HASANOVA P.M.

ON ONE MODELLING PROBLEM OF DISTRIBUTION PROCESS UNDER OVERLOAD OF VERTICAL HOLLOW THICK-WALLED CYLINDER INCLUDED IN RIGID ENVELOPE

Abstract

The mathematical modelling of dividing destruction process on action of gravity and some overload of vertical hollow thick-welled cylinder included in rigid thin envelope by using of analogous model cylinder is realized.

Consider a variant of modelling of delayed fracture process of hollow thick-walled cylinder fastened by face of weld with absolute rigid thin envelope occupying the vectoral position in the case of gravity action subject to some overboad.

We denote the interior radius of cylinder by a, and exterior one by b. If we assume that the cylinder is sufficiently long and we can ignore the end effects, then the main stress'll the tangent stress σ_{rz} depending only on the radial coordinate r. The quantity σ_{rz} is found in [1]. By using this we write the solution [1]

$$\sigma_{rz} = -\frac{\gamma n}{2}\left(r - \frac{a^2}{r}\right),\ \sigma_{rz}^{\max} = \frac{\gamma n}{2}\left(b - \frac{a^2}{b}\right). \tag{1}$$

Here γ is a specific weight of material of the cylinder $n = n(t)$ is an overload value. As it was noted in [1] the characteristics of the material of the cylinder is not contained in the solution (1), therefore it holds for any material.

We use the solution (1) and try to model the destruction process in time of considered cylinder. As in work [1] under the modelling we understand the creation on model construction for bounded quantum of time such quantity of accumulative damages that full-scale construction in operational process may have for a long time. As we know the kinetic conception is hold, it is assumed that the destruction process is characterized by some scalar quantity of damage accumulation and durability. In [2] for quantity of damage accumulation the following kinetic equation is constructed

$$\Pi(t,x) = H(t-t')\left\{-\frac{t_1^{1+m}(\sigma_*,T)}{t_0^{1+m}(\sigma_*,T) - t_1^{1+m}(\sigma_*,T)} + \right.$$
$$\left. + (1+m)\int_0^t \frac{(t-\tau)^m d\tau}{t_0^{1+m}(\sigma_*(x,\tau),T(x,\tau)) - t_1^{1+m}(\sigma_*(x,\tau),T(x,\tau))}\right\} \tag{2}$$

t is time, $(x) = (x_1, x_2, x_3)$ are coordinate points of body, $\Pi(t,x)$ is a quantity characterizing the damage accumulation, $\Pi(t') = 0$ (besides $\Pi(t) = 0$ for $0 \le t \le t'$), $\Pi(t_*) = 1$, where t' is appearance time in body the first damage, t_* is time before failure (continuity violation) of body for arbitrary given $\sigma_* = \sigma_*(x,t)$, $T = T(x,t)$. Here σ_* is some so called equivalent stress [2] being the functions of invariant quantities deviator of the stress tensor $\sigma_+ = (3s_{ij}s_{ij}/2)^{1/2}$; $\sigma = \sigma_{ij}\delta_{ij}/3$, $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$, σ_{ij}, σ is average voltage, δ_{ij} are the known Cronecker's symbols, $T = T(x,t)$ is temperature of body counted from some initial temperature. The functions $t_1 = t_1(\sigma_*, T)$, $t_0 = t_0(\sigma_*, T)$ are experimentally defined functions of material correspondingly the time before the appearance of damages in body and the time before failure for constants $\sigma_* = const$, $T = cons$, m is some experimentally defined constant of material, $H(t)$ is Heavyside's unit function. It's assumed that in the formula (2) besides the material functions and constants the stress field σ_* and the temperature T are known from solutions of corresponding problems.

Taking the material functions $t_i = t_i(\sigma_*, T)$ $(i = 1,2)$ according [2] in the form

$$t_i = t_{is} \exp\left[\beta_i\left(1 - \frac{\sigma_*}{\sigma_s}\right) + d_i\left(1 - \frac{T}{T_s}\right)\right], \quad (i = 1,2),$$

where $\beta_0, d_0 . \beta_1, d_1$ are constants of material, σ_s, T_s are some stress and temperature of reduction, t_{1s}, t_{0s} are times before appearance of damages in body and before failure for $\sigma_* = \sigma_s = const$, $T = T_s = const$ respectively, in [3] using the approximations $t_1(\sigma, T)/t_0(\sigma, T) \approx A = const$ [2], the following modelling destruction condition is obtained

$$\begin{gathered}\int_0^{t_M} (t_M - \tau)^{m_M} \exp\left[(1 + m_M)\beta_{0M} \frac{\sigma_*^M(\tau, x_\alpha)}{\sigma_s^M}\right] d\tau = \\ = C_1 + C_2 \int_0^t (t - \tau)^m \exp\left[(1 + m)\beta_0 \frac{\sigma_*(\tau, x_\alpha)}{\sigma_s}\right] d\tau .\end{gathered} \tag{3}$$

In (3) the quantities related to nature are left without index, and k models are denoted by using the index M.

In the relations (3) it's assumed the destruction process occurs isothermally. Besides here (x_α) is a point where the power of accumulative damages is maximal, C_1 and C_2 are some constants which are expressed by other constants of material in the next form:

$$C_1 = \frac{t_{osM}^{1+m_M}\left(A_M^{1+m_M} - A^{1+m}\right)}{(1 + m_M)(1 - A^{1+m})} \exp \beta_M , \tag{4}$$

$$C_2 = \frac{t_{osM}^{1+m_M}(1 + m)\left(1 - A_M^{1+m_M}\right)}{t_{os}^{1+m}(1 + m_M)(1 - A^{1+m})} \exp(\beta_M - \beta), \tag{5}$$

$$t_{1M}\left(\sigma_*^M, T_M\right)/t_{oM}\left(\sigma_*^M, T_M\right) \approx A_M = const, \ \beta_M = (1 + m_M)\beta_{0M}; \ \beta = (1 + m)\beta_0 \tag{6}$$

now by using the second formula of (1) we define

$$\sigma_* = \sigma_+ = \left(3 s_{ij} s_{ij} / 2\right)^{1/2} = \frac{\sqrt{3}}{2}\gamma_n\left(b - \frac{a^2}{b}\right). \tag{7}$$

It's clear that for σ_*^M we'll have

$$\sigma_*^M = \frac{\sqrt{3}}{2}\gamma_M n_M\left(b_M - \frac{a_M^2}{b_M}\right). \tag{8}$$

As we see, we assume that not only mechanical but also geometrical characteristics of full-scale and model cylinder distinguish.

In the modelling condition (3) we take into account (6)-(8).

In addition we obtain

$$\int_0^{t_M} (t_M - \tau)^{m_M} \exp\left[\frac{\sqrt{3}}{2}\beta_M \frac{\gamma_M}{\sigma_s^M}\left(b_M - \frac{a_M^2}{b_M}\right) n_M(\tau)\right] d\tau =$$

$$=C_1+C_2\int_0^t (t-\tau)^m \exp\left[\frac{\sqrt{3}}{2}\beta\frac{\gamma}{\sigma_s}\left(b-\frac{a^2}{b}\right)n(\tau)\right]d\tau . \qquad (9)$$

The equation (9) allows at prescribed value of overload of the model cylinder $n_M(t)$, to determine the modelling time t_M and vice versa. In addition the specific weight and also mechanical and geometrical characteristics of model and full-scale cylinder will enter to the obtained relations. We suppose that the overload value $n(t)$ changes by jump from the value $n=1$ ($n=1$ corresponds to the cylinder being under the influence of only specific weight) to the value $n=n_0>>1$, that holds in practice, then from the equation (9) we have

$$t_M^{1+m_M} n_0^M \frac{\sqrt{3}}{2}\beta_{0M}\frac{\gamma_M}{\sigma_s^M}\left(b_M-\frac{a_M^2}{b_M}\right)\exp\left[\frac{\sqrt{2}}{3}(1+m_M)\beta_{0M}\frac{\gamma_M}{\sigma_s^M}\left(b_M-\frac{a_M^2}{b_M}\right)\right]=$$
$$=C_1+C_2 t^{1+m} n_0\frac{\sqrt{3}}{2}\beta_0\frac{\gamma}{\sigma_c}\left(b-\frac{a^2}{b}\right)\exp\left[\frac{\sqrt{2}}{3}(1+m)\beta_0\frac{\gamma}{\sigma_s}\left(b-\frac{a^2}{b}\right)\right]. \qquad (10)$$

Here it's taken into account that for $n_0>>1$ we can take

$$1+L(n_0-1)\approx Ln_0;\quad 1+L_M\left(n_0^M-1\right)\approx L_M n_0^m ,$$

where

$$L=\frac{\sqrt{3}}{2}\beta_0\frac{\gamma}{\sigma_s}\left(b-\frac{a^2}{b}\right),\quad L_M=\frac{\sqrt{3}}{2}\beta_{0M}\frac{\gamma_M}{\sigma_s^M}\left(b_M-\frac{a_M^2}{b_M}\right).$$

From the relation (10) we can define the modelling time

$$t_m=\left[\frac{c_1+c_2 t^{1+m} n_0 L\exp[L(1+m)]}{n_0^M L_M\exp[L_M(1+m_M)]}\right]^{\frac{1}{1+m_M}} . \qquad (11)$$

The relation (10) also allows in explicit form to determine the overload value of model cylinder from the given modelling time b_M and from mechanical and geometrical characteristics of the considered cylinders

$$n_0^M=\frac{c_1+c_2 t^{1+m} n_0 L\exp[L(1+m)]}{t_M^{1+m_M} L_M\exp[L_M(1+m_M)]} . \qquad (12)$$

Now consider some particular cases. Assume that model and full-scale are from the same material $m_M=m$, $\beta_{0M}=\beta_0$, $A_M=A$, $\sigma_s^M=\sigma_s$, $t_{osM}=t_{os}$ and so on. In this case $c_1=0$, $c_2=1$.

Subject to mentioned relations (11) and (12) are transformed to the simplest relations respectively

$$\left(\frac{t_M}{t}\right)^{1+m}=\frac{n_0}{n_0^M}\frac{\gamma}{\gamma_M}\frac{b}{b_M}\frac{1-\left(\frac{a}{b}\right)^2}{1-\left(\frac{a_M}{b_M}\right)^2}\exp\left[\frac{\sqrt{3}}{2}(1+m)\frac{\beta_0}{\sigma_s}\left(\gamma\frac{b^2-a^2}{b}-\gamma_m\frac{b_M^2-a_M^2}{b_M}\right)\right], \qquad (13)$$

$$\frac{n_0^M}{n_0}=\left(\frac{t}{t_M}\right)^{1+m}\frac{\gamma}{\gamma_M}\cdot\frac{b}{b_M}\cdot\frac{1-\left(\frac{a}{b}\right)^2}{1-\left(\frac{a_M}{b_M}\right)^2}\exp\left[\frac{\sqrt{3}}{2}(1+m)\frac{\beta}{\sigma_3}\left(\gamma\frac{b^2-a^2}{b}-\gamma_M\frac{b_M^2-a_M^2}{b_M}\right)\right]. \qquad (14)$$

Now assume that the geometrical dimensions of full-scale and model cylinders also coincide. In this case $a_M = a$, $b_M = b$. Then the formulas (13) and (14) get the following form respectively

$$\frac{t_M}{t} = \left(\frac{n_0}{n_0^M}\right)^{\frac{1}{1+m}}, \qquad (15)$$

$$\frac{n_0^M}{n} = \left(\frac{t}{t_M}\right)^{1+m}. \qquad (16)$$

Consequently, for the given overload value of the full-scale cylinder n_0 accumulative in it damages for time t will equal to value of accumulative damages of model cylinder for the given overload n_0^m (and also for given geometrical and mechanical characteristic) during the time t_M which is determined from (11) or in considered particular cases from the relations (13) or (15). Besides if the modelling time t_M is given, even the corresponding overload value n_0^M allowing the equality of accumulative damages of full-scale and model cylinders, will be found from (12), (14), (16) depending on considered cases. In conclusion we note the following.

The variation interval of the time t is $0 \le t \le t_*$, where as is noted above, t_* is time before failure of full-scale cylinder. In addition if we take the quantity t_M as the failure time of model cylinder, then failure time of full-scale cylinder will be found in correspondence with one of the formulas (11), (13), (15) substituting t by t_*.

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Hasanova P.M.

Institute of Mathematics & Mechanics of NAS of Azerbaijan.

9, F.Agayev str., 370141, Baku, Azerbaijan.

Tel.: 39-47-20(off.).

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