ s(t,0)=w_1(t), \\ s(0,\vartheta)=w_2(\vartheta), \\ w_1(0)=w_2(0)=w_2 \neq 0; \quad 0\le\vartheta,\ \ t\le M. \end{cases}$$

In this case the theorem is remained valid and condition (2) takes the form

$$rank\left[Gs_{\overline{w}}(n-1,l),\ BGs_{\overline{w}}(n-2,l),\dots,B^{n-2}Gs_{\overline{w}}(1,l),B^{n-1}Gw_2(l)\right]=n,$$

where $s_{\overline{w}}(k,l)$ is the state of BSM at moment of time (k,l) for zero controls up to $l-1$-th range, if $l\ge 1$, and $s_{\overline{w}}(k,0)=w_1(k)$ for $l=0$. The recursion relations are changed for determination of $s_{\overline{w}}(k,l)$.

For $l=0$

$$s_{\overline{w}}(k,0)=w_1(k).$$

For $l\ge 1$

$$s_{\overline{w}}(k,l)=As_{\overline{w}}(k-1,l-1)+Ds_{\overline{w}}(k,l-1)+Bs_{\overline{w}}(k-1,l),$$

and if we denote

$$\gamma_{k-1}(\overline{w},l-1)=As_{\overline{w}}(k-1,l-1)+Ds_{\overline{w}}(k,l-1),$$

then

$$s_{\overline{w}}(k,l)=\gamma_{k-1}(\overline{w},l-1)+Bs_{\overline{w}}(k-1,l)$$

and finally

$$s_{\overline{w}}(k,l)=\sum_{\alpha=1}^{k}B^{\alpha-1}\gamma_{k-\alpha}(\overline{w},l-1)+B^k w_2(l).$$

Let two-parameter BSM A over field $GF(p)$ be described by the equation of state (1`) and the equation of output:

$$\begin{aligned} &y(t,\vartheta)=Cs(t,\vartheta), \\ &s(t,0)=w_1(t), \\ &s(0,\vartheta)=w_2(\vartheta), \qquad\qquad 0\le t,\quad \vartheta\le M, \\ &w_1(0)=w_2(0)=w_0\neq 0, \end{aligned} \tag{8}$$

where C is the matrix-row of dimension $(1\times n)$ and (1`) is obtained from (1) by substitution of initial conditions from (1) by the initial conditions from (8).

Definition 2. *Two-parameter BSM A described by equations (1`) and (8) is called reversible with respect to the state w_0, if there exists such positive integer $N\le M_1$ that for any input action $u(0,0)$ the sequence of test inputs* $\{u(k,l)\}_{\substack{k\le M\le N \\ l\le M\le N}}$, $M=M(w_0)$

under whose action signal $u(0,0)$ *is determined uniquely by output* $y(\bar{k},\bar{l})$, *where* $\max\{\bar{k},\bar{l}\} = M+1$.

For matrices of BSM A *the following natural conditions must be fulfilled*

I) $C \neq 0$. *If* $C = 0$, $y(t,\vartheta) = 0 \quad \forall t,\vartheta$, *i.e.* $y(t,\vartheta)$ *independent on* $u(0,0)$;

II) $Gw_0 \neq 0$. *If* $Gw_0 = 0$, *then* $s(1,1) = As(0,0) + Bs(0,1) + Ds(1,0)$,

i.e. all further conditions of BSM A *do not depend on* $u(0,0)$ *and therefore* $y(t,\vartheta)$ *does not depend on* $u(0,0)$.

Further, not stipulate that particularly we will consider the conditions I) and II) satisfied.

Theorem 2. *In order that two-parameter BSM* A *to be reversible with respect to state* w_0, *it is sufficient though one of two conditions to be fulfilled:*

I) $rank\left[s, Bs, \ldots, B^{n-1}S\right] = n \quad \forall s \neq 0$;

II) $rank\left[s, Ds, \ldots, D^{n-1}s\right] = n \quad \forall s \neq 0$.

Proof. We'll denote by V the vectors independent on $u(0,0)$, and by q the numbers independent on $u(0,0)$.

We have from (1) and (8)

$$s(1,1) = [A + u(0,0)G]s(0,0) + Bs(0,1) + Ds(1,0) = [A + u(0,0)G]w_0 + \\ + Bw_2(1) + Dw_1(1) = V + Gw_0u(0,0),$$

$$y(1,1) = Cs(1,1) = CV + CGw_0u(0,0) = q + CGw_0u(0,0).$$

Two cases are possible

i) $CGw_0 \neq 0$;

ii) $CGw_0 = 0$.

Suppose that case i) takes place. Then

$$u(0,0) = \frac{y(1,1) - q}{CGw_0}$$

and BSM A is reversible without test signals $(M = 0)$. Suppose now that case ii) takes place. Let's give test signal A to input BSM $u(1,0) = 0$. We have

$$s(1,2) = [A + u(1,0)G]s(1,0) + Bs(1,1) + Ds(2,0) = Aw_1(1) + B[V + Gw_0u(0,0)] + \\ + Dw_1(2) = V + BGw_0u(0,0),$$

$$y(2,1) = Cs(2,1) = CV + CBGw_0u(0,0) = q + CBGw_0u(0,0).$$

Two cases are possible

j) $CBGw_0 \neq 0$;

jj) $CBGw_0 = 0$.

If case j) takes place, then

$$u(0,0) = \frac{y(2,1) - q}{CBGw_0}$$

and BSM A is reversible with the help of input test signal $u(1,0) = 0$ $(M = 1)$.

If case jj) takes place, then let's give new test signal $u(2,0) = 0$ to input of BSM A. Then

$$s(3,1) = [A + u(2,0)G]s(2,0) + Bs(2,1) + Ds(3,0) = Aw_1(2) + B[V + BGw_0u(0,0)] + \\ + Dw_1(3) = V + B^2Gw_0u(0,0),$$

$$y(3,1)=Cs(3,1)=CV+CB^2Gw_0u(0,0)=q+CB^2Gw_0u(0,0).$$

If $CB^2Gw_0 \neq 0$, then

$$u(0,0)=\frac{y(3,1)-q}{CB^2Gw_0}$$

and BSM A is reversible with the help of input test signals

$$u(1,0)=u(2,0)=0 \ (M=2).$$

If $CB^2Gw_0=0$, then we continue the process. Let's prove that not longer than for n steps the initial control $u(0,0)$ will be found. Let's assume the contrary, then in our chain alternatives of type ii) and jj) exist, i.e.

$$CGw_0=0, \ CBGw_0=0, \ CB^2Gw_0=0, \ \ldots, \ CB^{n-1}Gw_0=0.$$

Let's denote Gw_0 by s. By condition $s \neq 0$. Let

$$\Gamma_1=s, \ \Gamma_2=Bs, \ \Gamma_3=B^2s, \ \ldots, \ \Gamma_n=B^{n-1}s.$$

Since by condition I) of theorem 2

$$rank[\Gamma_1,\Gamma_2,\ldots,\Gamma_n]=n,$$

then vectors $\Gamma_1,\Gamma_2,\ldots,\Gamma_n$ are linear independent and the equalities

$$C\Gamma_1=C\Gamma_2=\ldots=C\Gamma_n=0$$

imply $C=0$ and that is impossible.

Consequently, not longer than for n steps the initial control $u(0,0)$ will be found.

If condition II) of theorem 2 is fulfilled, then for $CGw_0=0$ we give test signal $u(0,1)=0$ to input of BSM and consider the condition $s(1,2)$

$$s(1,2)=[A+u(0,1)G]s(0,1)+Bs(0,2)+Ds(1,1)=Aw_2(1)+Bw_2(2)+$$
$$+D[V+Gw_0u(0,0)]=V+DGw_0u(0,0)$$
$$y(1,2)=CV+CDGw_0u(0,0)=q+CDGw_0u(0,0).$$

If $CDGw_0 \neq 0$, then

$$u(0,0)=\frac{y(1,2)-q}{CDGw_0}$$

and BSM A is reversible with the help of input test signal $u(0,1)=0$.

But if $CDGw_0=0$, then $u(0,0)$ we find by analogy way that not more than for n steps using condition II). The theorem has been proved.

Theorem 3. *Let matrices A,B and D are pairwise permutative. For reversibility of two-parameter BSM A with respect to condition w_0 it is sufficient that for any $s \neq 0$*

$$rank[(A+2BD)s,(2A+3BD)Bs,\ldots,(nA+(n+1)BD)B^{n-1}s]=n.$$

Proof. Let test signals $u(0,1)=u(1,0)=u(1,1)=0$ be given to input of BSM A.

Then

$$s(2,2)=[A+u(1,1)G]s(1,1)+BS(1,2)+Ds(2,1)=A(V+Gw_0u(0,0))+$$
$$B(V+DGw_0u(0,0))+D(V+BGw_0u(0,0))=V+[A+BD+DB]Gw_0u(0,0)=$$
$$=V+[A+2BD]Gw_0u(0,0),$$
$$y(2,2)=Cs(2,2)=q+C[A+2BD]Gw_0u(0,0).$$

If $C[A+2BD]Gw_0 \neq 0$, then

$$u(0,0)=\frac{y(2,2)-q}{C[A+2BD]Gw_0}$$

and BSM A is reversible with the help of input test signals $u(0,1)=u(1,0)=u(1,1)=0$ $(M=1)$.

Let $C[A+2BD]Gw_0=0$. Let's give the input $u(2,1)=0$ to input of BSM A. We have

$$s(3,2)=[A+u(2,1)G]s(2,1)+Bs(2,2)+Ds(3,1)=A[V+BGw_0u(0,0)]+$$
$$+B[V+(A+2BD)Gw_0u(0,0)]+D[V+B^2Gw_0u(0,0)]=$$
$$=V+[AB+B(A+2BD)+DB^2]Gw_0u(0,0)=V+[AB+AB+3DB^2]Gw_0u(0,0)=$$
$$=V+[2A+3DB]BGw_0u(0,0),$$
$$y(3,2)=q+C[2A+3BD]BGw_0u(0,0).$$

If $C[2A+3BD]BGw_0\neq 0$, then BSM A is reversible. If $C[2A+3BD]BGw_0=0$, then we give the test input signal $u(3,1)$ to the input of BSM. Then

$$s(4,2)=[A+u(3,1)G]s(3,1)+Bs(3,2)+Ds(4,1)=A[V+B^2Gw_0u(0,0)]+$$
$$B[V+(2A+3BD)BGw_0u(0,0)]+D[V+B^3Gw_0u(0,0)]=$$
$$=V+[AB^2+B(2A+3BD)B+DB^3]Gw_0u(0,0)=V+[AB^2+2AB^2+4DB^3]Gw_0u(0,0)=$$
$$=V+(3A+4DB)B^2Gw_0u(0,0),$$
$$y(4,2)=Gs(4,2)=q+C[3A+4BD]B^2Gw_0u(0,0).$$

If now $C[3A+4BD]B^2Gw_0\neq 0$, then BSM is reversible. And if $C[3A+4BD]B^2Gw_0=0$, then we continue the process.

Let's demonstrate that not longer than for n steps the initial control $u(0,0)$ will be found.

Let's assume the contrary. Then the following equalities have place:

$$C(A+2BD)Gw_0=0,\ C(2A+3BD)BGw_0=0,\dots,C(nA+(n+1)BD)B^{n-1}Gw_0=0.$$

Let's denote $Gw_0=s\neq 0$

$$\Gamma_1=(A+2BD)s,\ \Gamma_2=(2A+3BD)Bs,\ \dots,\ \Gamma_n=(nA+(n+1)BD)B^{n-1}s\ .$$

By the condition of the theorem the vectors $\Gamma_1,\Gamma_2,\dots,\Gamma_n$ are linear independent, i.e. equalities $C\Gamma_1=C\Gamma_2=\dots=C\Gamma_n=0$ imply $C=0$, and that is impossible.

Consequently, not longer than for n steps the initial control $u(0,0)$ will be found. Theorem has been proved.

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Received April 19, 2000; Revised September 28, 2000.
Translated by Soltanova S.M.