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TO THE STABILITY OF A CIRCULAR PLATE MANUFACTURED FROM CONTINUOUSLY-HETEROGENEOUS MATERIAL

Abstract

An axle-symmetrical task is examined in this article. It is supposed that the property of material is heterogeneous relatively to the coordinates of the thickness. The task is solved on the base of Bubnov-Galerkin method. A numerical calculation was carried out in the case of homogeneity. The results are presented in tables and diagrams.

As is known in many branches of machine and instrument – industry for projection and construction of different kinds of engineering structures circular plates manufactured from continuously-heterogeneous materials are widely used [1.2].

In the present paper the stability problem of an elasto-plastic plate under radial pressure is solved. The axially symmetric form of stability loss is considered.

It's assumed that the material of a plate is incompressible, linear hardening and only modulus of elasticity is a function of thickness coordinates, i.e.

$$\sigma_u = \lambda\sigma_s + E_0(1-\lambda)f(z)\varepsilon_u , \tag{1}$$

where

$$\sigma_u = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r\sigma_\theta} \ ,$$

$$\varepsilon_u = \frac{2}{\sqrt{3}}\sqrt{\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_r\varepsilon_\theta}$$

here E_0 is an elasticity modulus of homogeneous material, λ is a coefficient of linear hardening, σ_s is a yield point of material, σ_u and ε_u are stress and strain intensity respectively, σ_θ, σ_r and ε_r, ε_θ are radial and tangential stresses and strains.

Let a circular palate with the radius R be shrinked along the contour by surface pressure with the intensity P.

In the present case the following is valid

$$\sigma_u = -\sigma_r . \tag{2}$$

Subject to (2) in (1) w obtain

$$\sigma_r = -\lambda\sigma_s - E_0(-\lambda)f(z)\varepsilon_u .$$

In subcritical state the boundary of elasto-plastic zones is determined from the equation

$$1 - f(\rho_0)\bar{\varepsilon}_u = 0 \tag{3}$$

here $\bar{\varepsilon}_u = \varepsilon_u\varepsilon_s^{-1}$; $\rho_0 = z_0 h^{-1}$; $\rho = zh^{-1}$, ε_s corresponds to the value of σ_s.

In subcritical state ρ_0 and $\bar{\sigma}_\theta$ with the loading R are connected with the following relation

$$P^* = -\frac{P}{\sigma_s h} = \left[-\lambda - (1-\lambda)\bar{\varepsilon}_u\right]\int\limits_{\rho_2} f(\rho)d\rho + \bar{\varepsilon}_u \int\limits_{\rho_1} f(\rho)d\rho \tag{4}$$

here ρ_1 and ρ_2 are sizes of elastic and plastic zones along the thickness of a plate relatively, h is the thickness of a plate.

On the basis of relations introduced in [5] we can write the connection between the velocities of forces and moments through the velocities of strains and curvatures by the following form:

$$T_r = A_1 e_r + A_2 e_\theta + A_3 \chi_r + A_4 \chi_\theta ,$$
$$T_\theta = A_5 e_r + A_6 e_\theta + A_7 \chi_r + A_8 \chi_\theta , \tag{5}$$

$$M_r = B_1 e_r + B_2 e_0 + B_3 \chi_r + B_4 \chi_\theta ,$$
$$M_\theta = B_5 e_r + B_6 e_\theta + B_7 \chi_r + B_8 \chi_\theta , \tag{6}$$

where the following designations are introduced.

$$A_1 = A_6 = G_0 h \left[\int_{\rho_1} f(\rho) d\rho + 3\lambda \int_{\rho_2} f(\rho) d\rho \right],$$
$$A_2 = A_5 = 2 G_0 h \int_{\rho_1} f(\rho) d\rho , \tag{7}$$
$$A_3 = -G_0 h \left[\int_{\rho_1} f(\rho) \rho d\rho - 3\lambda \int_{\rho_2} f(\rho) \rho d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_r^2 (\rho_0 - \rho) d\rho \right],$$
$$A_4 = -G_0 h \left[2 \int_{\rho_1} f(\rho) \rho d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_r^2 \overline{\sigma}_\theta^2 (\rho_0 - \rho) d\rho \right],$$
$$A_7 = -G_0 h \left[\int_{\rho_1} f(\rho) \rho d\rho - 3\lambda \int_{\rho_2} f(\rho) \rho d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_\theta^2 (\rho_0 - \rho) d\rho \right],$$
$$A_8 = -G_0 h \left[2 \int_{\rho_1} f(\rho) \rho d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_r^2 \overline{\sigma}_\theta^2 (\rho_0 - \rho) d\rho \right].$$

Analogously we can establish that for B_1, B_2, ..., B_8 the following expressions hold

$$B_1 = B_6 = G_0 h^2 \left[\int_{\rho_1} f(\rho) \rho d\rho + 3\lambda \int_{\rho_2} f(\rho) \rho d\rho \right],$$
$$B_2 = B_5 = 2 G_0 h^2 \left[\int_{\rho_1} f(\rho) \rho d\rho \right],$$
$$B_3 = -G_0 h^3 \left[\int_{\rho_1} f(\rho) \rho^2 d\rho \right], \tag{8}$$
$$B_4 = -G_0 h^3 \left[2 \int_{\rho_1} f(\rho) \rho^2 d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_r \overline{\sigma}_\theta (\rho_0 - \rho) \rho d\rho \right],$$
$$B_7 = -G_0 h^3 \left[\int_{\rho_1} f(\rho) \rho^2 d\rho - 3\lambda \int_{\rho_2} f(\rho) \rho^2 d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_\theta^2 (\rho_0 - \rho) \rho d\rho \right],$$
$$A_8 = -G_0 h^3 \left[2 \int_{\rho_1} f(\rho) \rho^2 d\rho + 3\lambda \int_{\rho_2} \overline{\sigma}_r^2 \overline{\sigma}_\theta^2 (\rho_0 - \rho) \rho d\rho \right].$$

Using the principles of Ilyushin approximate theory [3] from (5) and (6) we can exclude e_r, ε_θ and represent the expression of moments velocities in the following form:

$$M_r = -\left(P_1\chi_r - P_2\chi_\theta\right),$$
$$M_\theta = -\left(P_3\chi_r + P_4\chi_\theta\right), \qquad (9)$$

where

Table 1.

$k_0 = 0{,}3$

$\lvert\bar{\varepsilon}_\theta\rvert$	ρ_1	$\lvert P^*\rvert$
0,446	0,4	1,011
0,449	0,375	1,013
0,452	0,35	1,014
0,458	0,3	1,015
0,461	0,275	1,016
0,465	0,25	1,018

$k_0 = 0{,}4$

$\frac{\lvert\bar{\varepsilon}_\theta\rvert}{2}$	ρ_1	$\lvert P^*\rvert$
0,431	0,4	1,015
0,435	0,375	1,018
0,439	0,35	1,021
0,446	0,3	1,022
0,450	0,275	1,023
0,455	0,25	1,024

$k_0 = 0{,}5$

$\frac{\lvert\bar{\varepsilon}_\theta\rvert}{2}$	ρ_1	$\lvert P^*\rvert$
0,417	0,4	1,019
0,421	0,375	1,022
0,425	0,35	1,024
0435	0,3	1,029
0,439	0,275	1,030
0,444	0,25	1,031

$k_0 = 0{,}6$

$\frac{\lvert\bar{\varepsilon}_\theta\rvert}{2}$	ρ_1	$\lvert P^*\rvert$
0,403	0,4	1,022
0,408	0,375	1,026
0,413	0,35	1,029
0,423	0,3	1,033
0,429	0,275	1,035
0,435	0,25	1,036

$k_0 = 0{,}7$

$\frac{\lvert\bar{\varepsilon}_\theta\rvert}{2}$	ρ_1	$\lvert P^*\rvert$
0,391	0,4	1,028
0,396	0,375	1,031
0,401	0,35	1,034
0,413	0,3	1,039
0,419	0,275	1,041
0,426	0,25	1,042

$$P_1 = B_1A_1 + B_2A_2 + B_3\,,$$
$$P_2 = B_1K_1 + B_2K_2 + B_4\,,$$
$$P_3 = B_5K_1 + B_6K_3 + B_7\,,$$
$$P_4 = B_5K_2 + B_6K_4 + B_8\,,$$
$$K_1 = \frac{A_6A_3 - A_2A_7}{A_6A_2 - A_1A_3}\,,\; K_2 = \frac{A_1A_6 - A_5A_2}{A_5A_2 - A_1A_6}\,,\; K_3 = \frac{A_1A_3 - A_2A_7}{A_2^2 - A_1^2}\,,\; K_4 = A_4A_2^{-1}\,. \qquad (10)$$

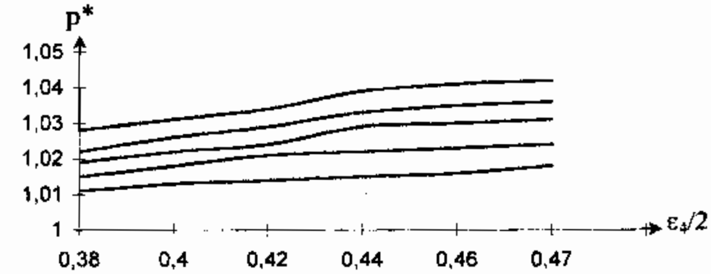

Fig. 1

Dependence between $\bar{\varepsilon}_4$ and ρ_1
for κ =0,3; 0,4; 0,5; 0,6; 0,7

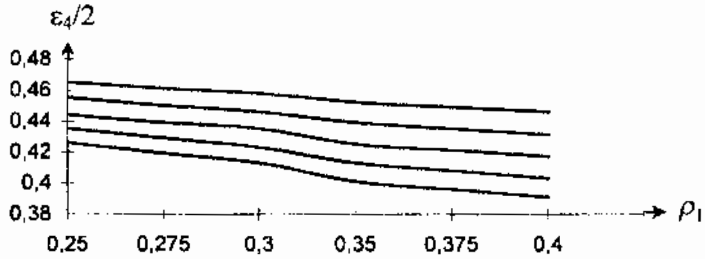

Fig. 2

Dependence between p*and ρ_1
for k =0,3; 0,4; 0,5; 0,6; 0,7

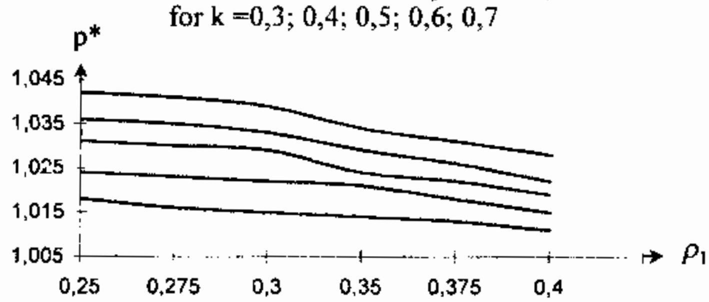

Fig. 3

Using the equations for an axially symmetric form of stability loss we can obtain:

$$P_1\frac{d^4\dot{W}}{dr^4}+P_1\frac{d^2}{dr^2}\left(\frac{1}{r}\frac{d\dot{W}}{dr}\right)+\frac{2P_1}{r}\frac{d^2\dot{W}}{dr^2}+P_2\frac{1}{r^2}\frac{d\dot{W}}{dr}-$$

$$-\frac{P_3}{r}\frac{d^3\dot{W}}{dr^3}-\frac{P_4}{r}\frac{d}{dr}\left(\frac{1}{r}\frac{d\dot{W}}{dr}\right)-Ph\frac{d^2\dot{W}}{dr^2}=0\,, \qquad (11)$$

$$W(r)=\sum_{i=1}^{n}A_i\theta_i(r) \qquad (12)$$

here $\theta_i(r)$ has to satisfy the corresponding boundary conditions.

Rejecting the elementary details we note that the critical loading is determined with the help of the next formula

$$P_k h=\frac{\sum_{i=1}^{n}A_i\int_0^R\left(\dot{Q}_1+\dot{Q}_2+\dot{Q}_3\right)\theta_i(r)rdr}{\sum_{i=1}^{n}A_i\int_0^R\frac{d^2\dot{Q}_i}{dr^2}\theta_i(r)dr}, \qquad (13)$$

where

$$\dot{Q}_1=\frac{d^2}{dr^2}\left[P_1\frac{d\theta_i}{dr^2}+P_2\frac{1}{r}\frac{d\theta_i}{d^2}\right],$$

$$\dot{Q}_2=\frac{2}{r}\left[P_1\frac{d^2\theta_i}{dr^2}+P_2\frac{1}{r}\frac{d\theta_i}{dr}\right], \qquad (14)$$

$$\dot{Q}_3=-\frac{1}{r}\frac{d}{d^2}\left[P_3\frac{d^2\theta_i}{dr^2}+\frac{P_4}{r}\frac{d\theta_i}{dr}\right].$$

Numerical calculation is led and the results are represented in the form of tables and diagrams. As it is obvious from the diagrams by increasing of a non-homogeneity parameter, the value of criticed loading sharply differs from critical loading for the linear elastic case.

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