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INVESTIGATION OF SINGULAR CONTROLS IN ONE DISCRETE SYSTEM WITH VARIABLE STRUCTURE AND DELAY

Abstract

In paper [1] one discrete problem with variable structure in investigated by the author. In the present paper for a discrete control problem with variable structure and delay the necessary conditions of the first order optimality are obtained and a special case is investigated. To this end the modified scheme of work [2] is used.

1. Consider a problem on minimum of the functional

$$S(u,v)=\varphi_1(x(t_1))+\varphi_2(y(T)) \tag{1}$$

under the following constraints

$$x(t+1)=f(t,x(t),x(t-h),u(t)),\ t\in Q_1=\{t_0,t_0+1,\dots,t_1-1\}, \tag{2}$$

$$x(t_0-h)=x_{t_0-h},\dots,x(t_0)=x_0, \tag{3}$$

$$y(t+1)=g(t,y(t),v(t)),\ t=Q_2=\{t_1,t_1+1,\dots,T-1\}, \tag{4}$$

$$y(t_1)=G(x(t_1)), \tag{5}$$

$$u(t)\in U\subset R^r,\quad t\in Q_1, \tag{6}$$

$$v(t)\in V\subset R^q,\quad t\in Q_2. \tag{7}$$

Here $\varphi_1(x),\varphi_2(y)$ are the given twice continuous differentiable scalar functions, $f(t,x,a,u)(g(t,y,v))$ is a given $n(m)$-dimensional vector-function continuous with respect to union of variables together with partial derivatives with respect to (x,a), (y) up the second order inclusively, $G(x)$ is a given twice continuous differentiable m-dimensional vector function, $u(t)\,(v(t))$ is $r(q)$-dimensional vector of control actions, $U(V)$ is a given non empty and bounded set, $t_0,t_1,T,\dots,x_{t_0-h},\dots,x_0$ are given, $(T=t_0+N)$, h,N be given natural numbers.

We'll call the pair $u(t)(v(t))$ satisfying the constrains (6), (7) admissible controls and the four $(u(t),v(t),x(t),y(t))$ an admissible process. Here $(x(t),y(t))$ is a solution of the system (2)-(7) corresponding to the admissible control $(u(t),v(t))$.

The admissible control $(u(t),v(t))$ being a solution of the problem on minimum of the functional (1) under the constraints (2)-(7) we'll call optimal control.

In the paper the necessary conditions of optimality in the considered problem are introduced.

2. Let $(u(t),v(t))$ be fixed and $(\overline{u}(t)=u(t)+\Delta u(t),\ \overline{v}(t)=u(t)+\Delta v(t))$ be arbitrary admissible controls.

Assume

$H(t,x,a,u,\psi)=\psi'\cdot f(t,x,a,u)$,

$M(t,y,v,\rho)=\rho'\cdot g(t,y,v),\ N(x(t_1))=\rho'(t_1-1)G(x(t_1))$,

$\Delta_{\overline{v}}H[t]\equiv H(t,x(t),a(t),\overline{v}(t),\psi(t))-H(t,x(t),a(t),u(t),\psi(t))$,

$f_x[t]\equiv f_x(t,x(t),x(t-h),u(t))$,

$H_x[t]\equiv\psi'(t)\cdot f_x[t]$,

$H_{xx}[t] \equiv H_{xx}(t, x(t), x(t-h), u(t), \psi(t))$.

Here $\psi(t)$ and $p(t)$ are n and m dimensional vector-functions being the solutions of the following problems

$$\begin{aligned} &\psi(t_1-1) = -\frac{\partial \varphi_1(x(t_1))}{\partial x} + N_x(x(t_1)), \\ &\psi(t-1) = H_x[t] + H_a[t+h] \\ &t = t_0, t_0+1, \ldots, t_1-h-1, \\ &\psi(t-1) = H_x[t],\ t = t_1-h, \ldots, t_1-1, \end{aligned} \tag{8}$$

$$\begin{aligned} &p(t-1) = M_y[t]\ ,\ \ t = t_1, \ldots, T-1, \\ &p(T-1) = -\frac{\partial \varphi_2(y(T))}{\partial y}. \end{aligned} \tag{9}$$

We'll call the problem (8)-(9) an adjoint system for the considered optimal control problem.

Using the relation (8)-(9) we can write the formula of increment of quality functional (1) corresponding to the admissible controls $(\bar{u}(t), \bar{v}(t))$ and $(u(t), v(t))$ in the following form

$$\begin{aligned} &\Delta S(u,v) = S(\bar{u},\bar{v}) - S(u,v) = -\sum_{t=t_0}^{t_1-1} \Delta_{\bar{u}} H[t] - \sum_{t=t_1}^{T-1} \Delta_{\bar{v}} M[t] + \frac{1}{2}\cdot \Delta x'(t_1) \frac{\partial^2 \varphi_1(x(t_1))}{\partial x^2} \Delta x(t_1) + \\ &+ \frac{1}{2}\cdot \Delta y'(T) \frac{\partial^2 \varphi_2(x(T))}{\partial y^2} \Delta y(T) - \sum_{t=t_0}^{t_1-1} [\Delta_{\bar{u}} H_x[t] \Delta x(t) + \Delta_{\bar{u}} H'_a(t) \Delta x(t-h)] - \\ &- \frac{1}{2} \sum_{t=t_0}^{t_1-1} [\Delta x'(t) H_{xx}(t) \Delta x(t) + \Delta x'(t) H_{xa}(t) \Delta x(t-h) + \Delta x'(t-h) H_{ax}[t] \Delta x(t) + \\ &+ \Delta x'(t-h) H_{aa}[t] \Delta x(t-h)] - \frac{1}{2}\cdot \Delta x'(t_1) N_{xx}(x(t_1)) \cdot \Delta x(t_1) - \sum_{t=t_1}^{T-1} \Delta_{\bar{v}} M'_y[t] \Delta y(t) - \\ &- \frac{1}{2} \sum_{t=t_1}^{T-1} \Delta y'(t) M_{yy}[t] \Delta y(t) + \eta_1(u,v;\Delta u, \Delta v), \end{aligned} \tag{10}$$

where the definition $\eta_1(u,v;\Delta u,\Delta v)$ is the residual of the increment formula whose explicit form is not here as being of no use.

On the other hand it's clear that $(\Delta x(t), \Delta y(t))$ is a solution of the following problem (linearized problem)

$$\begin{aligned} &\Delta x(t+1) = f_x[t] \Delta x(t) + f_a[t] \Delta x(t-h) + \Delta_{\bar{u}} f[t] + \eta_2(u;\Delta u), \\ &\Delta x(t_0-h) = 0, \ldots, \Delta x(t_0) = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &\Delta y(t+1) = g_y[t] \Delta y(t) + \Delta_{\bar{v}} g[t] + \eta_3(v;\Delta v), \\ &\Delta y(t_1) = G_x(x(t_1)) \Delta x(t_1) + o_1(\|x(t_1)\|), \end{aligned} \tag{12}$$

where by definition

$$\eta_2(u,\Delta u) = o_2(\|\Delta\alpha(t)\|) + \Delta_{\bar{u}} f_x[t] \Delta x(t) + \Delta_{\bar{u}} f_a[t] \Delta y(t),$$

$$\eta_3(v,\Delta v) = o_3(\|\Delta y(t)\|) + \Delta_{\bar{v}} g_y[t] \Delta y(t).$$

Here $\alpha = (x,a)'$ is $2n$-dimensional vector.

We write the representation of solutions of these linearized systems following papers [2, 4].

As a result we obtain

$$\Delta x(t)=\sum_{\tau=t_0}^{t-1}F(t,\tau)\Delta_{\bar u}f[\tau]+n_4(u;\Delta u), \tag{13}$$

$$\Delta y(t)=\Phi(t,t_1-1)\Delta y(t_1)+\sum_{\tau=t_1}^{t-1}\Phi(t,\tau)\cdot\Delta_{\bar v}g[\tau]+\eta_5(v;\Delta v). \tag{14}$$

Here $F(t,\tau)$ and $\Phi(t,\tau)$ are respectively $(n\times n)$ and $(m\times m)$ matrix functions being the solution of the following systems

$$\begin{aligned}&F(t,\tau-1)=F(t,\tau)f_x[\tau]+F(t,\tau+h)f_a[\tau+h],\\&F(t,t-1)=E_1,\ \ F(t,\tau)=0\ \ \tau>t-1,\\&\Phi(t,\tau-1)=\Phi(t,\tau)g_y[\tau],\\&\Phi(t,t-1)=E_2,\end{aligned} \tag{15}$$

where $E_i - i=1,2$ are unit matrices of corresponding dimensions, and $\eta_4(u,\Delta u),\eta_5(v,\Delta v)$ are remainder terms.

From (12) subject to (13) we have

$$\Delta y(t_1)=\sum_{\tau=t_0}^{t_1-1}G_x(x(t_1))F(t_1,\tau)\Delta_{\bar u}f[\tau]+\eta_6(u;\Delta u).$$

Taking into account this formula in (14) we obtain

$$\Delta y(t)=\sum_{\tau=t_0}^{t_1-1}\Phi(t,t_1-1)G_x(x(t_1))F(t_1,\tau)\Delta_{\bar u}f[\tau]+$$

$$+\sum_{\tau=t_1}^{t-1}\Phi(t,\tau)\Delta_{\bar v}g[\tau]+\eta_7(u;\Delta u)+\eta_8(v,\Delta v).$$

Here $\eta_i,\ i=6,7,8$ - remainder terms.

If we assume that in the formula of increment (10) $\Delta u(t)\neq 0$, and $\Delta v(t)=0$, then we get

$$\Delta S(u,v)=S(\bar u,\bar v)-S(u,v)=-\sum_{t=t_0}^{t_1-1}\Delta_{\bar u}H[t]+\frac{1}{2}\cdot\Delta x'(t_1)\left(\frac{\partial^2\varphi_1(x(t_1))}{\partial x^2}-N_{xx}(x(t_1))\right)\Delta x(t_1)+$$

$$+\frac{1}{2}\cdot\Delta y'(T)\frac{\partial^2\varphi_2(y(T))}{\partial y^2}\Delta y(T)-\sum_{t=t_0}^{t_1-1}\left(\Delta_{\bar u}H'_x[t]\Delta x(t)-\Delta_{\bar u}H'_a(t)\Delta x(t-h)\right)-$$

$$-\frac{1}{2}\sum_{t=t_0}^{t_1-1}\left[\Delta x'(t)H_{xx}(t)\Delta x(t)+\Delta x'(t)H_{xa}[t]\Delta x(t-h)+\Delta x'(t-h)H_{ax}[t]\Delta x(t)+\right.$$

$$\left.+\Delta x'(t-h)H_{aa}[t]\Delta x(t-h)\right]-\frac{1}{2}\sum_{t=t_1}^{T-1}\Delta y'(t)M_{yy}[t]\Delta y(t)+\eta_1(u,v;\Delta u,0). \tag{16}$$

If we assume that in the formula of increment (10) $\Delta u(t)=0,\ \Delta v(t)\neq 0$ then as a result we obtain that

$$\Delta S(u,v)=S(\bar u,\bar v)-S(u,v)=\sum_{t=t_1}^{T-1}\Delta_{\bar v}M[t]+\frac{1}{2}\cdot\Delta x'(t_1)\frac{\partial^2\varphi_2(y(T))}{\partial x^2}\Delta y(T)-$$

$$-\sum_{t=t_1}^{T-1}\Delta_{\bar v}M'_y[t]\Delta y(t)-\frac{1}{2}\sum_{t=t_1}^{T-1}\Delta y'(t)M_{yy}[t]\Delta y(t)+\eta_1(u,v;0,\Delta v). \tag{17}$$

It follows to note that in the case when $\Delta u(t)\neq 0,\ \Delta v(t)\equiv 0$ the formulas (13), (15) take the form

$$\Delta x(t)=\sum_{\tau=t_0}^{t-1}F(t,\tau)\Delta_{\bar u}f(\tau)+\eta_4(u;\Delta u), \quad (18)$$

$$\Delta y(t)=\sum_{\tau=t_0}^{t_1-1}Q(t,\tau)\Delta_{\bar u}f[\tau]+\eta_7(u;\Delta u), \quad (19)$$

where by definition

$$Q(t,\tau)=\Phi(t,t_1-1)G_x(x(t_1))F(t_1,\tau).$$

If $\Delta u(t)\equiv 0$, $\Delta v(t)\neq 0$ then we get that

$$\Delta x(t)\equiv 0, \quad (20)$$

$$\Delta y(t)=\sum_{\tau=t_1}^{t-1}\Phi(t,\tau)\cdot\Delta_{\bar v}g[\tau]+\eta_8(v;\Delta v). \quad (21)$$

Let's introduce in consideration the matrix functions $K(\tau,s)$ and $B(\tau,s)$ by means of the formulas

$$K(\tau,s)=\sum_{t=\max(\tau,s)+1}^{t_1-1}\big[F'(t,\tau)H_{xx}[t]F(t,s)+F'(t,\tau)H_{xa}[t]F(t-h,\tau)+F(t-h,\tau)\times$$

$$\times H_{ax}[t]F(t,s)+F'(t-h,\tau)H_{aa}[t]F(t-h,s)\big]-F'(t_1,\tau)\left(\frac{\partial^2\varphi_1(x(t_1))}{\partial x^2}-N_{xx}(x(t_1))\right)\times$$

$$\times F(t_1,s)-Q'(T,\tau)\frac{\partial^2\varphi_2(y(T))}{\partial y^2}Q(T,s)+\sum_{t=t_1}^{T-1}Q'(t,\tau)M_{yy}[t]Q(t,s), \quad (22)$$

$$B(\tau,s)=\sum_{t=\max(\tau,s)+1}^{T-1}\Phi'(t,\tau)M_{yy}[t]\Phi(t,s)-\Phi'(T,\tau)\frac{\partial^2\varphi_2(y(T))}{\partial y^2}\Phi(T,s). \quad (23)$$

Subject to the designations (22), (23) we can represent the formulas of increments (16), (17) in the form of

$$\Delta S_{\bar u}(u,v)=-\sum_{t=t_0}^{t_1-1}\Delta_{\bar u}H[t]-\frac{1}{2}\sum_{\tau=t_0}^{t_1-1}\sum_{s=t_0}^{t_1-1}\Delta_{\bar u}f'[\tau]K(\tau,s)\Delta_{\bar u}f[s]-$$

$$-\sum_{t=t_0}^{t_1-1}\left[\sum_{\tau=t_0}^{t-1}(\Delta_{\bar u}H'_x[t]F(t,\tau)+\Delta_{\bar u}H'_a[t]F(t-h,\tau))\Delta_{\bar u}f[\tau]\right]+\eta_9(u,v;\Delta u), \quad (24)$$

$$\Delta S_{\bar v}(u,v)=-\sum_{t=t_1}^{T-1}\Delta_{\bar v}M[t]-\frac{1}{2}\sum_{\tau=t_1}^{T-1}\sum_{s=t_1}^{T-1}\Delta_{\bar v}g[\tau]M(\tau,s)\Delta_{\bar v}g[s]-$$

$$-\sum_{t=t_1}^{T-1}\left[\sum_{\tau=t_1}^{t-1}\Delta_{\bar v}M_y[t]\Phi(t,\tau)\Delta_{\bar v}g[\tau]\right]+\eta_{10}(u,v;\Delta v). \quad (25)$$

Here η_9, η_{10} are remainder terms.

3. Let's pass to derive the necessary conditions of optimality. It's valid

Theorem 1. *Let along the process* $(u(t),v(t),x(t),y(t))$ *the sets*

$$f(t,x(t),x(t-h),U)=\{l_1;l_1=f(t,x(t),x(t-h),v),v\in U\}, \quad (26)$$

$$g(t,y(t),V)=\{l_2;l_2=g(t,y(t),w),w\in V\} \quad (27)$$

be convex. Then for the optimality of the admissible control $(u(t),v(t))$ *it's necessary that the relations*

$$\sum_{t=t_0}^{t_1-1}\Delta_{\vartheta(t)}H[t]\le 0 \qquad (28)$$

for all $\vartheta(t)\in U,\ t\in Q$

$$\sum_{t=t_1}^{T-1}\Delta_{w(t)}M[t]\le 0\,, \qquad (29)$$

for all $w(t)\in V\,,\ t\in Q_2$ *be satisfied.*

We'll call the inequality (28), (29) analogue of discrete maximum principle [5] for the considered problem.

The formulas of increment (24), (25) allow also to investigate the case of degeneration of discrete maximum principle.

Definition. *We'll call the admissible control* $(u(t),v(t))$ *singular in the sense of Pontryagin's maximum principle in the considered problem, if the identity*

$$\sum_{t=t_0}^{t_1-1}\Delta_{\vartheta(t)}H[t]\equiv 0$$

for all $\vartheta(t)\in U,\ t\in Q_1$

$$\sum_{t=t_1}^{T-1}\Delta_{w(t)}M[t]\equiv 0$$

for all $w(t)\in V\,,\ t\in Q_2$ *are satisfied.*

In singular case it's valid

Theorem 2. *If the sets (26), (27) are convex, then for the optimality of the singular, in the sense of Pontryagin's maximum principle, control* $(u(t),v(t))$ *it's necessary that the relations*

$$\sum_{\tau=t_0}^{t_1-1}\sum_{s=t_0}^{t_1-1}\Delta_{\vartheta(\tau)}f'[\tau]K(\tau,s)\Delta_{\vartheta(s)}f[s]-\sum_{t=t_0}^{t_1-1}\left[\sum_{\tau=t_0}^{t-1}\left(\Delta_{\vartheta(t)}H_x'[t]F(t,\tau)+\Delta_{\vartheta(t)}H_a'[t]\times\right.\right.$$

$$\left.\left.\times F(t-h,T)\right)\Delta_{\vartheta(\tau)}f[\tau]\right]\le 0 \qquad (30)$$

for all $\vartheta(t)\in U,\ t\in Q_1$

$$\sum_{\tau=t_1}^{T-1}\sum_{s=t_1}^{T-1}\Delta_{w(\tau)}g'[\tau]M(\tau,s)\Delta_{w(s)}g[s]+\sum_{t=t_1}^{T-1}\left[\sum_{\tau=t_1}^{t-1}\Delta_{w(t)}M_y'[t]\Phi(t,\tau)\Delta_{w(\tau)}g[\tau]\right]\le 0 \qquad (31)$$

for all $w(t)\in V\,,\ t\in Q_2$ *be satisfied.*

4. In this point it's assumed that the sets U and V are open, and $f(t,x,a,u)$ $(g(t,y,v))$ is continuous with respect to union of variables together with partial derivatives with respect to $(x,a,u)\,((y,v))$ up the second order inclusively.

On made assumptions following [2, 3] we can show that the first and second variations (in classical sense) of the functional $S(u,v)$ have the following or respectively

$$\delta^1 S(u,v;\delta u,\delta v)=-\sum_{t=t_0}^{t_1-1}H_u'[t]\delta u(t)-\sum_{t=t_1}^{T-1}M_v'[t]\delta v(t)\,, \qquad (32)$$

$$\delta^2 S(u,v;\delta u,\delta v)=\delta x'(t_1)\frac{\partial^2\varphi_1(x(t_1))}{\partial x^2}\delta x(t_1)+\delta y'(T)\frac{\partial^2\varphi_{21}(y(T))}{\partial y^2}\delta y(T)-$$

$$-2\sum_{t=t_0}^{t_1-1}(\delta u'(t)H_{ux}[t]\delta x(t)+\delta u'(t)H_{ua}[t]\delta x(t-h))-\sum_{t=t_0}^{t_1-1}[\delta x'(t)\cdot H_{xx}[t]\delta x(t)+\delta x'(t)\times$$
$$\times H_{xa}[t]\delta x(t-h)+\delta x'(t-h)H_{ax}[t]\delta x(t)+\delta u'(t)H_{uu}[t]\delta u(t)+\delta x'(t-h)H_{aa}[t]\delta x(t-h)]-$$
$$-\delta x'(t_1)N_{xx}(x(t_1))\delta x(t_1)-2\sum_{t=t_1}^{T-1}\delta v'(t)M_{vy}[t]\delta y(t)-$$
$$-\sum_{t=t_1}^{T-1}[\delta y'(t)\cdot M_{yy}[t]\delta y(t)+\delta v'(t)\cdot M_{vv}[t]\delta v(t)]. \qquad (33)$$

Here $\{\delta u(t)\in R^r,\ t\in Q_1,\ \delta v(t)\in R^q,\ t\in Q_2\}$ are arbitrary vector-functions (variations of controls), and $(\delta x(t),\delta y(t))$ is a variation of the trajectory $(x(t),y(t))$ being a solution of a system of equations in the variations

$$\begin{aligned}&\delta x(t+1)=f_x[t]\delta x(x)+f_a[t]\delta x(t-h)+f_u[t]\delta u(t),\\&\delta x(t_0-h)=0,\dots,\delta x(t_0)=0,\end{aligned} \qquad (34)$$

$$\begin{aligned}&\delta y(t+1)=g_y[t]\delta y(t)+g_v[t]\delta v(t),\\&\delta y(t_1)=G_x(x(t_1))\delta x(t_1).\end{aligned} \qquad (35)$$

From the relation (32) it follows that for the optimality of the admissible equation $(u(t),v(t))$ it's necessary that the relations

$$H_u[t]=0,\ \ \forall t\in Q_1\ ,$$
$$M_v[t]=0,\ \ \forall t\in Q_2$$

be satisfied.

These relations are called Euler analogue for the considered problem and represented themselves as a necessary condition of the first order optimality.

We give a necessary condition of the second order optimality.

As it's obvious the systems (34), (35) are linear non-homogeneous difference equations.

Therefore their solutions allow the following representations

$$\delta x(t)=\sum_{\tau=t_0}^{t-1}F(t,\tau)f_u[\tau]\delta u(\tau), \qquad (36)$$

$$\delta y(t)=\Phi(t,t_1-1)\delta y(t_1)+\sum_{\tau=t_1}^{t-1}\Phi(t,\tau)g_v[\tau]\delta v(\tau), \qquad (37)$$

where $F(t,\tau)$ and $\Phi(t,\tau)$ are the solutions of the problem (15), (16) respectively. With the help of the formula (36), (37) the second variation of the functional (1) is represented in the form of

$$\delta^2 S(u,v;\delta u,\delta v)=-\sum_{\tau=t_0}^{t_1-1}\sum_{s=t_0}^{t_1-1}\delta u(\tau)f_u'[\tau]K(\tau,s)f_u[s]\delta u(s)-$$
$$-2\sum_{t=t_0}^{t_1-1}\left[\sum_{\tau=t_0}^{t-1}\delta u'(t)\cdot[H_{ux}[t]F(t,\tau)+H_{ua}[t]F(t-h,\tau)]f_u[\tau]\delta u(\tau)\right]-\sum_{t=t_0}^{t_1-1}\delta u'(t)H_{uu}(t)\delta u(t)-$$
$$-\sum_{\tau=t_1}^{T-1}\sum_{s=t_1}^{T-1}\delta v'(\tau)g_v'[\tau]B(\tau,s)g_v[s]\delta v(s)-2\sum_{t=t_1}^{T-1}\left[\sum_{\tau=t_1}^{t-1}\delta v'(t)M_{vy}[t]\Phi(t,\tau)g_v[\tau]\delta v(\tau)\right]-$$
$$-\sum_{t=t_1}^{T-1}\delta v'(t)M_{vv}[t]\delta v(t). \qquad (38)$$

With the help of formula (38) it's proved

Theorem 3. *For the optimality of the admissible control* $(u(t), v(t))$ *in the considered problem it's necessary that the relations*

$$\sum_{\tau=t_0}^{t_1-1}\sum_{s=t_0}^{t_1-1}\delta u'(\tau)f_u'[\tau]K(\tau,s)f_u[s]\delta u(s)+$$

$$+2\sum_{t=t_0}^{t_1-1}\left[\sum_{\tau=t_0}^{t-1}\delta u'(t)(H_{ux}[t]F(t,\tau)+H_{ua}[t]F(t-h,\tau))f_u[\tau]\delta u(\tau)\right]+\sum_{t=t_0}^{t_1-1}\delta u'(t)H_{uu}[t]\delta u(t)\le 0 \quad (39)$$

for all $\delta u(t)\in R^r$, $t\in Q_1$

$$\sum_{\tau=t_1}^{T-1}\sum_{s=t_1}^{T-1}\delta v'(\tau)g_v[\tau]B(\tau,s)g_v[s]\delta v(s)+$$

$$+2\sum_{t=t_1}^{T-1}\left[\sum_{\tau=t_1}^{t-1}\delta v'(t)M_{uy}[t](t,\tau)g_v[\tau]\delta u(\tau)\right]+\sum_{t=t_1}^{T-1}\delta v'(t)M_{vv}(t)\delta v(t)\le 0\,. \quad (40)$$

for all $\delta v(t)\in R^q$, $t\in Q_2$ *be satisfied.*

The corollary of theorem 3 is

Theorem 4. *Along the singular optimal classical extremal the relations*

$$\vartheta'[f_u'[\theta]K(\theta,\theta)f_u[\theta]+H_{uu}]\upsilon\le 0 \quad (41)$$

for all $\vartheta\in R^r$, $\theta\in Q_1$

$$w'[g_v[\theta]B(\theta,\theta)g_v[\theta]+M_{vv}[\theta]]w\le 0 \quad (42)$$

for all $w\in R^g$, $\theta\in Q_2$ *are satisfied.*

We'll call the optimality conditions (41), (42) analogous of Gabasov-Kirillova's optimality condition from [6].

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