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THE NUMERICAL MODELING OF THE RADIAL FLOW PROBLEM FOR THE GAS CONDENSATE MIXTURE WITH PHASE TRANSITIONS

Abstract

Filtration of gas condensate mixture in differ from poor gas goes with continuous change of the composition of phases caused by retrogreade phenomena. Real gas condensate mixture is many component and many phase. Its filtration is a complex physical with the phase transformations and it is described by the system of the complex non-linear differential equations with the variable coefficients of the second order.

Here the isothermic flow of the gas condensate mixture to the central well, i.e. the radial flow problem of the gas condensate mixture is considered in the gas regime with the phase transformations taking into account the real properties of the fluids and the medium on the base binary model. The problem has been solved numerically. The numerical calculations have been carried aut for the data taken for the concrete deposit. On the base of the results of calculations the conclusions have been made.

Non-stationary isothermic plane-radial filtration of gas condensate mixture to the central well is numerically modeled as a binary system with regard to real properties of fluids and rock in exhaustion condition.

Mathematical statement of the problem is as follows. It's necessary to solve the following system of the second order partial differential equations with variable coefficients:

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\frac{K_z(\rho_k)P\beta[1-C(P)\gamma(P)]}{\mu_z(P)Z(P)P_{at}}+\frac{K_k(\rho_k)S_k(P)}{\mu_k(P)a_k(P)}\right]\frac{\partial P}{\partial r}\right\}=$$

$$=\frac{m}{k}\frac{\partial}{\partial t}\left\{\frac{(1-\rho_k)P\beta}{Z(P)P_{at}}[1-C(P)\gamma(P)]+\rho_k\frac{S_k(P)}{a_k(P)}\right\}, \qquad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\frac{K_k(\rho_k)}{\mu_k(P)a_k(P)}+\frac{K_z(\rho_k)PC(P)\beta}{\mu_z(P)Z(P)P_{at}}\right]\frac{\partial P}{\partial r}\right\}=$$

$$=\frac{m}{k}\frac{\partial}{\partial t}\left\{\frac{\rho_k}{a_k(P)}+(1-\rho_k)\frac{P\beta C(P)}{Z(P)P_{at}}\right\}, \qquad (2)$$

under the following initial and boundary conditions

$$\left.\begin{array}{l}
P=P(r)\ (\text{fig.2});\quad \rho_k=0\quad \text{for}\quad t=0,\ r_c\le r\le R_k,\\
2\pi r_c hk\left(\frac{K_z(\rho_k P\beta[1-C(P)\gamma(P)])}{\mu_z(P)Z(P)P_{at}}+\frac{K_k(f_k)S_k(P)}{\mu_k(P)a_k(P)}\right)\frac{\partial P}{\partial r}=-q_r\ \text{for}\quad r=r_c,\\
\frac{\partial P}{\partial r}=0\ \text{for}\quad r=R_k,\ P_k<P_0\\
P_1=P_2;\rho_{k1}=\rho_{k2};\frac{\partial P_1}{\partial r}=\frac{\partial P_2}{\partial r}\ \text{for}\ r=r_2(t),\\
P_2=P_3=P_{nk};\ \frac{\partial P_2}{\partial r}=\frac{\partial P_3}{\partial r};\ \rho_{k2}=\rho_{k3}=0\quad \text{for}\quad r=r_2(t),
\end{array}\right\}, \qquad (3)$$

where k, m are absolute permeability and porosity of the formation which is regarded as homogeneous; $K_г(\rho_k)$ $K_k(\rho_k)$ are relative phase permeabilities for gas and liquid phases; $\mu_г(P)$, $\mu_k(P)$ are viscosities of gas and liquid phases; β and $Z(P)$are coefficients of temperature correction and over compressibility for the gas phase; $C(P)$ is a content of condensate in gas phase; $\gamma(P)$ is a relation of specific weight of condensate in liquid and gas phases at normal conditions; $S_k(P)$ is the quantity of dissolved gas in liquid phase; $a_k(P)$ is a volume coefficient of liquid phase; P is current pressure; ρ_k is condensate saturation of formation; $P_{ат}$ is atmospheric pressure; t is time; r_c is a radius of a well; R_k is a radius of drainage of a well; r is a radial coordinate; P_c is a bottom-hole pressure; P_k is pressure on contour of drainage of a well; $P_{нк}$ is initial condensation; P_0 is the initial seam pressure; $q_г$ is gas consumption; h seam thickness; P_i, ρ_{ki}, $i = 1,2,3$ are pressure and condensate-saturation in zones formed in filtration process and namely in bottom-hole (zone of maximum condensation) intermediate and contour zones with the radii $r_2(t)$, $r_1(t)$ respectively (fig.1).

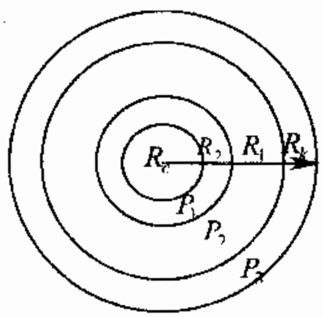

Fig. 1.

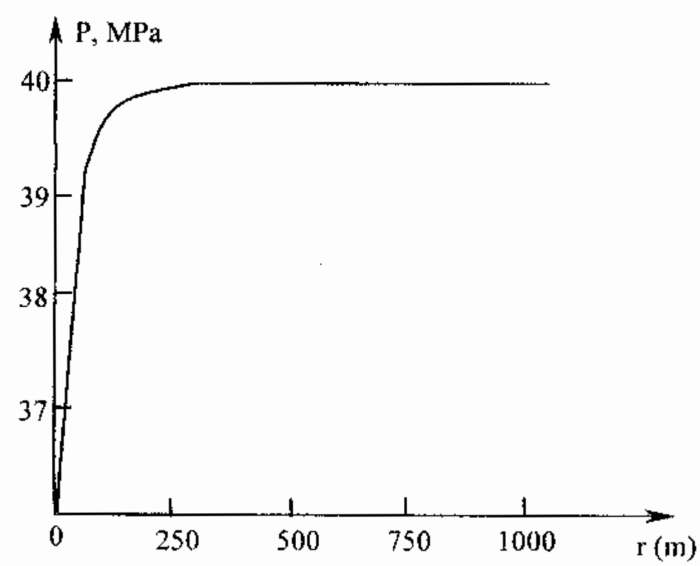

Fig. 2

The problem is solved numerically. For this we write the system (1)-(2) and the condition (3) in the next vector from:

$$\frac{\partial}{\partial t}[AU]=\frac{k}{rm}\frac{\partial}{\partial r}\left[B\frac{\partial U}{\partial r}\right], \tag{4}$$

$$\left.\begin{aligned}
&U=\begin{Vmatrix}P(r)\\0\end{Vmatrix} \text{ for } t=0,\ r_c\le r\le R_k,\\
&U=\begin{Vmatrix}P_k-\Delta P\\0\end{Vmatrix}=\begin{Vmatrix}P_c\\\rho_k\end{Vmatrix} \text{ for } r=r_c,\\
&B\frac{\partial U}{\partial r}=\begin{Vmatrix}0\\\rho_k\end{Vmatrix} \text{ for } r=R_k,\\
&U_1=U_2;\ \frac{\partial U_1}{\partial r}=\frac{\partial U_2}{\partial r} \text{ for } r=r_2(t),\\
&U_2=U_3=\begin{Vmatrix}P_{uk}\\0\end{Vmatrix} \text{ for } r=r_1(t),
\end{aligned}\right\} \tag{5}$$

where

$$U=\begin{Vmatrix}P\\\rho_k\end{Vmatrix},$$

$$A=\begin{Vmatrix}\frac{1-\rho_k}{Z(P)P_{at}}\beta[1-C(P)\gamma(P)] & \frac{S_k(P)}{a_k(P)}\\ \frac{1-\rho_k}{Z(P)P_{at}}\beta C(P) & \frac{1}{a_k(P)}\end{Vmatrix},$$

$$B=\begin{Vmatrix}\frac{rK_z(\rho_k)P}{\mu_z(P)Z(P)P_{at}}\beta[1-C(P)\gamma(P)]+\frac{rK_k(\rho_k)S_k(P)}{\mu_k(P)a_k(P)} & 0\\ \frac{rK_k(\rho_k)}{\mu_k(P)a_k(P)}+\frac{rK_z(\rho_k)C(P)P\beta}{\mu_z(P)Z(P)P_{at}} & 0\end{Vmatrix}.$$

If we apply to the vector equation (4) complemented with the initial and boundary conditions (5), an implicit two layer scheme, then the error of approximation of this scheme has the order $O(\tau+h^2)$ and the scheme is stable. In this case if we keep count with large step with respect to τ, then in this addition the exactness is lost. Therefore we'll apply to this problem an implicit three layer scheme. This scheme has a second order exactness with respect to both variables, i.e. $\varepsilon_{i,j}=O(\tau^2+h^2)$. In the present case we'll keep calculations with large step without loss of exactness.

Applying on nodes of the grid (r_i,t_j) an implicit three layer scheme to the system (4) we obtain

$$\frac{3}{2}A_{i,j+1}U_{i,j+1}-2A_{i,j}U_{i,j}+\frac{1}{2}A_{i,j-1}U_{i,j-1}=$$
$$=\frac{k\tau}{r_imh^2}\left[B_{i+1,j}\left(U_{i+1,j}-U_{i,j}-U_{i-1,j}\right)\right]. \tag{6}$$

Let's introduce on the grid (r_i,t_j) the next designations:

$$\left.\begin{aligned}&(AU)_t=\frac{(AU)_{j+1}-(AU)_j}{\tau};\ (AU)_{\bar t}=\frac{(AU)_j-(AU)_{j-1}}{\tau},\\&(AU)_{\overset{\circ}{t}}=\frac{1}{2}\left[(AU)_t+(AU)_{\bar t}\right]=\frac{(AU)_{j+1}-(AU)_{j-1}}{2\tau},\\&(AU)_{\bar t t}=\frac{(AU)_t-(AU)_{\bar t}}{\tau}=\frac{(AU)_{j+1}-2(AU)_j+(AU)_{j-1}}{\tau^2}.\end{aligned}\right\}\qquad(7)$$

Then from (7) we have

$$\left.\begin{aligned}&A_{i,j-1}U_{i,j-1}=AU-\tau(AU)_{\overset{\circ}{t}}+\frac{1}{2}\tau^2(AU)_{\bar t t},\\&A_{i,j+1}U_{i,j+1}=AU+\tau(AU)_{\overset{\circ}{t}}+\frac{1}{2}\tau^2(AU)_{\bar t t},\end{aligned}\right\}\qquad(8)$$

where $A_{i,j}U_{i,j}=(AU)_{i,j}=AU$ is designated.

Similarly we obtain

$$\left.\begin{aligned}&B_{i-1,j}U_{i-1,j}=BU-h(BU)_{\overset{\circ}{r}}+\frac{1}{2}h^2(BU)_{\bar r r},\\&B_{i+1,j}U_{i+1,j}=BU+h(BU)_{\overset{\circ}{r}}+\frac{1}{2}h^2(AU)_{\bar r r}.\end{aligned}\right\}\qquad(9)$$

Regarding to (8) and (5) in the left hand side of the equality (6) we have

$$\frac{3}{2}A_{i,j+1}U_{i,j+1}-2A_{i,j}U_{i,j}+\frac{1}{2}A_{i,j-1}U_{i,j-1}=\tau(AU)_{\overset{\circ}{t}}+\tau^2(AU)_{\bar t t},$$

from the right hand side we get

$$\frac{k\tau}{r_i mh^2}\left[B_{i+1,j}\left(U_{i+1,j}-U_{i,j}\right)-B_{i,j}\left(U_{i,j}-U_{i-1,j}\right)\right]=\frac{k\tau}{r_i m}(BU)_{\bar r r}.$$

Thus we pass to the following

$$(AU)_{\overset{\circ}{t}}+\tau(AU)_{\bar t t}-\frac{k\tau}{r_i m}(BU)_{\bar r r}=0.\qquad(10)$$

Since $r_c\le r_i\le R_k$, then from (10) it follows that the scheme (6) approximates an input differential problem with the velocity $O\left(\tau^2+h^2\right)$.

Now consider under which conditions the scheme (6) is stable.

Denoting

$$\alpha_i^j=A_{i,j}U_{i,j}U\beta_i^j=-B_{i+1,j}\left(U_{i+1,j}-U_{i,j}\right)+B_{i,j}\left(U_{i,j}-U_{i-1,j}\right)$$

we write the system (6) in the form of

$$\frac{3}{2}\mathrm{E}\alpha_i^{j+1}-2\mathrm{E}\alpha_i^j+\frac{1}{2}\mathrm{E}\alpha_i^{j-1}=-\frac{k\tau}{r_i mh^2}\beta_i^j,\qquad(11)$$

where $\mathrm{E}=\begin{Vmatrix}1&0\\0&1\end{Vmatrix}$ is a unit matrix.

Regarding to (7) in (11) we have

$$D\alpha_{\overset{\circ}{t}}+\tau^2\mathrm{E}\alpha_{\bar t t}+\frac{k\tau}{r_i mh^2}\mathrm{E}\beta^j=0,\qquad(12)$$

where $D=\begin{Vmatrix}\tau&0\\0&\tau\end{Vmatrix}$ is a diagonal matrix and denoted $\alpha=\alpha_i^j$, $\beta^j=\beta_i^j$.

Since $\min r_i = r_c$, then by satisfying the inequality $\tau < \frac{4mr_c h^2}{k}$, the scheme (12) and consequently the scheme (6) are stable by initial data.

For solving the system of algebraic equations (6) we apply Newtonian method for a system of non-linear equations. For this we write the system (6) in expanded form

$$\left.\begin{aligned}
&A_{i,1}U_{i,1} - A_{i,0}U_{i,0} - \frac{k\tau}{mr_i h^2}\left[B_{i+1,0}\left(U_{i+1,0} - U_{i,0}\right) - B_{i,0}\left(U_{i,0} - U_{i-1,0}\right)\right] = \\
&= f_0^i\left(U_1, U_2, \dots, U_N\right) = 0, \\
&\frac{3}{2}A_{i,2}U_{i,2} - 2A_{i,1}U_{i,1} + \frac{1}{2}A_{i,0}U_{i,0} - \frac{k\tau}{mr_i h^2}\left[B_{i+1,2}\left(U_{i+1,2} - U_{i,2}\right) - \right. \\
&\left. - B_{i,2}\left(U_{i,2} - U_{i-1,2}\right)\right] = f_1^i\left(U_1, U_2, \dots, U_N\right) = 0, \\
&\frac{3}{2}A_{i,3}U_{i,3} - 2A_{i,2}U_{i,2} + \frac{1}{2}A_{i,1}U_{i,1} - \frac{k\tau}{mr_i h^2}\left[B_{i+1,3}\left(U_{i+1,3} - U_{i,3}\right) - \right. \\
&\left. - B_{i,3}\left(U_{i,3} - U_{i-1,3}\right)\right] = f_2^i\left(U_1, U_2, \dots, U_N\right) = 0, \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\frac{3}{2}A_{i,N}U_{i,N} - 2A_{i,N-1}U_{i,N-1} + \frac{1}{2}A_{i,N-2}U_{i,N-2} - \frac{k\tau}{mr_i h^2}\left[B_{i+1,N}\left(U_{i+1,N} - U_{i,N}\right) - \right. \\
&\left. - B_{i,N}\left(U_{i,N} - U_{i-1,N}\right)\right] = f_{N-1}^i\left(U_1, U_2, \dots, U_N\right) = 0.
\end{aligned}\right\} \quad (13)$$

Denote $U = \left(U_1, U_2, \dots, U_N\right)^T \in H$

$$F(U) = \left(f_0^i(U), f_1^i(U), \dots, f_{N-1}^i(U)\right)^T$$

and we write the system (13) in the form of the operator equation

$$F(U) = 0,$$

where $F : H \to H$ is mapping from H in H.

Applying Newtonian method to the system (13) we obtain a system of the linear vector equations

$$F\left(U^R\right)\left(U^{R+1} - U^R\right) + F\left(U^R\right) = 0, \quad R = 0,1,\dots, \quad (14)$$

where

$$F(U) = \left\| \begin{matrix}
\frac{f_0^i(U_1+\varepsilon, U_2, \dots, U_N) - f_0^i(U_1, U_2, \dots, U_N)}{\varepsilon} & \dots & \frac{f_0^i(U_1, U_2, \dots, U_N+\varepsilon) - f_0^i(U_1, U_2, \dots, U_N)}{\varepsilon} \\
\frac{f_1^i(U_1+\varepsilon, U_2, \dots, U_N) - f_1^i(U_1, U_2, \dots, U_N)}{\varepsilon} & \dots & \frac{f_1^i(U_1, U_2, \dots, U_N+\varepsilon) - f_1^i(U_1, U_2, \dots, U_N)}{\varepsilon} \\
\dots\dots\dots & & \dots\dots\dots \\
\frac{f_{N-1}^i(U_1+\varepsilon, U_2, \dots, U_N) - f_{N-1}^i(U_1, U_2, \dots, U_N)}{\varepsilon} & \dots & \frac{f_{N-1}^i(U_1, U_2, \dots, U_N+\varepsilon) - f_{N-1}^i(U_1, U_2, \dots, U_N)}{\varepsilon}
\end{matrix} \right\|$$

ε is a sufficiently small positive number.

For solving the system of the linear vector equations (14) Gauss method was applied with choice of the main element by row.

Numerical calculations were introduced. As an example data of concrete gas condensate deposit [1] are used

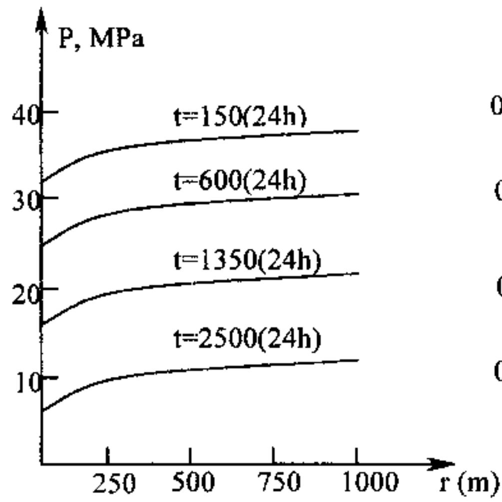

Fig. 3

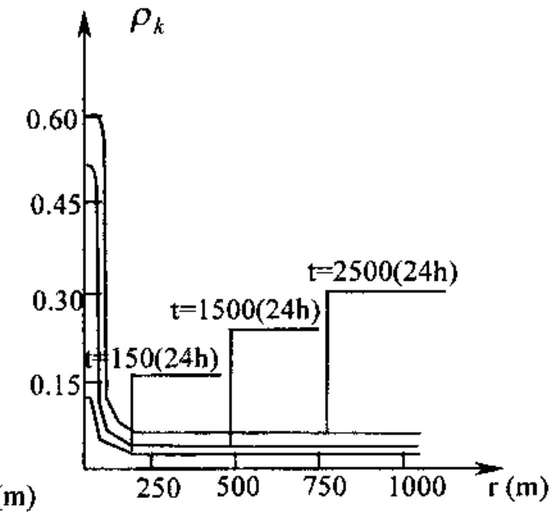

Fig. 4

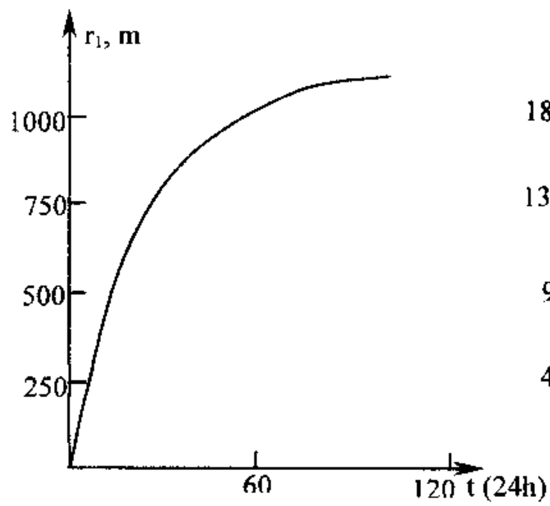

Fig. 5

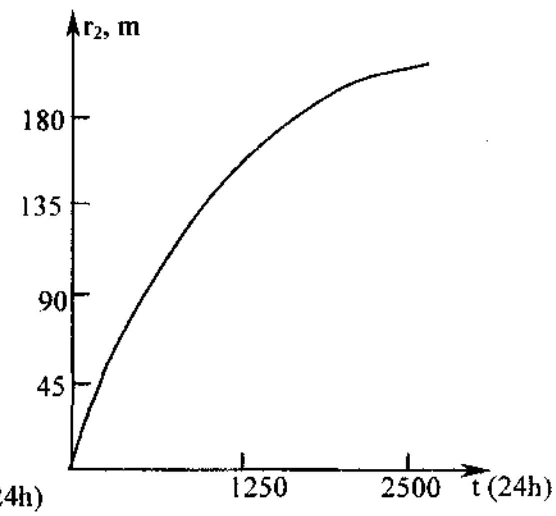

Fig. 6

$P_0 = 40$ MPa, $m = 0{,}2$; $h = 20\text{m}$; $k = 0{,}05\ \text{mkm}^2$;

$R_k = 1000$ m ; $r_c = 0{,}1$ m ; $P_{nk} = 39{,}23$ MPa ; $q_r = 5 \cdot 10^5\ \text{m}^3/24\text{h.}$;

$C_0 = 3 \cdot 10^{-4}\ \text{m}^3/\text{m}^3$; $\beta = 0{,}8$.

On the basis of calculation the graphs of distribution of pressure (fig.3) and condensate-saturation for different times, and graphs of travel of boundary between the zones (fig. 5 and 6) are constructed. The calculations showed that condensate-saturation along the formation is very non-uniformly distributed, sharply increases in bottle-hole zone reaching the end of exploitation at walls well the value 0,60. The boundary of condensate separation very quickly reaches the contour of formation for 120 days. The boundary of maximum condensate zone the end of exhaustion of layer for 2500 days reaches 180 m.

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