APPLIED PROBLEMS OF MATHEMATICS AND MECHANICS

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THE METHOD OF OPTIMAL DISTRIBUTION OF USERS' TIME ON INTRANET

Abstract

The method of determining of optimum redistribution of time allocated to users with aim to increase efficiency of Intranet functioning has been considered. The resolution of this problem has been reduced to finding solution to the problem of whole-number programming with Boolean variables under certain restrictions.

Creation of a big number of local networks over the recent time and connection of them to "worldwide web" environment have led to substantial development of Internet.

Intranet is a local network restricted to limits of one enterprise and using toolkits and technologies of Internet to provide an access both, to the resources of enterprise and external information resources from any point of planet [1]. Under Internet technology the network solutions with the use of TCP/IP protocol are meant. It can be said that such solutions use Internet as a means of traffic between various clusters of a local network of enterprise.

It is obvious that the number of users working in parallel within the given interval of time determine the workload on communication channel, connecting the Intranet to Internet. The considerable fraction of disconnections and failures during information transfer mode falls on "peak" periods of Intranet usage. When the total demand of users for information resources exceeds real capacity of Intranet, this usually results in substantial decrease of network efficiency. However, such situations can be corrected in terms of redistribution of users' time in a way that leads to keeping the number of users working in parallel within given interval of time possibly close to the number corresponding to appropriate even distribution of users.

The proposed method is based on determining the optimum redistribution of time available to users with aim to increase efficiency of Intranet functioning.

Let M is the number of users and interval [T1, T2] is time of Intranet operation per day. The access time available to i user will be denoted as τ_i, $i = \overline{1,M}$. Let's introduce the concept of contour in a way similar to that accepted on cellular phone networks. The contour is a unit of measurement of time available to users, which is equal in length to Δt, the minimum interval of time established by Intranet provider. It is obvious, that condition $0 < \Delta t \le \tau_i$, $i = \overline{1,M}$ should be satisfied. Then the quantity of contours k_i available to i user will be determined as:

$$k_i = \left[\frac{\tau_i}{\Delta t} \right], \quad i = \overline{1,M},$$

Where $[a]$ is a whole part of a number a.

Now, let's break down an interval $[T1,T2]$ by points $t_i = T_1 + i\,\Delta t$, $i = 0,1,...,N$, $t_0 = T_1$, $t_N = T_2$, into sub-intervals Δt of equal length:

$$N = \left[\frac{T_2 - T_1}{\Delta t}\right].$$

Let's note, that τ_i satisfies natural condition of:

$$\Delta t \le \tau_i \le T_2 - T_1, \quad i = \overline{1,M},$$

And a sum

$$K = \sum_{i=1}^{M} k_i, \quad i = \overline{1,M}$$

Of contours allocated to all M users falls within limits:

$$M \le K \le NM.$$

As far, in cases $\tau_i = \Delta t$, $i = \overline{1,M}$; $K = M$ and $\tau_i = T_2 - T_1$, $i = \overline{1,M}$; $K = NM$ the problem becomes trivial, we shall consider a case, when:

$$M < K < NM$$

In this particular case, the problem of optimization arises.

Let's introduce a matrix: $X = \left\| x_{ij} \right\|$, $\left(i = \overline{1,M};\ j = \overline{1,N}\right)$, the elements of which are determined as follows:

$$x_{ij} = \begin{cases} 1, & \text{if for i subscriber in interval } [t_{j-1}, t_j] \\ & \text{giving a time of access;} \\ 0, & \text{in opposite case.} \end{cases}$$

In parallel, any pair $(x_{i_0 l}, x_{i_0 s};\ \ x_{i_0 l} \wedge x_{i_0 s} = 1,\ l \ne s)$ should satisfy a condition of binding:

$$\left|l - s\right| < K_{i_0},\ i_0 = \overline{1,M};\ \ l,s = \overline{1,N}.$$

This condition means that contours allocated to each user are bound, i.e. the contours allocated to one user are placed in consecutive time intervals $\left[t_{i-1}, t_i\right]$, $i = \overline{1,N}$.

The number corresponding to even distribution of users across temporary intervals $\left[t_{i-1}, t_i\right]$, $i = \overline{1,N}$ from total time $[T1, T2]$ will be:

$$C = \frac{K}{N}.$$

Taking into account a statistics of previous time (days, months), it is possible to determine λ_i, intensity of inquiries flow from i user. As, the sum of intensities of inquiries flow from all users in one time interval $\left[t_{i-1}, t_i\right]$, $i = \overline{1,N}$ should not exceed μ, intensity of inquiries servicing in Intranet, it is necessary to introduce the following restriction:

$$\sum_{i=1}^{M} x_{ij}\lambda_i < \mu, \; j = \overline{1,N}.$$

Thus, the problem of optimum distribution of users' time is reduced to identification of such $x_{i\,j}$, $i = \overline{1,M}$, $j = \overline{1,N}$, which would minimize the functional:

$$\sum_{j=1}^{N}\left(\sum_{i=1}^{M} x_{ij} - C\right)^2 \rightarrow \min$$

And satisfy conditions:

$$\sum_{j=1}^{n} x_{i\,j} = K_i, \quad i = \overline{1,M};$$

$$\sum_{i=1}^{M} x_{ij}\lambda_i < \mu, \quad j = \overline{1,N};$$

$$\left|l - s\right| < K_{i_0},\; i_0 = \overline{1,M},\; l,s = \overline{1,N}, \text{ for } \forall \left(x_{i_0 l}, x_{i_0 s};\quad x_{i_0 l} \wedge x_{i_0 s} = 1,\; l \neq s\right).$$

It is obvious, that the model given above relates to the class of problems of whole-number programming with Boolean variables, which can be resolved through "branches and borders" method [3].

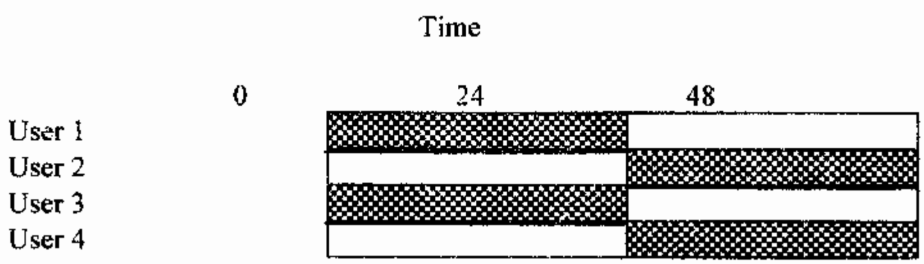
It is necessary to mention, that reduction of interval Δt down to reasonable limits, reduces number of intersections between contours of various users, thus increasing number of options for selection of optimum solution. But, on the other hand, conditions of binding for each user's contours reduce their quantity. Therefore, the issues of Δt selection with account of binding or partial binding of each user's contours requires further research in that direction.

We consider it necessary to illustrate the application of the method given above with one simple example:

$$M = 4, \quad \Delta t = 0.5\ h., \quad [T_1, T_2] = [0,24], \quad \tau_1 = \tau_2 = \tau_3 = \tau_4 = 12\ h.,$$

$$K_1 = K_2 = K_3 = K_4 = 24, \quad N = 48, \quad C = 2.$$

Ignoring the condition of restriction of all users' inquiries intensity sum and with account of minimal value of the functional to be equal to zero, the solution of the problem given above will look graphically as follows:



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