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HARNACK'S INEQUALITY FOR THE SOLUTIONS OF KOLMOGOROV'S EQUATION

Abstract

In the work the non-divergent equation by Kolmogorov whose coefficient satisfies Cordes condition is considered. For the non-negative solutions of this equation in parabolic cylinders by Harnack type inequality is proved.

In the present work it is considered the operator

$$L = a(x,y,t)\frac{\partial^2}{dx^2} - x\frac{\partial}{\partial y} - \frac{\partial}{\partial t}, \qquad (1)$$

where $(x,y,t)\in R^3$ and with respect to the coefficient $a(x,y,t)$ the fulfillment of measurability and ellipticity conditions

$$0 < C_1 \le a(x,y,t) \le C_2 \qquad (2)$$

is assumed.

The equation $Lu = 0$ is called Kolmogorov's equation.

The work is devoted to the proof of the inequality by Harnack for the non-negative solution of Kolmogorov's equation.

Let's consider the equations by Cordes, i.e. such equations

$$Lu = 0, \qquad (3)$$

for which the constants C_1 and C_2 of inequality (2) satisfy the condition

$$\frac{2C_2}{C_1} < 3.$$

1^0. Let's denote by $K_{x_0,R}^{t_1,t_2;y_1y_2}$ cylinder determined by inequalities

$$t_1 < t < t_2,\ \ y_1 < y < y_2,\ \ |x - x_0| < R.$$

Let positive numbers S, β and R be given.

Let's assume

$$b_0 = \frac{1}{5\beta S},$$

and let $0 < b \le b_0$. Let's consider three cylinders (fig.1):

$$K_1 = K_{R,R}^{0,bR^2;0,2bR^3}, \quad K_2 = K_{R,\frac{R}{16}}^{\frac{31}{32}bR^2,bR^2;\frac{31}{16}bR^3,2bR^3},$$

$$K_3 = K_{R,\frac{1}{16}R}^{0,\frac{2}{32}bR^2;\frac{15}{16}bR^3,\frac{17}{16}bR^3}.$$

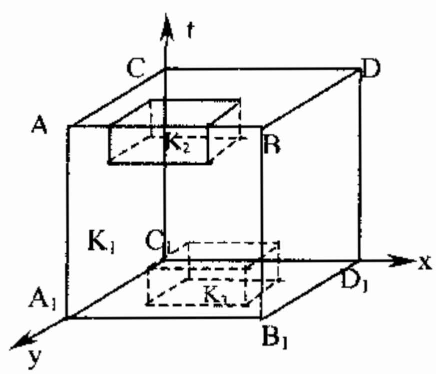

Fig. 1.

Denote by S_1 the unification of three bounds, i.e. $S_1 = ACC_1A_1 \cup CDD_1C_1 \cup BDD_1B_1$.

By definition of singular boundary of domain $S_1 \subset \Gamma(K_1)$ (see [2]).

Let's determine a function of the following form

$$U(x,y,t)=\int_E g_{S,\beta}(x,\xi;y,\zeta;t-\tau)d\mu(\xi,\zeta,\tau),$$

where $E\subset K_3$ is B-set and the function

$$g_{S,\beta}(x,\xi;y,\zeta;t-\tau)=$$
$$=\begin{cases}\dfrac{1}{(t-\tau)^S}\exp\left[-\dfrac{(x-\xi)^2}{4\beta(t-\tau)}-\dfrac{3}{\beta(t-\tau)^3}\left(y-\zeta-(t-\tau)\dfrac{x+\xi}{2}\right)^2\right] & for\ t-\tau>0\\ 0\quad for\ \ t-\tau\le 0\quad (except\ t-\tau=0,\ x-\xi=0,\ y-\zeta=0)\end{cases}$$

is a fundamental solution of the equation $Lu=0$, when $S=2$ and $\beta=1$.

Let's give without the proof the following inequalities from work [2]:

$$\sup_{S_1}U\le\left(bR^2\right)^{-S}\exp\left[-\frac{1}{5\beta b}\right]\mu(E), \tag{4}$$

$$\inf_{K_2}U\ge\left(bR^2\right)^{-S}\exp\left[-\frac{1}{5{,}3\beta b}\right]\mu(E). \tag{5}$$

2^0. At first we will proof auxiliary lemma being a simple corollary from lemma of increase [2]. Let's assume

$$b=\min\left(\frac{1}{10C_2},1\right).$$

Lemma 1. *Let in the cylinder* $K_{\xi,\xi}^{0,b\xi^2;0,2b\xi^3}$ *the domain* D *intersecting the cylinder* $K_{\xi,\frac{\xi}{2}}^{\frac{1}{2}b\xi^2,b\xi^2;b\xi^3,2b\xi^3}$ *and having limiting points on parabolic boundary of the cylinder* $K_{\xi,\xi}^{0,b\xi^2;0,2b\xi^3}$ *be located. Denote by* Γ *that part of boundary* D, *which is located strictly inside of cylinder* $K_{\xi,\xi}^{0,b\xi^2;0,2b\xi^3}$. *Let in* D *a solution of the equation (3), continuous in* $\overline{D}$, *positive in* D *and reducing to zero on* Γ *is determined.*

Then for any $h>0$ *there exists* $\delta>0$, *dependent on* h,C_1 *and* C_2 *such that from the inequality*

$$mesD<\delta\xi^6 \tag{6}$$

it follows

$$\frac{\sup_{\overline{D}}u}{\sup_{D\cap K_{\xi,\frac{\xi}{2}}^{\frac{1}{2}b\xi^2,b\xi^2,b\xi^3,2b\xi^3}}u}>h. \tag{7}$$

Proof. Let's take $\beta=C_1$ and $S=\dfrac{2C_2}{C_1}$. Then $b_0=\dfrac{1}{10C_2}$, and by the condition $b=\min\left(\dfrac{1}{10C_2},1\right)$, i.e. the condition $b\le b_0$ is satisfied.

Let η be a constant of lemma 3 [2]. Then η in this case depends only on C_1 and C_2. Let further m is such least natural number that

$$\left(1+\frac{\eta}{2}\right)^{m} > h .$$

Let's assume

$$\delta = \frac{b^2}{4^9 m^6} .$$

Divide the difference

$$K_{\xi,\xi}^{0,b\xi^2;0,2b\xi^3} \setminus K_{\xi,\frac{\xi}{2}}^{\frac{1}{2}b\xi^2,b\xi^2,b\xi^3,2b\xi^3}$$

into m parts by parabolic boundaries Γ_i of cylinders,

$$K^{(i)} = K_{\xi,\frac{\xi}{2}\left(1+\frac{i}{m}\right)}^{\frac{1}{2}b\xi^2\left(1-\frac{i}{m}\right),b\xi^2;b\xi^3\left(1-\frac{i}{m}\right),2b\xi^3} , \quad i = 0,1,\dots,m-1 .$$

Γ_0 coincides with parabolic boundary $K_{\xi,\frac{\xi}{2}}^{\frac{1}{2}b\xi^2,b\xi^2;b\xi^3,2b\xi^3}$. Let's assume

$$M_i = \max_{D\cap\Gamma_i} u , \quad i = 0,1,\dots,m-1 .$$

Let M_i be reached at a point $(x^i, y^i, t^i) \in \Gamma_i$. Consider

$$K_1^{(i)} = K_{x^i,\frac{\xi}{2m}}^{t^i-b\left(\frac{\xi}{2m}\right)^2,t^i;y^i+\left(x^i-\frac{\xi}{2m}\right)(t-t^i)-2b\left(\frac{\xi}{2m}\right)^3,y^i+\left(x^i-\frac{\xi}{2m}\right)(t-t^i)} .$$

It is easy to show that $K_1^{(i)} \subset K^{(i+1)}$, $i = 0,1,\dots,m-1$.

In the cylinder $K_1^{(i)}$ we will consider the cylinders

$$K_2^{(i)} = K_{x^i,\frac{\xi}{32m}}^{t^i-\frac{b}{2}\left(\frac{\xi}{8m}\right)^2,t^i,y^i+\left(x^i-\frac{\xi}{2m}\right)(t-t^i)-\frac{b}{2}\left(\frac{\xi}{4m}\right)^3,y^i+\left(x^i-\frac{\xi}{2m}\right)(t-t^i)}$$

and

$$K_3^{(i)} = K_{x^i,\frac{\xi}{32m}}^{t^i-b\left(\frac{\xi}{2m}\right)^2,t^i-15b\left(\frac{\xi}{8m}\right)^2;y^i+\left(x^i-\frac{\xi}{2m}\right)(t-t^i)-17\frac{b}{2}\left(\frac{\xi}{4m}\right)^3,y^i+\left(x^i-\frac{\xi}{2m}\right)(t-t^i)-15\frac{b}{2}\left(\frac{\xi}{4m}\right)^3} .$$

It is clear that

$$mesK_3^{(i)} = \frac{b^2\xi^6}{2\cdot 4^8 m^6} . \tag{8}$$

Let's denote the set $D \cap K_1^{(i)}$ by D'. If now we will apply the lemma 3 [2] to the cylinders $K_1^{(i)}$, $K_2^{(i)}, K_3^{(i)}$ and domain D', then we obtain that

$$\sup_{D'} u \geq \left[1+\eta\frac{mesE}{mesK_3^{(i)}}\right] \sup_{D'\cap K_2^{(i)}} u . \tag{9}$$

On the other side

$$\sup_{D'} u = \sup_{D'\cap K_1^{(i)}} u \leq \sup_{D\cap K_2^{(i+1)}} u = u\left(x^{i+1}, y^{i+1}, t^{i+1}\right) = M_{i+1} . \tag{10}$$

Besides a point $(x^i, y^i, t^i) \in D' \cap K_2^{(i)}$, therefore

$$\sup_{D'\cap K_2^{(i)}} u \geq u\left(x^i, y^i, t^i\right) = M_i . \tag{11}$$

Then from (9), (10) and (11) we obtain that

$$M_{i+1} \ge \left[1+\eta \frac{mesE}{mesK_3^{(i)}}\right] M_i . \tag{12}$$

Now, taking into account now (8), we obtain

$$mesE = mes\left(K_3^{(i)} \setminus D'\right) > mes\left(K_3^{(i)}\right) - mesD > \frac{1}{2} mes\left(K_3^{(i)}\right).$$

Thus from (12) we conclude

$$M_{i+1} \ge \left(1+\frac{\eta}{2}\right) M_i .$$

If we will repeat all these operations when $i = 0,1,\dots,m-1$, then we will receive that

$$M_m > \left(1+\frac{\eta}{2}\right)^m M_0$$

so by a maximum principle [2]

$$\sup_D u > h \sup_{K_{\xi,\frac{\xi}{2}}^{\frac{1}{2}b\xi^2,b\xi^2,b\xi^3,2b\xi^3}} u .$$

Lemma has been proved.

Let's consider the following transformation

$$x' = x + x_0 - \eta,\ t' = t + t_0 - b\eta^2,\ y' = y + (x_0-\eta)(t - b\eta^2) + y_0 - 2b\eta^3 . \tag{13}$$

It is easy to check that the operator (1) remains invariant at transformation (13). If we apply transformation (13) to cylinders

$$K_{\eta,\eta}^{0,b\eta^2,0,2b\eta^3} \quad \text{and} \quad K_{\eta,\frac{\eta}{2}}^{\frac{1}{2}b\eta^2,b\eta^2;b\eta^3,2b\eta^3},$$

then these cylinders will pass correspondingly to the following

$$\left.\begin{aligned} K_1 &= K_{x_0,\eta}^{t_0-b\eta^2,t_0;y_0+(x_0-\eta)(t-t_0)-2b\eta^3,y_0+(x_0-\eta)(t-t_0)}, \\ K_2 &= K_{x_0,\frac{\eta}{2}}^{t_0-\frac{1}{2}b\eta^2,t_0;y_0+(x_0-\eta)(t-t_0)-bR^3,y_0+(x_0-\eta)(t-t_0)}. \end{aligned}\right\} \tag{14}$$

Then we can receive the statement similar to lemma 1 in cylinders (14).

Lemma 2. *Let all conditions of lemma 1 be satisfied in cylinders (14).*

Then for any $h > 0$ there exists $\delta > 0$ dependent on h, C_1 *and* C_2 *such that from* the inequality $mesD < \delta\eta^6$ it follows the inequality

$$\frac{\sup_{\overline{D}} u}{\sup_{D \cap K_2} u} > h . \tag{15}$$

Now it is possible to formulate an inequality by Harnack for the non-negative solution of the equation (3).

Theorem 1 (Harnack's inequality). *Let in cylinder* $K_{R,R}^{0,bR^2;0,2bR^3}$ *the non-negative solution* $u(x,y,t)$ *of the equation (3) be determined.*

Then

$$\sup_{K_{R,\frac{1}{32}R}^{\frac{1}{32}bR^2,\frac{2}{32}bR^2;bR^3,\frac{17}{16}bR^3}} u(x,y,t) \Big/ \inf_{K_{R,\frac{1}{32}R}^{\frac{31}{32}bR^2,bR^2;\frac{31}{16}bR^3,2bR^3}} u(x,y,t) < C_3, \tag{16}$$

where $C_3 > 0$ is a constant dependent on C_1 and C_2.

Proof. It is clear that it is enough to prove the theorem for the case $R = 1$. Let's designate for convenience

$$K_1 = K_{1,1}^{0,b;0,2b}, \quad K_2 = K_{1,\frac{1}{32}}^{\frac{31}{32}b,b;\frac{31}{16}b,2b}, \quad K_3 = K_{1,\frac{1}{32}}^{\frac{1}{32}b,\frac{2}{32}b;b,\frac{17}{16}b}.$$

Then the proved inequality (16) will be of the following form

$$\sup_{K_3} u / \inf_{K_2} u < C.$$

The theorem will be proved, if from the assumption

$$\sup_{K_3} u = 2$$

will follow

$$\inf_{K_3} u > v,$$

where $v > 0$ is a constant dependent on C_1 and C_2.

Let's assume

$$\widetilde{K}_3 = K_{1,\frac{1}{16}}^{0,\frac{1}{32}b;\frac{15}{16}b,\frac{17}{16}b}.$$

Let's denote by G_1 the set of points $(x,y,t) \in \widetilde{K}_3$, where $u(x,y,t) > 1$. Let's assume in Lemma 2 $h = 2^7$ and we will find corresponding δ. Further assume

$$\varepsilon_0 = \left(\frac{1}{4096}\right)^6 \delta. \tag{17}$$

Let's consider two cases separately:

$$mes\ G_1 \geq \varepsilon_0$$

and

$$mes\ G_1 < \varepsilon_0.$$

Case 1. $mes\ G_1 \geq \varepsilon_0$.

Let $S = \dfrac{2C_2}{C_1}$, $\beta = C_1$. The equation (3) is a Cordes type equation by, therefore $S < 3$. Besides that $G_1 \subset K_{1,1}^{0,b;0,2b} \subset K_{1,1}^{0,1;0,2}$. Then by property of (S,β)-capacity $\gamma_{S,\beta}(G_1) \geq C mes G_1 \geq C\varepsilon_0$ (see [2]).

Let μ be admissible measure on G_1 such that

$$\mu(G_1) > \frac{1}{2}\gamma_{S,\beta}(G_1) > \frac{C}{2} mes G_1.$$

Let's assume

$$V(x,y,t) = \int_{G_1} g_{S,\beta}(x,\xi;y,\zeta;t-\tau)\,d\mu(\xi,\zeta,\tau) - b^{-S}\exp\left[-\frac{1}{5\beta b}\right]\mu(G_1).$$

Then using from increase lemma and a maximum principle (see [2]) it is easy to obtain that

$$u\big|_{K_2} \geq \mathrm{v}\big|_{K_2} \geq b^{-S}\exp\left[-\frac{1}{5{,}3\beta b}\right]\mu(G_1) - b^{-S}\exp\left[-\frac{1}{5\beta b}\right]\mu(G_1) >$$
$$\geq \frac{C\varepsilon_0 b^{-S}}{2}\left(\exp\left[-\frac{1}{5{,}3\beta b}\right] - \exp\left[-\frac{1}{5\beta b}\right]\right).$$

Thus in case 1 for v we can take

$$\frac{C\varepsilon_0 b^{-S}}{2}\left(\exp\left[-\frac{1}{5{,}3\beta b}\right] - \exp\left[-\frac{1}{5\beta b}\right]\right).$$

Case 2. $mes\ G_1 < \varepsilon_0$.

Let's assume

$$K^{(\rho)} = K_{1,\frac{1}{32}+\rho}^{\frac{b}{32}(1-\rho^2),\frac{2}{32}b;b(1-\rho^3),\frac{17}{16}b}$$

It is clear that $K^{(0)} = K_3$ and $K^{\left(\frac{1}{32}\right)} \subset \widetilde{K}_3$.

Let's assume

$$G_\rho^{(1)} = G_1 \bigcap \left(K^{(\rho)} \setminus K^{(0)}\right).$$

by virtue of (17) we have

$$mesG_{\frac{1}{64}}^{(1)} < \left(\left(\frac{1}{64}\right)^2\right)\delta .$$

On the other side

$$mes\, G_\rho^{(1)} \geq O\left(\rho^6\right) \quad for \quad \rho \to 0 .$$

Then

$$mes\, G_\rho^{(1)} \geq O\left(\rho^{12}\right) \quad for \quad \rho \to 0 ,$$

that is why for sufficiently small $\rho > 0$

$$mes\, G_\rho^{(1)} > \left(\rho^2\right)^6 \delta .$$

Therefore there exists such ρ_1, $0 < \rho_1 < \dfrac{1}{64}$, that

$$mes\, G_{\rho_1}^{(1)} = \left(\rho_1^2\right)^6 \delta . \tag{18}$$

Let's find on parabolic boundary of the cylinder $K^{(\rho_1^2)}$ a point $(x^1, y^1, t^1) \in G_1$, in which $u(x^1, y^1, t^1) \geq 2$.

Let's take the cylinder

$$K_{(1)} = K_{x^1,\rho_1^2}^{t^1-b\rho_1^4,t^1;y^1-2b\rho_1^6+(x^1-\rho_1^2)(t-t^1),y^1+(x^1-\rho_1^2)(t-t^1)}.$$

It is easy to show that this cylinder is located in a gap between the cylinders $K^{(0)}$ and $K^{(\rho_1)}$.

Let's assume

$$\mathrm{v}_1(x, y, t) = u(x, y, t) - 1 .$$

We have $\mathrm{v}_1(x^1, y^1, t^1) \geq 1$ and $\mathrm{v}_1(x, y, t) > 0$ in G_1, $\mathrm{v}_1(x, y, t) \leq 0$ outside of G_1. Let's denote by $D_{(1)}$ that component of the set $G_1 \bigcap K_{(1)}$, which contains a point (x^1, y^1, t^1).

Let's apply to the cylinder $K_{(1)}$ and to domain $D_{(1)}$ in it lemma 2 that is possible by virtue of (18):

$$\sup_{D_{(1)}} u > \sup_{D_{(1)}} \mathrm{v}_1 \geq 2 \cdot 2^6 .$$

Let's denote by G_2 the set of points $(x, y, t) \in \widetilde{K}_3$, where

$$u(x, y, t) > 2^6 .$$

Let's consider $K^{(\rho_1+\rho)}$, $0 < \rho < \frac{1}{32} - \rho_1$. Let's assume

$$G_\rho^{(2)} = G_2 \bigcap \left(K^{(\rho_1+\rho)} \setminus K^{(\rho_1)}\right).$$

From the relation $0 < \rho < \frac{1}{32} - \rho_1$ follows that $\rho_1 < \rho + \rho_1 < \frac{1}{32}$, that is why $K^{(\rho_1+\rho)} \in \widetilde{K}_3$. Since $\rho_1 < \frac{1}{64}$ and $G_2 \subset G_1$, then by virtue of (17)

$$mesG_{\frac{1}{64}}^{(2)} < \left(\left(\frac{1}{64}\right)^2\right)^6 \delta .$$

Again it will be

$$mesG_\rho^{(2)} \geq O\left(\rho^6\right) \ for \quad \rho \to 0 ,$$

consequently,

$$mesG_\rho^{(2)} \geq O\left(\rho^{12}\right) \ for \quad \rho \to 0 .$$

Then for sufficiently small positive ρ on those by the previous reasons

$$mesG_\rho^{(2)} \geq \left(\rho^2\right)^6 \delta .$$

Therefore there exists such ρ_2 from an interval $\left(0, \frac{1}{64}\right)$ that

$$mesG_{\rho_2}^{(2)} = \left(\rho_2^2\right)^6 \delta . \tag{19}$$

Let's find on parabolic boundary $K^{\left(\rho_1+\rho_2^2\right)}$ a point $\left(x^2, y^2, t^2\right)$, in which $u\left(x^2, y^2, t^2\right) > 2 \cdot 2^6$. Let's take the cylinder

$$K_{(2)} = K_{x^2, \rho_2^2}^{t^2 - h\rho_2^4, t^2, y^2 - 2h\rho_2^6 + \left(x^2 - \rho_2^2\right)\left(t - t^2\right), y^2 + \left(x^2 - \rho_2^2\right)\left(t - t^2\right)},$$

since that this cylinder is located in gap between cylinders $K^{(\rho_1)}$ and $K^{(\rho_1+\rho_2)}$.

Let's assume

$$\mathrm{v}_2(x, y, t) = u(x, y, t) - 2^6 ,$$

thus $\mathrm{v}_2\left(x^2, y^2, t^2\right) > 2^6$ and $\mathrm{v}(x, y, t) > 0$ in G_2, $\mathrm{v}(x, y, t) \leq 0$ outside of G_2. Let's designates by $D_{(2)}$ that component of the set $G_2 \bigcap K_{(2)}$, which contains a point $\left(x^2, y^2, t^2\right)$.

Applying to the cylinder $K_{(2)}$ and domain $D_{(2)}$ in it lemma 2, we will find

$$\sup_{D_{(2)}} u > \sup_{D_{(2)}} \mathrm{v} \geq 2^6 \cdot 2^7 = 2 \cdot 2^{2 \cdot 6} .$$

If $\rho_1+\rho_2<\dfrac{1}{64}$, then we will continue the process. Let's denote by G_3 the set of points $(x,y,t)\in\widetilde{K}_3$, where $u>2^{2\cdot 6}$.

Let's consider $K^{(\rho_1+\rho_2+\rho)}$, $0<\rho<\dfrac{1}{64}-\rho_1-\rho_2$. This implies that $\rho_1+\rho_2<\rho+\rho_1+\rho_2<\dfrac{1}{64}$, that is why $K^{(\rho_1+\rho_2+\rho)}\subset\widetilde{K}_3$.

Let's assume

$$G_\rho^{(3)}=G_3\bigcap\left(K^{(\rho_1+\rho_2+\rho)}\setminus K^{(\rho_1+\rho_2)}\right)$$

and we will find such $\rho_3<\dfrac{1}{64}$ that

$$mes\, G_{\rho_3}^{(3)}=\left(\rho_3^2\right)^6\delta\ ,$$

and on parabolic boundary $K^{(\rho_1+\rho_2+\rho_3^2)}$ the point (x^3,y^3,t^3) is contained, where $u(x^3,y^3,t^3)>2\cdot 2^{2\cdot 6}$. Then in gap between $K^{(\rho_1+\rho_2)}$ and $K^{(\rho_1+\rho_2+\rho_3)}$ we will find the cylinder $K_{(3)}$ and in it the domain $D_{(3)}$ and so on.

We will continue this process until it becomes for the first time $\rho_1+\rho_2+...+\rho_k\geq\dfrac{1}{64}$. Further such moment will come: when the sum $\rho_1+...+\rho_k$ will exceed $\dfrac{1}{64}$ otherwise we could continue this process infinitely and as at each its step the value u increases more than 2^6 times, the function would appear unbounded in $\widetilde{K}_3$.

So, let

$$\rho_1+...+\rho_{k-1}<\frac{1}{64}$$

and

$$\rho_1+...+\rho_k\geq\frac{1}{64}. \tag{20}$$

For every number $i, i=1,2,...,k$ there corresponds a set $G_{(\rho_i)}^{(i)}\subset\widetilde{K}_3$ satisfying the equality

$$mes G_{(\rho_i)}^{(i)}=\left(\rho_i^2\right)^6\delta \tag{21}$$

and

$$u\big|_{G_{(\rho_i)}^{(i)}}=2^{(i-1)6}. \tag{22}$$

From (20) it follows that there exists such i_0, that

$$\rho_{i_0}>\frac{1}{2^{\frac{i_0}{2}\cdot 8}}.$$

Otherwise

$$\rho_1+\rho_2+...+\rho_k \le \frac{1}{2^{\frac{1}{2}+8}}+\frac{1}{2^{\frac{2}{2}+8}}+...+\frac{1}{2^{\frac{k}{2}+8}}=$$

$$=\frac{1}{2^8}\left(\frac{1}{2^{\frac{1}{2}}}+\frac{1}{2^{\frac{2}{2}}}+...+\frac{1}{2^{\frac{k}{2}}}\right)<\frac{1}{64},$$

but it is impossible by virtue of (20).

Then (21), (22) give us

$$mesG_{\rho_{i_0}}^{(i_0)}=\left(\rho_{i_0}^2\right)^6\delta>\left(\frac{1}{2^{i_0+16}}\right)^6\delta=2^{-6(i_0+16)}\delta, \tag{23}$$

$$u\Big|_{G_{\rho_{i_0}}^{(i_0)}}>2^{(i_0-1)6}. \tag{24}$$

Let μ be an admissible measure determined on $G_{\rho_{i_0}}^{(i_0)}$ and such that $\mu\left(G_{\rho_{i_0}}^{(i_0)}\right)>\frac{\gamma_{S,\beta}\left(G_{\rho_{i_0}}^{(i_0)}\right)}{2}$, so owing to property of (S,β)- capacity (see [2])

$$\mu\left(G_{\rho_{i_0}}^{(i_0)}\right)>\frac{C}{2}mesG_{\rho_{i_0}}^{(i_0)},$$

i.e.

$$\mu\left(G_{\rho_{i_0}}^{(i_0)}\right)>\frac{C}{2}2^{-6(i_0+16)}\delta. \tag{25}$$

Let's consider the function

$$\mathrm{v}(x,y,t)=2^{(i_0-1)6}\left[\int_{G_{\rho_{i_0}}^{(i_0)}}g_{S,\beta}(x,\xi;y,\zeta;t-\tau)d\mu(\xi,\zeta,\tau)-\right.$$

$$\left.-b^{-S}\exp\left[-\frac{1}{5\beta b}\right]\mu\left(G_{\rho_{i_0}}^{(i_0)}\right)\right].$$

In lower base of the cylinder K_1 outside of $\overline{G}_{\rho_{i_0}}^{(i_0)}$ it is negative, on lateral bounds, which are parabolic boundaries is negative owing to lemma 2 (see [2]). Outside of set $G_{\rho_{i_0}}^{(i_0)}$ in K_1 it does not exceed $2^{(i_0-1)6}$. So by a maximum principle it does not exceed and everywhere in $K_1\setminus\overline{G}_{\rho_{i_0}}^{(i_0)}$.

Applying the inequality (5)

$$\inf_{K_2}u>\inf_{K_2}\mathrm{v}\ge 2^{(i_0-1)6}b^{-S}\left(\exp\left[-\frac{1}{5{,}3\beta b}\right]-\exp\left[-\frac{1}{5\beta b}\right]\right)\mu\left(G_{\rho_{i_0}}^{(i_0)}\right)>$$

$$>2^{-103}C\delta b^{-S}\left(\exp\left[-\frac{1}{5{,}3\beta b}\right]-\exp\left[-\frac{1}{5\beta b}\right]\right).$$

The number staying in the right side of the last inequality we will take as v. It obviously depends on C_1 and C_2.

The theorem has been proved.

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