MAMEDOV I.S.

THE BIG DEFORMATIONS OF STRINGS UNDER THE ACTION OF VARIABLE INTENSITY NORMAL LOAD

Abstract

The analytic solution of non-linear boundary problem on great axially symmetric elastic deformations of flat strings under the action of variable intensity normal load is getten, determined the tensions, deformations, horizontal and vertical components of removal vectors.

Let's consider the problem of equilibrium of the big (unit order and more) asymptotic elastic deformation of flexible plane strings constant up to the deformation of cross-section whose ends are rigidly attached. The strings are under the motion of axially-symmetric applied (with respect to middle of attaching points of the ends of the thread) on all length of normal loads of variable intensity; specific weight of the strings is ignored.

It is supposed that the material of the string before the destruction can be strongly hardly deformed.

Such problems for the strings, membranes and momentless rotation cover at different statements, limitation and assumptions were considered at papers [1]-[10] and in [11]. At above statement the problems of string balance under the action of normal load of constant intensity and some inverse problems on form-change of strings at deformation process by preassigned rule were considered.

The main dependencies and equations. Let's take the thread to the rectangle coordinate system of XOY. Let the axis OX cross through the points of attached ends of string and the axis OY through the middle of the segment line connecting the last and perpendicularly to this line and the form of thread the ends of which are rigidly attached on the same level before and after deformation are given correspondingly by the equations

$$\zeta=\zeta(\xi); \quad Y=Y(X), \tag{1}$$

where ξ, X and ζ, Y are abscissa and ordinates, correspondingly of some points of strings before and after deformation.

The balance equations of string's elements will be

$$\frac{d}{dx}\left[T_*\left(1+Y'^2\right)^{-1/2}\right]-q_0(X)Y'=0; \quad \frac{d}{dx}\left[T_*Y'\left(1+Y'^2\right)^{-1/2}\right]-q_0(X)=0 \ , \tag{2}$$

where T_* is the tension of the string and $q_0(X)$ is intensity of normal load, acting on unit length of thread. For the material of thread between the real tension and logarithmic deformations the linear dependence is taken in the form [4], [5]

$$\varepsilon_1=\frac{1}{E}\sigma=\frac{1}{E}\frac{T_*}{F_*},$$

$$\varepsilon_2=-\frac{\nu}{E}\sigma=-\frac{\nu}{E}\frac{T_*}{F_*}=-\nu\varepsilon_1 \ , \tag{3}$$

$$\varepsilon_3=-\frac{\nu}{E}\sigma=-\frac{\nu}{E}\frac{T_*}{F_*}=-\nu\varepsilon_1 \ ,$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are main logarithmic deformations; ε_1 is longitudinal and $\varepsilon_2, \varepsilon_3$ are radial and ring deformation of the thread of cross-section, cross deformations if the thread of square or rectangular section σ is real normal strain, E is a modulus of elasticity of material thread, ν is a coefficient of cross deformations, F_x is a area of cross-section of the string after the deformation.

For the round, square and rectangular cross-section strings correspondingly we can write

$$\ln\frac{F_x}{F_0} = \left\{ \begin{array}{l} \ln\dfrac{\rho^2}{\rho_0^2} = 2\ln\dfrac{\rho}{\rho_0} = 2\varepsilon_2 \\ \ln\dfrac{a^2}{a_0^2} = 2\ln\dfrac{a}{a_0} = 2\varepsilon_2 \\ \ln\dfrac{\alpha\cdot\beta}{\alpha_0\cdot\beta_0} = \ln\dfrac{\alpha}{\alpha_0} + \ln\dfrac{\beta}{\beta_0} = \varepsilon_2 + \varepsilon_3 \end{array} \right\} = -2\nu\varepsilon_1 \,, \qquad (4)$$

where F_0 is a area of cross-section of the string before the deformation, ρ_0 and ρ are radius of round, a_0 and the length of sides of the square and rectangular cross-section of the strings correspondingly before and after the deformation.

The equation of tangent string before and after the deformation will be

$$\frac{d\zeta}{d\zeta} = tg\psi; \quad \frac{dY}{dX} = tg\varphi, \qquad (5)$$

where ψ, φ are angles between the tangent string and the positive direction of axis OX at some points before and after the deformation.

Let's introduce dimensionless variables and values

$$r = \frac{\xi}{l}; \; \eta = \frac{\zeta}{l}; \; x = \frac{X}{l}; \; y = \frac{Y}{l}; \; T = \frac{T_x}{EF_0}; \qquad (6)$$
$$q(x) = \frac{q_0(X)l}{EF_0}; \; F = \frac{F_x}{F_0} \,,$$

where l is a half of distance between the points of fixing of thread-end.

Using (6) from (1)-(5) we'll get

$$\eta = \eta(r); \; y = y(x), \qquad (7)$$

$$\frac{d}{dx}\left[T\left(1 + y'^2\right)^{-1/2}\right] + q(x)y' = 0 \,, \qquad (8)$$
$$\frac{d}{dx}\left[T\cdot y'\left(1 + y'^2\right)^{-1/2}\right] - q(x) = 0 \,,$$

$$\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1, \qquad (9)$$

$$F = \exp(-2\nu\varepsilon_1), \qquad (10)$$

$$T = \varepsilon_1\exp(-2\nu\varepsilon_1), \qquad (11)$$

$$\frac{d\eta}{dr} = tg\psi; \; \frac{dy}{dx} = tg\varphi. \qquad (12)$$

Taking into account the axis symmetry and right fixing of thread-ends the boundary conditions we can write as

$$x = 0; \; y = y_0; \varphi = 0; T = T_0 \;\; for \;\; r = 0 \;\; or \;\; \psi = 0 \qquad (13)$$
$$x = 1; \; y = 0; \varphi = \varphi_1; T = T_1 \;\; for \;\; r = 0 \;\; or \;\; \psi = \psi_1$$

where y_0 is a deflection of string in the center after the deformation , $y_0, y_1', \varphi_1, T_0$ and T_1 are unknown values. At given $\eta = \eta(r)$ the value ψ_1 is defined from the relation

$$\psi_1 = \left(arctg \frac{d\eta}{dr} \right)\Big|_{r=1} .$$

For the longitudinal deformation ε_1 we can write

$$\varepsilon_1 = \ln \frac{ds}{ds_0} = \ln \frac{dx \cos\psi}{dr \cos\varphi} = \ln \left[\left(\frac{dx}{dr} \right) \left(\frac{1 + y'^2}{1 + \eta'^2} \right)^{1/2} \right], \tag{14}$$

where ds_0, ds are the length of the elements of strings before and after the deformation.

From the system (8) we'll get

$$\frac{dT}{dx} = 0; \quad T = const . \tag{15}$$

Taking into account (15) system (8) we can write so

$$\begin{aligned} &\frac{d}{dx}\left[\left(1 + y'^2\right)^{-1/2} \right] + \frac{q(x)}{T} y' = 0 , \\ &\frac{d}{dx}\left[y'\left(1 + y'^2\right)^{-1/2} \right] - \frac{q(x)}{T} = 0 . \end{aligned} \tag{16}$$

From (15) it follows that at axially symmetric deformation of thread under the normal load action and constant and variable intensivity, independently, tension everywhere is constant [11].

Taking into account (15) and (11) relation (14) we can write so

$$\begin{aligned} &\varepsilon_1 = \ln \frac{\int_0^x ds}{\int_0^r ds_0} = \ln \frac{\int_0^1 ds}{\int_0^1 ds_0} = const \qquad or \\ &\varepsilon_1 = \ln \frac{\int_0^x \left(1 + y'^2\right)^{1/2} dx}{\int_0^r \left(1 + \eta'^2\right)^{1/2} dr} = \frac{\int_0^1 \left(1 + y'^2\right)^{1/2} dx}{\int_0^1 \left(1 + \eta'^2\right)^{1/2} dr} = const . \end{aligned} \tag{17}$$

From (17) we'll get

$$\int_0^x \left(1 + y'^2\right)^{1/2} dx = \left(\frac{\int_0^1 \left(1 + y'^2\right)^{1/2} dx}{\int_0^1 \left(1 + \eta'^2\right)^{1/2} dr} \right) \int_0^r \left(1 + \eta'^2\right)^{1/2} dr . \tag{18}$$

At the known $\eta = \eta(r)$ and $y = y(x)$ from the relation (18) it is defined the dependence $x = x(r)$. At solution of inverse problem (the inverse problem is the problem where initial and finite forms of strings are given $\eta = \eta(r)$ and $y = y(x)$ and the load are defined under the action which happens in advance given form change of thread and other unknown values-tension and deformation) the dependencies $\eta = \eta(r)$ and $y = y(x)$ are known and from (18) we can find the dependence $x = x(r)$ and at solving the straight problem the dependence $\eta = \eta(r)$ is given and the dependence $x = x(r)$ we can find only after the determination of dependence $y = y(x)$. Let's denote that $x = x(r)$ may be one-valued and many-valued function. The function $x=x(r)$ at 1) $\psi_1<\frac{\pi}{2}$, $\varphi_1<\frac{\pi}{2}$ and 2) $\psi_1>\frac{\pi}{2}$, $\varphi_1>\frac{\pi}{2}$ will be one-valued and at 3) $\psi_1<\frac{\pi}{2}$, $\varphi_1>\frac{\pi}{2}$ will be many-valued; in the first case to the one value r from the $0\le r\le 1$ corresponds one value x from the $0\le x\le 1$, in the second case to the one value x from the $0\le r\le r^*$ corresponds the one value x from the $0<x<x^*$, in the third case to the two-valued r from the $0\le r\le 1$ corresponds one value from the $0\le x\le x^*$ where r^*,x^* is maximal value r and x (moreover $r^*>1$, $x^*>1$).

From the first or the second equation of the system (16) we'll get

$$y''\left(1+y'^2\right)^{3/2}=\frac{q(x)}{T}. \tag{19}$$

From (19) at bounded conditions (13) we'll get

$$y'=\pm\left(\int_0^x\frac{q(x)}{T}dx\right)\left[1-\left(\int_0^x\frac{q(x)}{T}dx\right)^2\right]^{-1/2}, \tag{20}$$

$$y=\pm\int_1^x\left(\left(\int_0^x\frac{q(x)}{T}dx\right)\left[1-\left(\int_0^x\frac{q(x)}{T}dx\right)^2\right]^{-1/2}\right)dx. \tag{21}$$

Following [4] and [5] from (9)-(11) we'll get

$$\varepsilon_1=\varepsilon_1^*=\frac{1}{2\nu};\ \varepsilon_2=\varepsilon_2^*=-\frac{1}{2};\ \varepsilon_1=\varepsilon_1^*=\frac{1}{2\nu};\ \varepsilon_3=\varepsilon_3^*=-\frac{1}{2}; \tag{22}$$

$$F=F^*=\frac{1}{e};\quad T=T^*=\frac{1}{2\nu}, \tag{23}$$

where ε_1^*,T^* is maximal and $\varepsilon_2^*,\varepsilon_3^*,F^*$ is minimal value ε_1,T and $\varepsilon_2,\varepsilon_3,F$.

Using the solution of the inverse problem the analytical expression of normal load of variable intensity we can give so [11]

$$q(x)=\frac{T}{bCh^2\frac{x}{b}}, \tag{24}$$

where b,T (T is tension) are constant values.

Substituting (24) in (20) and (21) we'll get

$$y'=\pm Sh\frac{x}{b}, \tag{25}$$

$$y=\pm b\left(Ch\frac{x}{b}-Ch\frac{1}{b}\right). \tag{26}$$

Since for the points of thread before and after the deformation holds the $\eta\le 0$ when $0\le r\le 1$, $y\le 0$ and $Ch\frac{x}{b}<Ch\frac{1}{b}$ when $0\le x\le 1$, then when $b>0$ in the right parts of the formulas (25) and (26) in the further the sigh plus is used, i.e.

$$y'=Sh\frac{x}{b}, \tag{27}$$

$$y = b\left(Ch\frac{x}{b} - Ch\frac{1}{b} \right). \tag{28}$$

The length of string (the ends of string are rigidly attached on the points $(-1;0),(1;0)$) before and after the deformation are defined from the relation:

$$L_0 = 2\int_0^1 \left(1 + \eta'^2\right)^{1/2} dr \tag{29}$$

$$L_1 = 2\int_0^1 \left(1 + y'^2\right)^{1/2} dx \tag{30}$$

where L_0, L_1 are dimensionless length of string before and after the deformation.

Substituting (27) in (30) we'll get

$$L_1 = 2\int_0^1 \left(1 + y'^2\right)^{1/2} dx = 2bSh\frac{1}{b}. \tag{31}$$

From the (26)-(31) it follows that the thread of length L_0 the end of which are rigidly attached at the points $(-1;0),(1;0)$ defomed under the acting of normal load intensity (24) gets the form of catenary of length $L_1 = 2bSh\frac{1}{b}$. .

Let's denote that flexible homogeneous not deformable string of length $L_1 = 2bSh\frac{1}{b}$., the ends which are rigidly attached at the points $(-1;0),(1;0)$ under the action of force of gravity also will take the form of chain length (28) [12].

Using (27) from (17) and (18) we'll get:

$$\varepsilon_1 = \ln\frac{bSh\frac{x}{b}}{\int_0^r \left(1 + \eta'^2\right)^{1/2} dr} = \ln\frac{bSh\frac{1}{b}}{\int_0^1 \left(1 + \eta'^2\right)^{1/2} dr} = const, \tag{32}$$

$$x = x(r) = b\ln\left(\lambda \pm \sqrt{\lambda^2 + 1} \right), \tag{33}$$

where $\lambda = \dfrac{Sh\frac{1}{b}}{\int_0^1 \left(1 + \eta'^2\right)^{1/2} dr} \int_0^r \left(1 + \eta'^2\right)^{1/2} dr$.

Since $\exp\left(\frac{x}{b}\right) > 0$ and $\lambda \geq 0$, then in (33) in front of the root the sing plus is taken, i.e.

$$x = x(r) = b\ln\left(\lambda + \sqrt{\lambda^2 + 1} \right). \tag{34}$$

Let the straight string $(\eta = \eta(r) = 0;\ \eta' = 0)$ the ends of which are attached in the points $(-1;0),(1;0)$ is deformed under the load (24). Then from the (32)-(34) we'll get

$$\varepsilon_1 = \ln\frac{bSh\frac{x}{b}}{r} = \ln\left(bSh\frac{1}{b} \right) = const, \tag{35}$$

$$x = x(r) = b\ln\left[\left(rSh\frac{1}{b}\right) + \sqrt{\left(rSh\frac{1}{b}\right)^2 + 1}\right]. \qquad (36)$$

From (36) follows that $x = 0$ where $r = 0$ and $x = 1$ when $r = 1$, i.e. the solution (36) satisfies the bounded condition for x from (13).

From (21) and (35) we'll get

$$\ln\left(b^* Sh\frac{1}{b^*}\right) = \frac{1}{2v}, \qquad (37)$$

where b^* is the limit value of parameter b.

At $v = 0{,}5$ from (23) and (37) we'll get

$$b^* Sh\frac{1}{b^*} = e;\ \ b^* \approx 0{,}3725;\ T \approx 0{,}3679. \qquad (38)$$

Taking into account (23) and (38) from (24) we'll get

$$q^*(x) = \frac{T^*}{b^* Ch^* \dfrac{x}{b^*}} = \frac{1}{2veb^* Ch^2 \dfrac{x}{b^*}} \approx \frac{1}{1{,}0126 Ch^2 2{,}6846x}, \qquad (39)$$

$$q^*(0) \approx 0{,}98756;\quad q^*(1) \approx 0{,}01823,$$

where $q^*(x)$ is limited allowed intensivity applied to the thread of the load $q(x)$.

At $0 \le q(x) \le q^*(x)$ the balance of deformed thread developing from the initial stable-increasing of deformation ε_1, from zero until the maximally allowing ε^* corresponds to the increase of the tension T and intensivity of the load $q(x)$ from zero until the maximally allowing T^* and $q^*(x)$; the further supercritical increasing of tension and intensity of load but their decreasing [4]-[7], [11].

At fig.1 the graphic of the function $q^* = q^*(x)$ is described.

Taking into account (38) from (28) we'll get:

$$y^* = y^*(x) = b^*\left(Ch\frac{x}{b^*} - Ch\frac{1}{b^*}\right) \approx 0{,}3725 Ch 2{,}6846x - 2{,}7437, \qquad (40)$$

$$y^*(0) \approx -2{,}3612;\ \ y^*(1) = 0,$$

where $y^* = y^*(x)$ is the equation of deformed thread at the load $q^* = q^*(x)$.

Horizontal and vertical components of vectors transforming the thread points will be:

$$u = u(r) = x(r) - r = \left(b\ln\left[\left(r\,Sh\frac{1}{b}\right) + \sqrt{\left(r\,Sh\frac{1}{b}\right)^2 + 1}\right]\right) - r, \qquad (41)$$

$$u(0) = 0;\ \ u(1) = 0,$$

$$\text{v} = \text{v}(r) = y(x(r)) - \eta(r) = \left[b\left(Ch\frac{x(r)}{b} - Ch\frac{1}{b}\right)\right] - \eta(r),$$

$$\text{v}(0) = b\left(1 - Ch\frac{1}{b}\right) - \eta(0);\ \text{v}(1) = 0, \qquad (42)$$

where u, v are horizontal and vertical components of vector of transforming points of thread. Thus at the load (24) the problem is solved analytically.

If at the given $\eta=\eta(x)$ and $q=q(x)$ are integrals $\int_0^r \left(1+\eta'^2\right)^{1/2} dr$, $\int_0^x \frac{q(x)}{T}dx$, $\int_0^x \left(1+y'^2\right)^{1/2} dx$ and (21) aren't expressed at the elementary functions, then the problem is solved by the approximate methods.

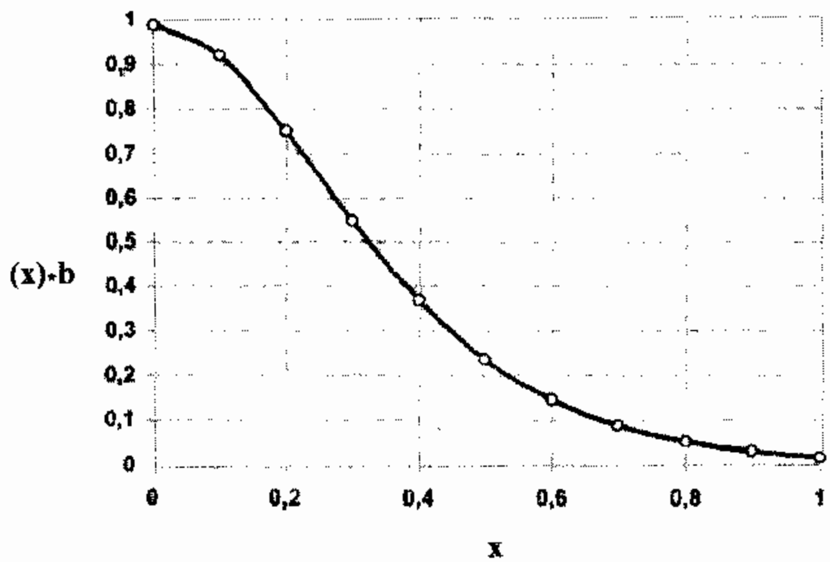

Fig. 1.

Let's denote that the load applied to the thread as in the case of membrane and momentless covers must be either constant intensity or intensity being smooth variable function of coordinate [13].

References

[1]. Birger I.A., Panovka Ya.G. *Hardness, stability, vibrations.* Pub.: "Engineering", v.1, 1968, 832p.
[2]. Alekseyev N.I. *Static's and steady motion of thread.* M., Legindustriya, 1970, 263p.
[3]. Alekseyev S.A. Indz. Sb. AN SSSR, v.10, 1951, p.71-80.
[4]. Pnanisin A.R. The equilibration stability of elastic systems. Gostekhizdat, 1955, 476p.
[5]. Panovko Ya.G., Gubanova I.I. The stability and vibrations of elastic systems, M., "Nauka", 1979, 284p.
[6]. Grigoryev A.S. Rounding of seventh All Union conference on theorem of envelope and flakes. Dnepropetrovsk, 1969, M., "Nauka", 1970, p.X 89.
[7]. Feodosyev V.I. PMM, 1968, v.32, issue 3, p.339-344.
[8]. Novojilov V.V. The static and dynamics of elastic systems. M., "Nauka", 1987, 340p.
[9]. Mushtari Kh.M. Nonlinear theory of shells. M., "Nauka", 1990, 224p.
[10]. Mamedov I.S. Indz. J. AN SSSR, v.5, issue 5, p.927-935.
[11]. Mamedov I.S. Trans. AS Az., ser. phys. tech. tech. and math. sci., v.XIX, №5, 1999, p.184-191.
[12]. Bukhgolc N.N. Main course of mechanical theory. p.1, M., "Nauka", 1972, 468p.
[13]. Novojilov V.V., Chernikh K.F., Mikhaylovsky. The linear theory of thin shells. Leningrad. "Polytechnics", 1991, 656p.

Mamedov I.S.
Institute of Mathematics & Mechanics of NAS of Azerbaijan.
9, F.Agayev str., 370141, Baku, Azerbaijan.
Tel.: 39-47-20(off.).

Received December 11, 2000; Revised March 30, 2001.
Translated by Mamedova V.A.